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HYDRAULICS

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PREFACE

This book is written to supply a nucleus for a basic course in the *hydraulics of engineering.* It is not intended as an exhaustive treatise on any phase of the subject, nor as including a short course on any one of the professional fields, such as water supply, water power engineering, or the economics of general hydraulic engineering; neither is it a manual for hydraulic measurements in the field or in the testing laboratory, nor yet a storehouse of information on highly specialized problems involving hydraulics phenomena of various sorts.

Nevertheless the book has been written in the atmosphere of these practical engineering activities, and may serve both for an introductory course to the more specialized studies and as a text and reference book on everyday hydraulics problems.

The treatment has four stages: first, description of the physical phenomena, particularly from the engineer's viewpoint; second, development when necessary of the fundamental laws into useful formulas, but with the emphasis kept on basic facts and on the rectification of approximate theory by coefficients from experimental data; third, presentation of numerous examples showing good form in the analysis and good methods in the solutions of problems, with many comments on salient features; fourth, inclusion of many typical problems mostly drawn from practical engineering cases.

Since the contents and arrangement of the book doubtless reflect more or less the teaching experience and auxiliary methods of the authors, it seems appropriate to state their belief that students will be given needed orientation and stimulus by preliminary demonstration lectures or parallel laboratory work. Moreover, a schedule containing a computing period of two hours or longer, say once a week, in addition to and following two or more recitations on the text has been found of great value for driving home the fundamental principles and for developing

initiative and judgment in solving minor engineering problems similar to the longer ones given at the ends of chapters.

There is somewhat more material in this book than could usually be covered thoroughly in a four-hour course for one semester. If the time available for a first course in hydraulics is only about one-half of that, a short course might consist of Chapters I, II, VII, VIII, IX, XI, XII, and XV, with a few additions according to circumstances.

In the chapters on orifices, weirs, and pipes, results of new experimental data have been presented. Short chapters are included on logarithmic plotting and on laws of variation with associated percentage effects of small changes, because the subject matter is important, not only especially in hydraulics, but also generally in technical activities, and is not commonly given adequately to students in earlier courses.

The text introduces several new types of diagrams to facilitate computations and to give a more comprehensive view.

The authors wish particularly to acknowledge their indebtedness to Professor I. P. Church's texts on *Mechanics of Engineering* and *Hydraulic Motors.* Anyone who critically examines both earlier and more recent books on hydraulics will appreciate on the one hand the masterly improvements made by Professor Church and on the other hand the extensive credit due to him.

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NOMENCLATURE

The following symbols are used in this text. Various subscripts and accents are added to the letters as the occasion demands. Where the same symbol is used for two or more things there can seldom be confusion; the rest of the formula or the text will make it clear which meaning is proper. Additional uses, other symbols, and further definitions are detailed throughout the text.

A = Area of cross-section (sometimes surface area).
A_V = Area of projection on a vertical plane.
A_H = Area of projection on a horizontal plane.
B or b = Breadth or width.
c = Compressive stress per unit area.
C (capital letter) = Coefficient, usually referred to by distinctive phrase, such as "discharge coefficient, Chezy coefficient," etc.
D = Wheel diameter, also used for gage difference.
d = Pipe, jet, or other diameter, also used for depth.
$\left.\begin{matrix} a \\ b \\ c \\ d \end{matrix}\right\}$ = Angles involved in turbine and pump theory; also used for various distances.
e = Efficiency; also eccentricity.
F = Total force.
f = Friction factor in pipe formula; also common coefficient of sliding friction.
g = Acceleration constant of gravity, usually taken as 32.2 ft. per sec., per sec. $2g$ = 64.4 ft. per $\overline{\text{sec.}}^2$, $\sqrt{2g} = 8.02$.
H = Total head or height; also used for total horizontal reaction.
h = Height or head, *always measured vertically*.
h_F = "Friction head" or loss of head due to friction.
k = Cancelling or corrective vertical gage distance.
L, l = Length.
n = Kutter's coefficient of roughness, also an exponent of velocity.
N_s = Specific speed.
P = Total pressure (usually in some one direction).
P_H = Horizontal component of total pressure.

P_V = Vertical component of total pressure.
p = Pressure per unit of area (over and above atmospheric pressure unless otherwise stated or clearly implied).
Q = "Discharge," volume per unit of time (rate of flow).
R = "Hydraulic radius," or "hydraulic mean depth."
r = Geometric radius.
S = Total surface; *e.g.*, rubbing surface.
s = Slope (hydraulic slope); also used for peripheral velocity (rim speed) of a wheel.
T = Temperature (particularly in gas problems); also used for total tension, or total thrust.
t = Time (duration); also used for thickness.
u = Tangential component of absolute velocity of water; also used for coefficient of obsolute viscosity (c.g.s. units).
V = Volume, particularly in gas and flotation problems; also used for total vertical reaction.
v = Velocity, usually in feet per second.
W = Total weight.
w = Weight of a unit volume of water.
w', w_1, etc. = Weights of unit volumes of other liquids, gases, or solids.
x = Variable distance or dimension, usually horizontal.
y = Variable distance or dimension, usually vertical.
z = Variable distance, also "potential head."
θ = Angle.
ϕ = "Speed ratio" for turbines and pumps; also used for "backwater function."

HYDRAULICS

CHAPTER I

PRESSURE-HEADS, PRESSURES, AND BALANCING COLUMNS

1. The word **hydraulics,** derived from two Greek words meaning water pipe, and formerly referring only to that branch of engineering science which treats of liquids in motion, is now usually interpreted as including *hydrostatics* as well.

HYDROSTATICS

2. Hydrostatics, as here treated, deals with the equilibrium of *fluids* at rest or practically so, and with the pressures exerted, particularly as applicable to the usual problems of engineering practice. The fluids commonly dealt with are *liquids,* but gases are also included in the subject. Textbooks on physics give definitions of liquids and gases, and discussions of their characteristic properties. Certain properties are here restated briefly.

3. Definition of a Liquid.—A liquid is a substance that, like solids, has a definite volume, but, unlike solids, not a definite shape. Its particles move relatively to each other so readily that, when unconfined at the sides or at any part of the bottom, the action of gravity causes the liquid to *flow* and to seek the lowest possible level (hence the word "fluid"). Therefore **a liquid conforms to the shape of the containing vessel** or reservoir, and, **when at rest,** presents a **level upper surface** unless restrained by the walls of a completely filled container.

4. Pascal's Law.—*Pressure* if exerted anywhere upon a mass of liquid: (1) is *transmitted undiminished in all directions;* (2) acts with the *same force* on all *equal surfaces;* and (3) acts in a direction **at right angles** to those surfaces. [It is to be understood that the liquid is confined, and that proper allowance is made for the

influence of gravity in cases where different levels within the mass of liquid are involved when applying parts (1) and (2) of Pascal's law. (See law of variation of liquid pressure stated below.)]

5. Law of Variation of Hydrostatic Liquid Pressure within a Mass of the Liquid Due to the Action of Gravity.—As far as the action of gravity is concerned, the increase of pressure with increase of depth in the liquid obeys the same law as for a vertical prism of a solid.

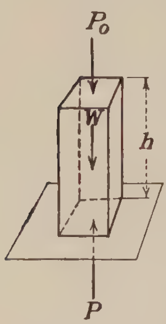


Fig. 1.

Figure 1 represents a solid column, say of concrete or brickwork. Let W be its total weight; w' the weight per unit volume or the "heaviness" of the material; A the area of the horizontal cross-section; P the total pressure on the bottom; p the pressure per unit area or the "intensity of pressure" (assuming support at all points of the bottom of the column). Assume a total load P_o on top of the column, this load being uniformly distributed and of an "intensity" p_o per unit area. (Note the use of **capital letters** for **total values** and of **small letters** for **unit values.**)

For equilibrium the upward forces must balance the downward forces, or $P = P_o + W = p_oA + Ahw'$. Also $p = \frac{P}{A}$. Therefore $p = p_o + hw'$. If there is no load on top of the column, $p = hw'$. In words this may be stated: "The **increase in pressure** per unit area equals the **height times the heaviness** (or unit weight)," and this is the same as the weight of a prism of the material of unit cross-section and height h.

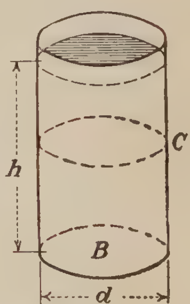


Fig. 2.

The same reasoning applies to a liquid column, which, of course, must be supported at the sides as well as at the bottom. The side support may be given either by the walls of a container or by the pressures from surrounding liquid. In Fig. 2, showing an open standpipe filled with water, the reaction pressures of the vertical sides against the liquid are everywhere horizontal, and their resultant in any horizontal direction is zero. The entire

weight of the vertical column of water is supported by the bottom. Moreover, in this case, none of the weight is transmitted down to the bottom by the vertical sides. The sides confine the liquid and resist its horizontal pressure but do not transmit any weight except that of the metal.

Then for the intensity of liquid pressure (above atmospheric) at depth h, we write, just as for the solid prism of Fig. 1,

$$p = hw. \tag{1}$$

By common usage in hydraulics the word "pressure" used without qualifications usually **means** pressure per unit area or "intensity" of pressure, *e.g.*, pounds per square inch, or kilograms per square centimeter, **above local atmospheric pressure.**

6. Example: What is the pressure of the water at level C in Fig. 2, 25 ft. below the surface?

Solution: From Eq. (1), $p = hw$, we have $p = 25 \times 62.4 = 1560$ lb. per sq. ft., 62.4 lb. being the weight of a cubic foot of fresh water at ordinary low temperatures. Or, in *customary units,* $\frac{1560}{144} = 10.85$ *lb. per sq. in.* Note that $\frac{62.4}{144} = 0.433$. Hence the **Rule: "Each foot depth of water causes a pressure of 0.433 lb. per sq. in."** The problem then could be solved, $p = 25 \times 0.433 = 10.85$ lb. per sq. in.

Another Example: What is the pressure corresponding to a head of 30 in. of mercury (whose sp. gr. = 13.6)?

Solution: $p = \frac{30}{12} \times \frac{13.6 \times 62.4}{144} = 14.7$ lb. per sq. in.

Obvious deductions include:

1. Pressure varies *uniformly* with the depth of liquid.[1]
2. Within a mass of liquid the pressure increases with descent and decreases with ascent.

7. Pressure-head, or Head, of Liquid.—Reversing Eq. (1),

$$h = \frac{p}{w}. \tag{2}$$

[1] Liquids are practically incompressible. The density of water at a depth of one mile ($p = 2285$ lb. per sq. in., or 156 atmospheres) has increased only ¾ of 1 per cent. This is in contrast to gases. In the earth's atmosphere, at one mile above sea-level, the density of the air has decreased 18 per cent.

Example: What depth (or overlying height) of water is necessary to cause a pressure of 60 lb. per sq. in.?

Solution: $h = \frac{(60 \times 144)}{62.4} = 60 \times 2.31 = 138.5$ ft.

Note that to get h *in feet* we must have p *in pounds* per *square foot* and w in *pounds per cubic foot.* Also note that $\frac{144}{62.4} = 2.31$, the reciprocal of 0.433. Hence the **Rule: "Each pound per square inch corresponds to a depth** or height **of 2.31 ft. of water."**

The term *pressure-head* (or simply "head") is used to designate the vertical height of a static water column (or a column of any liquid or gas concerned) corresponding to the state of pressure or degree of compression of the fluid at the point in question.

8. Illustrations of Pressure-head.—In Fig. 3, if an open water column is caused to exist by connecting a riser pipe at B, the water

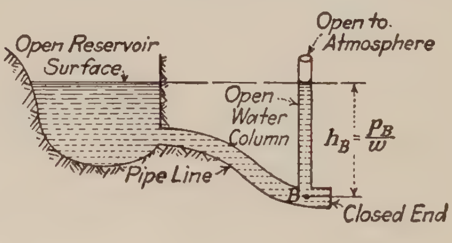


FIG. 3.

will rise in it to the reservoir level when flow ceases and equilibrium is established. Now it frequently occurs in engineering problems that information concerning the intensity of pressure, say in pounds per square inch or kilograms per square centimeter, is given, but it is desired to know the corresponding height or "head." This height $\left(= \frac{p}{w}\right)$ that would result from (or else that is necessary to cause) a certain liquid pressure p, is therefore called, as stated above, the *pressure-head,* or simply the *head,* of the liquid under pressure. Hence a term of the form and equivalent to $\frac{p}{w}$ stands for a vertical height of the liquid in question—a linear dimension.

In Fig. 4, the piston acts as a load on the column of water confined in the cylinder, and causes a fluid pressure in the cylinder greater than that due merely to the head h_1, so that, if there were an open liquid column connected at B, the height h_2 would exceed h_1. In fact, $h_2 = h_1 + \frac{\frac{P_o}{A}}{w} = h_1 + \frac{p_o}{w}$, where A is the area of the piston. In this case the pressure-head is due mostly to the mechanical pressure of the piston. Of course, if the open column is real rather than imaginary, the pressure-head on the piston can be regarded as due to the head $h_2 - h_1$.

Problem for the Student: If h_1 is 8 ft., P_o = 1000 lb., diameter = 12 in., what is the pressure-head at level B?

Ans. 28.4 ft. of water.

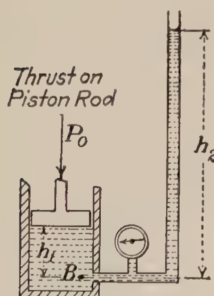


FIG. 4.

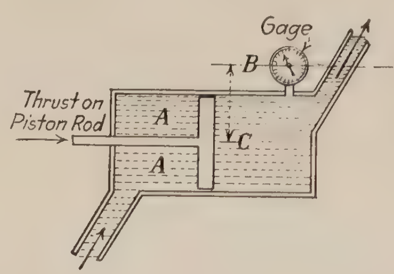


FIG. 5.

Considering the pressure of the water in the cylinder of a water pump (Fig. 5—diagrammatic only, the pump valves and other mechanical details being omitted from the sketch), if the pressure gage at B reads 50 lb. per sq. in. (when the pump is running), this means that the water pressure at the right of the piston is enough to balance a water column. $\frac{50 \times 144}{62.4} = 50 \times 2.31 = 115.5$ ft. high, plus height BC (see also example on p. 4). It does not necessarily mean that there is a free water surface of a tank or reservoir 115.5 ft. above the gage. In fact, the pressure in the cylinder may be necessary *in part* to overcome the friction of *flowing* water in a pipe line, or to force open the pump valves, or to force water through a nozzle, etc. But it is known that, as far as the state of pressure in the right end of the

cylinder at the gage is concerned, it is the same as if 115.5-ft. head of water were causing it instead of the force of the piston.

9. Atmospheric Pressure—When and How to Be Considered and Mentioned.—In most problems of hydraulic engineering the atmosphere has access to both sides, or both ends, or to the top and bottom of the structure or the mass of liquid (see Figs. 1 to 5). Consequently, in common everyday engineering usage **the local atmospheric pressure** is taken as a **datum** and is called **zero pressure.** Almost all gages are graduated to read zero when exposed to *local* atmospheric pressure.

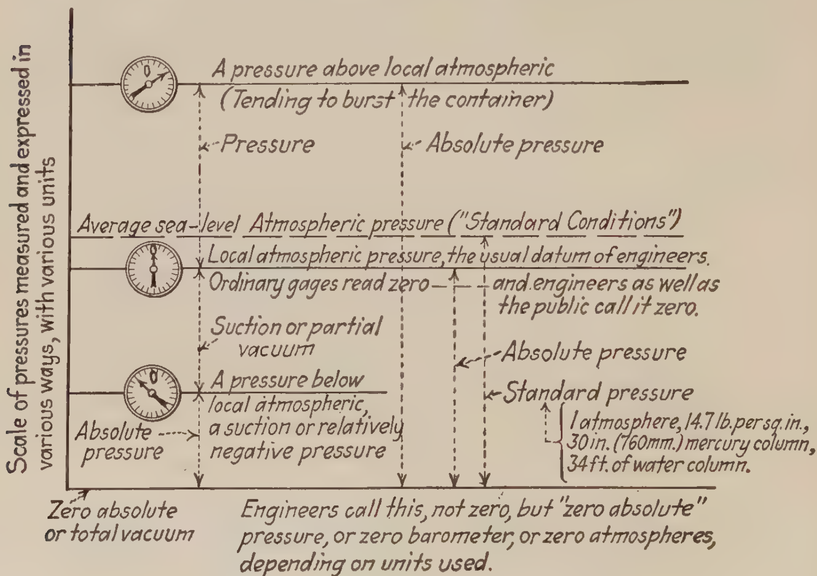


Fig. 6.

When the pressure of a liquid or a gas is less than local atmospheric[1] (*i.e.*, somewhere between this and a vacuum, being what for gases is termed "a partial vacuum"), the numerical value of such a pressure must be distinguished by some one of several significant words or phrases. The object is to indicate

[1] The standard atmospheric pressure (average pressure at sea level) balances a barometer column of 760 mm. of mercury at freezing temperature, or $\frac{760}{25.40} = 29.92$ in. For mercury at 60°F. the value is 30 in., which is commonly used.

clearly whether to measure *down* from the atmospheric pressure or *up* from absolute zero pressure. The student should carefully note the following expressions (see Fig. 6).

10. If intensities of pressure (either liquid or gaseous) are represented by a vertical scale as at the left of Fig. 6, the several methods of measurement may be illustrated graphically. The student should particularly note that "*zero pressure*" in ordinary engineering as well as *popular usage is* **not a vacuum,** but is simply the local prevailing atmospheric pressure (usually differing appreciably from the "standard atmospheric pressure").

In Fig. 6 the local atmospheric pressure is indicated as being *less* than standard, but it is often *above* standard at localities within a few hundred feet of sea level, depending on the weather. Note carefully, in Fig. 6 and in the example below that the *unusual* (but sometimes desirable) *method* of referring a pressure to vacuum (or absolute) datum implies the use of a word signifying this fact, *e.g.*, "above vacuum," "absolute," "barometer," "atmospheres." If such a word is not used after the numerical value and units, the datum is understood to be local atmospheric.

For example, if a confined liquid or a gas has a "suction" pressure of just "10 in. of mercury column," when the barometer reads 29.0 in., the intensity may be properly designated in any one of the following ways (starting in each case with the given mercury measure):

(*a*) $29 - 10 = 19$ *in. absolute mercury column.*

(*b*) $\frac{19}{12} \times 13.6 = 21.53$ *feet height of water absolute.*

(*c*) $\frac{19}{12} \times 13.6 \times \frac{62.4}{144} = 1.583 \times 13.6 \times 0.433 = 9.33$ lb. per sq. in. *absolute pressure.*

(*d*) $\frac{29 - 10}{30} = \frac{19}{30}$ of an *atmosphere.* This unit always implies that vacuum is the datum or zero pressure.

(*e*) $\frac{10}{12} \times 13.6 = 0.833 \times 13.6 = 11.33$ *ft. water suction.*

(*f*) $\frac{10}{12} \times 13.6 \times \frac{62.4}{144} = 0.833 \times 13.6 \times 0.433 = 4.91$ lb. per sq. in. *suction.*

(*g*) 10 in. *mercury vacuum* is a common way of designating the given suction pressure. Note that *vacuum* is here used interchangeably with *suction*.

Notice that in each of these expressions a significant word is used to qualify the numerical value and units. It is necessary to designate in this manner all pressures less than atmospheric, also all pressures of any magnitude whatsoever when referred to vacuum as datum, in order to avoid misunderstandings and errors in everyday engineering business, in conversation, correspondence, and calculations.

If, for example, in case (*c*) above, the word *absolute* were omitted it would be understood to mean 9.33 lb. per sq. in. ordinary gage pressure, *i.e.*, above atmospheric, which is (at sea level) 9.33 + 14.7 = 23.03 lb. per sq. in. absolute, thus causing about 150 per cent error.

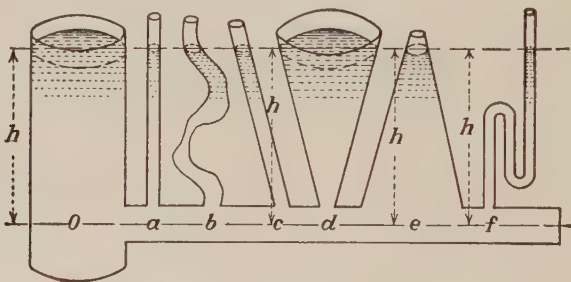


Fig. 7.

N. B: A pressure may be measured up or down from atmospheric (either local or standard), or up from vacuum, and it may be expressed in weight per unit area (the common unit being pounds per square inch), in feet of water column, in inches of mercury column, etc., or in atmospheres. Hence the need of care in expressing unusual conditions.

11. It is an axiom that "**water seeks its level.**" A corollary is that "**at all points at the same level in the same mass of liquid, at rest, pressures per unit area are equal.**"

Question for the Student: Why is this true? Figure 7, adapted from a common physics textbook illustration, is significant in this connection.

In Eq. (1), $p = hw$, a little consideration of Fig. 7 and of everyday experience with the physical properties of liquids will make it clear that "h" refers only to *vertical extent* and not to the *length*, inclined or sinuous, of water column, because horizontal elements do not affect the pressure. Neither does the size nor shape of the communicating passageway make any difference, if only the head be the same, as far as pressure per unit area is concerned. Thus in Fig. 8, $h =$ simply $h_1 + h_2 + h_3 + \ldots h_6$.

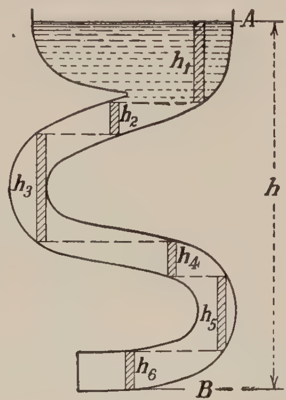


FIG. 8.

If there be enough water removed from the apparatus illustrated by Fig. 7 to lower any of the levels 1 ft., it will be found that each and all of the other levels have also been lowered 1 ft., although the volumes and the weights of water removed from the several columns differ considerably. The reason for this will be discussed further partly under the text references to Figs. 31 and 32 (Hydrostatic Paradox), and partly on page 26 under Components in Any Direction of liquid pressure.

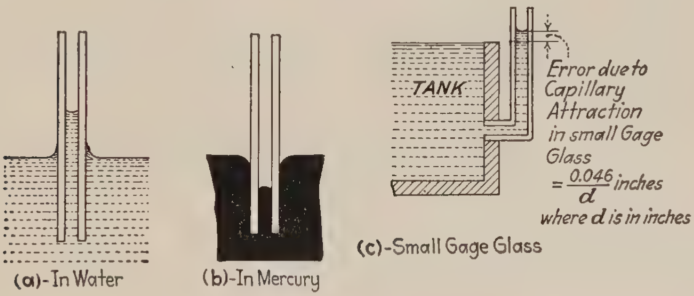


FIG. 9.—Capillary attraction.

12. Capillary Attraction. Cohesion, Adhesion, and Surface Tension.—The exception to the otherwise general statement that the upper surface of a free body of liquid at rest is level consists in the condition at the edges of the surface area, close to a bounding solid. If the liquid wets the solid (*e.g.*, water and

clean glass), it is because there is a greater attraction between the liquid and the solid than between particles of the liquid, or adhesion is stronger than cohesion, and conditions are as shown by (*a*) Fig. 9. On the other hand, if the liquid does not wet the solid (*e.g.*, mercury and glass), cohesion is stronger than adhesion and conditions are as shown by (*b*) Fig. 9. The curved upper surface is called a **meniscus.**

If small-bore glass tubes are used in gages, the effects of capillarity will cause water to stand higher and mercury lower than with large glass tubes, and even with the large tubes there is a curved surface where the liquid touches the glass. For water, the extra height is about $\frac{0.046}{d}$ in., where d is the internal diameter of the tube in inches. Thus, if a differential water-air gage (Fig. 17) has two glass columns nominally both ⅛-in. internal diameter, but actually one tube 0.01 in. larger and the other 0.01 in. smaller, the water will stand about 1⁄16 in. higher in the smaller tube than in the larger, when a common hydrostatic pressure would lead one to expect them to stand at exactly the same level. In experimental work this possible difficulty may usually be avoided by using larger glass tubes, say of ⅜-in. internal diameter. *All readings should be taken at the level of the middle of the meniscus*, *i.e.*, the bottom of the curve for water and the top for mercury. These positions are away from the maximum effects of capillary attraction and are nearest to the proper level.

BALANCING LIQUID COLUMNS

13. Simple U-tube Gages.—In Fig. 10, the mercury below level 0—0 is in independent equilibrium, and pressures at 1 and 2 are equal, these points being at the same level in the same mass of liquid at rest. Hence

$$p_1 = p_2, \text{ or } Hw = H_m w_m, \text{ or } \frac{H}{H_m} = \frac{w_m}{w}. \tag{3}$$

If such a U-tube is used as a gage (Fig. 11) and it is necessary to calculate (from the difference shown by the columns of the U-tube) the *pressure-head* at A, *i.e.*, the height above A to which the liquid in the pipe would rise in an open column, then, for conven-

ience in analysis, such a column may be imagined, as shown in Figs. 12 and 13.

For Figs. 11 and 12, as for Fig. 10, $Hw = H_m w_m$, or $(h - K)w = H_m w_m$. Hence

$$h = \left(\frac{w_m}{w}\right) H_m + K \tag{4}$$

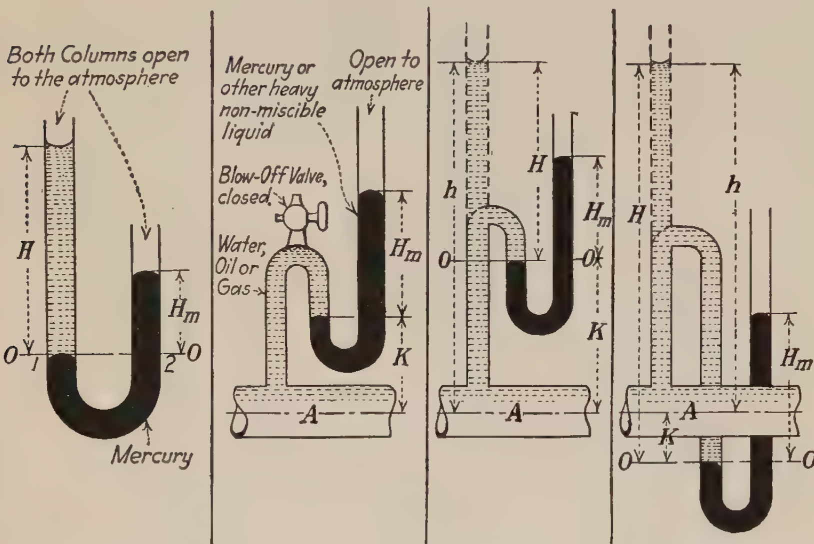


Fig. 10. Fig. 11. Fig. 12. Fig. 13.

Fig. 10.—U-tube with mercury column balancing a water column, (or water column balancing an oil column, etc.).

Fig. 11.—Simple U-tube gage as used to measure pressure-head. If the pressure in the pipe or vessel at A is less than atmospheric, the left column is the higher.

Figs. 12 and 13.—These two sketches illustrate the relation of pressure-head at point A to the indications of the U-tube. [The dotted open water (or other liquid) columns showing directly the pressure-head h do not exist ordinarily, but they may always be imagined for convenience.]

If the U-tube gage is so located that the low reading (top of low column) is below the point A where the head is to be found (Fig. 13), then $(h + K)w = H_m w_m$, or

$$h = \left(\frac{w_m}{w}\right) H_m - K. \tag{5}$$

The ratio $\frac{w_m}{w}$ is the *specific gravity* of mercury or whatever liquid is used in the gage. If the liquid in the pipe at A is not water but some other fluid, then the ratio will be the same as the ratio

of specific gravities. For mercury the sp. gr. = 13.6, both liquids near 32°F. (0°C.). For water and mercury both at 50°F. (10°C.), the ratio = 13.57. Hence for a simple U-tube

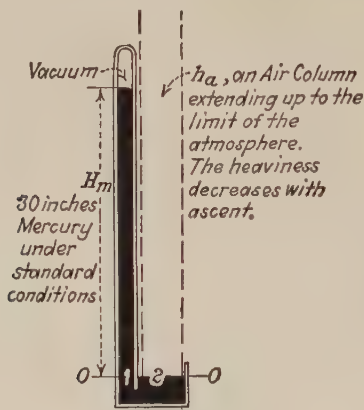


FIG. 14.

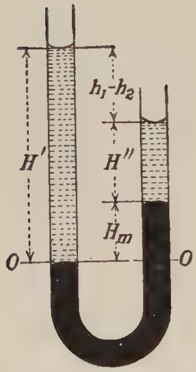


FIG. 15.

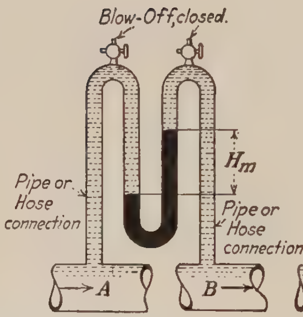


FIG. 16.

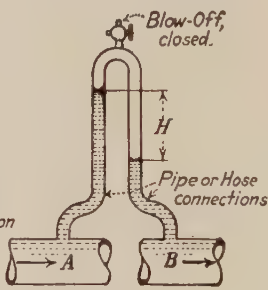


FIG. 17.

FIG. 15.—Shows a U-tube with mercury column acting as a balancer or a differential in *balancing two unequal water columns.*

FIG. 16.—Shows a U-tube as used in practice as a differential gage with heavy liquid in the bend of the U-tube, as compared to the light liquid or gas in the pipes A and B. The reason for the existence of a difference of pressure between A and B (shown at the same level) may be either because A and B are not in a *continuous* mass of liquid *i.e.* are separate sources of pressure, or because the liquid although continuous is not at rest but is *flowing.*

FIG. 17.—Shows an *inverted U-tube* used as a differential gage, with a light liquid (non-miscible) or a gas in the bend of the U, and a heavier liquid in the pipes A and B.

gage used to measure head of water, $h = (13.6 \times H_m) \pm K$, using the + sign when low column is *above* A, and the − sign when it is *below* A.

The common barometer using mercury or other liquid is a case of balancing fluid columns, one side being an air column and the other a liquid column. In Fig. 14, at the level 0—0 in the mercury the pressures at 1 and 2 are equal. Then the atmospheric pressure $p_a = p_2 = p_1 = H_m w_m$. For mercury w_m = 0.49 lb. per cu. in. When the barometer reads 30 in. (H_m = 30 in.), with vacuum above the column, then $p_a = 30 \times 0.49$ = 14.7 lb. per sq. in. *absolute*, the standard value at sea level.

14. Differential U-tube Gages.—Using the gage (Fig. 16), it is desired to compute (from the "gage difference" H_m) the differ-

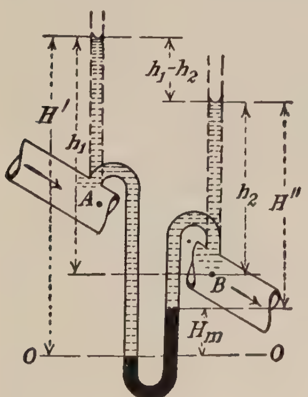


FIG. 18.

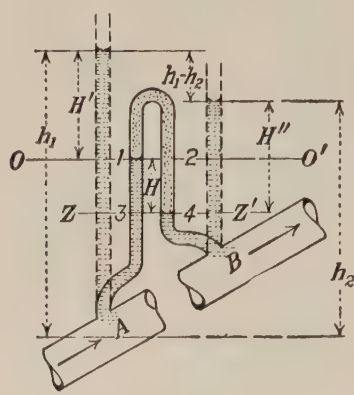


FIG. 19.

ence[1] between the water (or other liquid) heads at A and B. For the present, A and B may be thought of as separate pipes containing liquid at rest under different pressures. (Later it will be found that *flowing water* may have a difference of pressures at the same level due to friction or due to acceleration.)

For convenience imagine two open water columns attached, Fig. 18. It is seen that conditions are similar to those of Fig. 15, where a mercury column acts as a differential to balance two unequal water columns.

For Figs. 15 and 18, above the level 0—0, the water column at the left balances (at the right) the mercury column and also the water column, *i.e.*, $H'w = H_m w_m + H''w$, or $H' - H'' = H_m \times$

[1] In hydraulic engineering at times it is necessary to measure accurately the *difference* between two heads without wanting to know the exact value of either head by itself.

$\frac{w_m}{w}$. From Figs. 15 and 18 it is seen that $H' - H'' = (h_1 - h_2) + H_m$. Then, by substitution,

$$(h_1 - h_2) = H_m\left(\frac{w_m}{w} - 1\right). \tag{6}$$

If the two liquids are mercury and water, $h_1 - h_2 = 12.6H_m$

For a liquid in the U with sp. gr. of 1.30, and water in the pipe and connections to the U, $h_1 - h_2 = 0.3H_m$. In such a case the gage multiplies or magnifies the difference of water heads, while with mercury the gage difference H_m is much smaller than the water difference. One may thus select liquids for the gage to suit the degree of sensitiveness required in the measurements to be made.

For the case shown in Fig. 17, assuming water in the pipe and connections, and oil in the summit bend of the inverted U, it may again be imagined that open liquid columns are attached at A and B, as shown in Fig. 19. As in the previous analysis, we may start *within the gage* at the same level in the same continuous mass of liquid, namely, at points 1 and 2 where the pressures are equal. Calling the pressures at 0, 1, 2, etc., p_0, p_1, p_2, etc., then $p_1 = p_2$. But $p_1 = p_0 = H'w$, and $p_2 = p_4 - Hw_o$, where w_o is the unit weight of the oil. But $p_4 = p_{Z'}, = H''\,w$, whence $p_2 = H''w - Hw_o$. Therefore, since $p_1 = p_2$, $H'w = H''w - Hw_o$, or $H'' - H' = H\left(\frac{w_o}{w}\right)$. From Fig. 19, as a matter of adding distances, $H' + H = H'' + (h_1 - h_2)$, or $H'' - H' = H - (h_1 - h_2)$, so that $H - (h_1 - h_2) = H\left(\frac{w_o}{w}\right)$,

and
$$h_1 - h_2 = H\left(1 - \frac{w_o}{w}\right). \tag{7}$$

Or, with considerably less algebraic detail, it may be noted that at the low column level of Fig. 17 (corresponding to points 3 and 4 of Fig. 19)[1] there is the same difference of pressures as between A and B, since we have gone upwards equal distances in the same liquid. (This remark applies also to the high column level of Fig. 16.)

At the high column level of Fig. 17 (corresponding to points 1 and 2 of Fig. 19) this A-B difference of pressures has been

[1] For the U-tube gages in Figs. 18 and 19, note that the "gage differences" H_m and H depend only on $(h_1 - h_2)$ and are *independent* of the *differences of elevation* between points A and B.

changed to an equality by going up the same distance H *in different liquids,* so that *the A-B difference of pressure is balanced by* a column of the heavy liquid minus a column of the light liquid of the same height. Then, for Fig. 16, $(h_1 - h_2)w = H_m w_m - H_m w$, or $h_1 - h_2 = H_m\left(\frac{w_m}{w} - 1\right)$; and for Fig. 17, $(h_1 - h_2)w = Hw - Hw_o$, or $h_1 - h_2 = H\left(1 - \frac{w_o}{w}\right)$. If, for example, with water in the pipes, oil of sp. gr. 0.80 be used in the gage, then $h_1 - h_2 = 0.2H$, and $H = 5(h_1 - h_2)$.

Problem for the Student: In Fig. 16, let the liquid in the U be water and the liquid in the pipes and connections be oil. Required to compute the *difference of oil heads* $h_A - h_B$ in terms of the gage difference and the specific gravity of the oil, first algebraically, then for gage difference = 3.0 ft. and sp. gr. = 0.85. *Ans.*

$$(h_A - h_B) = D\left(\frac{1}{\text{sp. gr.}} - 1\right) = 0.529 \text{ ft.}$$

(of oil).

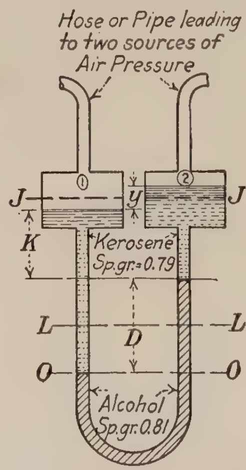


FIG. 19A.—Sensitive differential U-tube gage.

Problem: Figure 19*A* is a sketch of a sensitive differential U-tube gage for gas pressures. The cross-sectional area of the two upper enlargements or reservoirs have 100 times the internal sectional area of the glass tubes of the U. When the two gas pressures at (1) and (2) are equal, the liquids stand at common levels at J—J and L—L.

Question: When $D = 2.00$ ft., what is the difference in air pressures between (1) and (2) in: (*a*) feet height of water column; (*b*) pounds per square inch; (*c*) feet height of air of weight 0.081 lb. per cu. ft.?

Solution: Let h_1 and h_2 = air pressures at surface of kerosene in tubes (1) and (2) respectively, measured in feet-height water column. Pressures 0—0 balance; and, expressing all pressures in feet height of water, $h_1 + (K + D)0.79 = h_2 + (y + K)0.79$

$+ D \times 0.81$. But the section areas of reservoirs are 100 times the sectional areas of the glass tubes of the U.

$$\text{Therefore } y = \frac{D}{100},$$

so that $h_1 + 0.79k + 0.79D = h_2 + 0.79 \times \frac{D}{100} + 0.79k + 0.81D$

$$(h_1 - h_2) = (0.81 + 0.0079 - 0.79)D = 0.0279D = 0.0279 \times 2.0 = 0.0558 \text{ ft. water.} \qquad \textit{Ans. to (a).}$$

also $\quad 0.0558 \times \frac{62.4}{144} = 0.0242$ lb. per sq. in. *Ans.* to (*b*).

and $\quad 0.0242 \times \frac{144}{0.081} = 43$ ft. air column. *Ans.* to (*c*).

Problems

1. Show the derivation of the constant used in converting "feet head of water" to pressure in "pounds per square inch."

2. Show the derivation of the constant used in converting pressure in "pounds per square inch" to "feet head of water."

3. Standard atmospheric pressure is measured by a barometer column of 30 in. of mercury. The specific gravity of mercury is 13.6. (*a*) Compute the corresponding barometer height of water column. (*b*) Compute the corresponding pressure in pounds per square inch.

4. What is the pressure at a point 100 ft. below the surface of Cayuga Lake? Barometer = 29 in. mercury.

Ans. 57.5 lb. per sq. in. absolute, or 43.3 lb. per sq. in. gage.

5. Truncated circular conical tank 10 ft. high, 4 ft. diameter at top, 2 ft. diameter at bottom. (*a*) What is the pressure-head on the bottom? (*b*) What is the unit pressure on the bottom? (*c*) What is the total pressure on the bottom?

Ans. (*a*) 10 ft.
(*b*) 4.33 lb. per sq. in.
(*c*) 1960 lb.

6. Tank of problem 5 with diameter 2 ft. at top, 4 ft. at bottom, and 10 ft. high, full of water. (*a*) What is the pressure-head on the bottom? (*b*) What is the unit pressure on the bottom? (*c*) What is the total pressure on the bottom?

Ans. (*a*) 10 ft.
(*b*) 4.33 lb. per sq. in.
(*c*) 7840 lb.

7. An oil line is under a pressure of 20 lb. per sq. in., sp. gr. oil = 0.80. (*a*) What is the pressure-head expressed in feet of water? (*b*) What is the pressure-head expressed in feet of oil?

Ans. (*a*) 46.2 ft. water.
(*b*) 57.7 ft. oil.

8. A gage connected to a tank containing a gas under pressure registers 73.5 lb. per sq. in. What is the pressure of the gas in atmospheres?

Ans. 6 atmospheres.

9. *ABCD* (Fig. P.1) is a pipe line connecting the two reservoirs. Flow had been established through the line and then a valve at *D* was closed, so that the entire pipe line is full of water. The elevations given are all above the same datum plane. (*a*) Compute the intensity of pressure in pounds per square inch at *B*. (*b*) Compute the intensity of pressure in pounds per square inch at *C*. (*c*) What is the pressure-head at *B*?

Ans. (*a*) 26 lb. per sq. in.
(*b*) −4.33 lb. per sq. in.
(*c*) 60 ft.

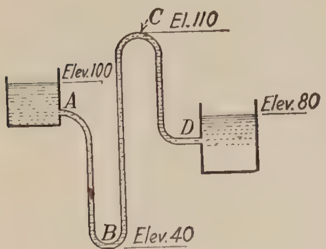


FIG. P.1.

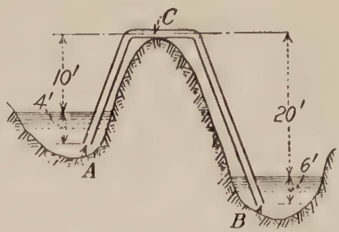


FIG. P.2.

10. The siphon pipe (Fig. P.2) is full of water and the end *A* is closed, *B* open. What is the pressure at *C* in (*a*) atmospheres; (*b*) inches of mercury vacuum? If, now, end *B* is closed and *A* opened, what is the pressure at *C* in (*c*) pounds per square inch; (*d*) feet head of water absolute? (Barometer, 30 in. mercury.)

Ans. (*a*) 0.412 atmosphere.
(*b*) 17.64 in. vacuum.
(*c*) −4. 33 lb. per sq. in.
(*d*) 24 ft. absolute.

11. The upper part of the inverted U-tube gage (Fig. 17) contains air. The gage difference "*H*" is 30 in. of water column. What is the difference in pressure between points *A* and *B* in pounds per square inch? (Air weighs 0.0808 lb. per cubic foot.) Can the weight of air be neglected in ordinary engineering practice?

Ans. 1.082 lb. per sq. in.
Yes.

12. Tank *A* (Fig. P.3) at first contains air under the pressure shown. Enough air is then removed until the gage on left (combination pressure and vacuum gage) records a vacuum of 10 in. mercury. (*a*) What is the pressure in tank in pounds per square inch under second condition? (*b*) Draw a sketch of the mercury gage on right under this second condition.

Ans (*a*) −4.91 lb. per sq. in.

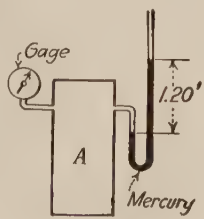


FIG. P.3.

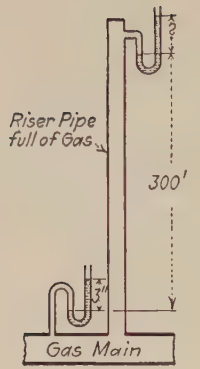


Fig. P.4.

13. The pressure of illuminating gas is commonly measured and expressed in "inches of water." In Fig. P.4 the U-tube at the gas main shows a pressure of 3 in. of water. What difference should the upper U-tube show, in inches of water? The right-hand columns of both gages are open to the atmosphere. The density of both air and gas may be assumed constant throughout the 300 ft. of elevation. Air weighs 0.080 in this locality and gas weighs 0.035 lb. per cu. ft.

Ans. 5.6 in. of water.

14. (*a*) Suction pipe of a pump (Fig. P.5). Water extends up to *A*. There is air above *A* in pipe and pump cylinder. What is the pressure in atmospheres in the pipe 10 ft. below level *A* (standard barometric conditions)? (*b*) What is the absolute pressure in the pipe at level *B*?

Ans. (*a*) 0.618 atmosphere.
(*b*) 1 atmosphere.

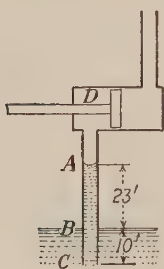


Fig. P.5.

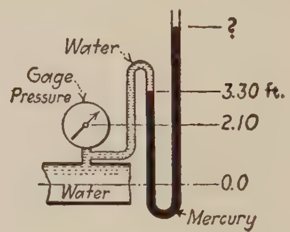


Fig. P.6.

15. A mercury U-tube gage is connected and shows readings as in Fig. P.6. If the correct reading of the pressure-gage is 30 lb. per sq. in. what should be the reading of the right column of the mercury gage?

Ans. 8.31 ft.

16. For oil of sp. gr. 0.85 (Fig. P.7) what is the difference of surface levels in the oil tanks when the gage difference *D* is 3.50 ft? *Ans.* 0.618 ft.

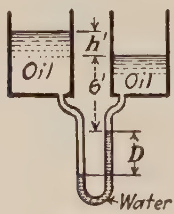


Fig. P.7.

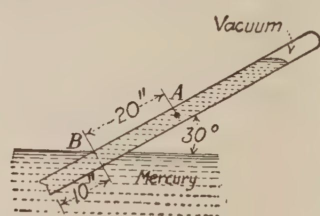


Fig. P.8.

17. Figure P.8 shows an inclined mercury barometer tube. What is the pressure of the mercury at *A* in atmospheres? (*a*) Assume standard atmos-

pheric conditions. (*b*) Assume that the local weather bureau barometer reads 25 inches.
Ans. (*a*) ⅔ atmos.
(*b*) ½ atmos.

18. *A* and *B* (Fig. 16) both contain oil with specific gravity of 0.75. Pressure in *A* is 20 lb. per sq. in. What is the pressure in *B*? *A* and *B* are at the same level. The U-tube contains water with a gage difference $H_m = 30$ in.
Ans. 19.73 lb. per sq. in.

19. Mercury U-tube gage scale readings are as shown in Fig. P.9. What is the head of water in feet in the pipe at level *A*?
Ans. 23.4 ft. *suction head*, or 10.6 ft. absolute for standard atmosphere.

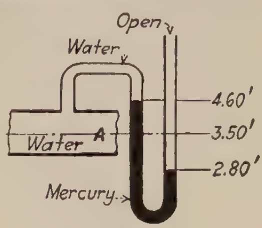


Fig. P.9.

20. What is the pressure of the gas in the tank Fig. P.3 expressed in atmospheres if the difference in levels of the mercury in the U-tube is 5 ft.?
Ans. 3 atmospheres.

21. An oil gage tester consists of a small metal U-tube containing oil. In one branch a weighted piston is placed, and to the other branch at same level the gage is attached. The piston has a cross-section = ⅕ sq. in. For a total weight of 4 lb. on the piston, what is the pressure-head in feet of the confined oil of sp. gr. = 0.90?
Ans. 51.4 ft. oil.

22. (*a*) If the barometer reads 28 in., what correct reading should an ordinary pressure-gage show when connected to the tester as described in problem 21? (*b*) What is the corresponding absolute pressure in pounds per square inch?
Ans. (*a*) 20 lb. per sq. in.
(*b*) 33.7 lb. per sq. in. absolute.

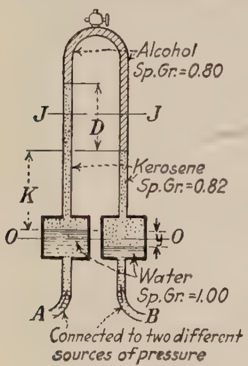


Fig. P.10.

23. Balancing liquid columns (Fig. P.10). (*a*) Express difference of heads (water) $h_A - h_B$ in terms of observed gage difference = *D* ft. The reservoir cross-sections are $n(= 100)$ times as large as those of the glass columns. With equal pressures at *A* and *B*, the tops of kerosene columns stand at the common level *J*—*J*; and the water levels in the reservoirs stand at 0—0. (*b*) When $D = 1.00$ ft., what is $h_A - h_B$ in feet of water?
Ans. (*b*) 0.0218 ft.

24. A long small-bore steel pipe supported vertically in the elevator shaft of the Standard Oil Company building in New York was used for a mercury column to test high-pressure gages. Top of column open to atmosphere. If a gage under test showed 1200 lb. per sq. in. for a mercury column 195 ft. high, what correction, plus or minus, should be applied to the gage reading to make it correct?
Ans. −54 lb. per sq. in.

25. A barometer reads 29.4 in. What is the atmospheric pressure?

Ans. 14.41 lb. per sq. in. absolute.

26. What is the pressure of gas in the upper part of the closed tube of Fig. P.11? *Ans.* 6.04 lb. per sq. in. absolute.

27. Referring to Fig. 10: (*a*) If $H_m = 10$ in., find H in feet. (*b*) What is the pressure at 0—0? *Ans.* (*a*) 11.33 ft.

(*b*) 4.91 lb. per sq. in.

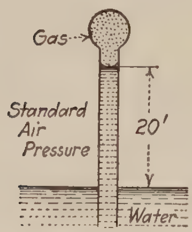


FIG. P.11.

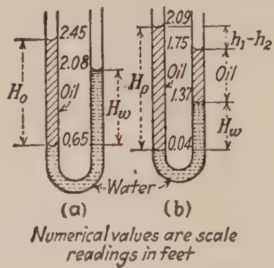


FIG. P.12.

28. Same kind of oil in the two gages sketched in Fig. P.12. (*a*) From data of Fig. *a*, determine specific gravity of oil (numerical value). (*b*) Referring to Fig. *b*, first express algebraically $h_1 - h_2$, the difference of oil column heights, in terms of H_w and specific gravity of oil. Derive the final equation by stated logical steps. *Secondly*, show that the numerical values of Fig. 2 closely verify the equation. *Ans.* (*a*) 0.795.

29. Tank A is known to contain a gas under a pressure of 2 atmospheres. It is desired to find the pressure in tank B, filled with gas, by means of a connecting U-tube filled with mercury. It is found that the right-hand column of mercury connected to tank B stands 42¼ in. lower than the left hand. What would an ordinary gage read if it could be attached to tank B?

Ans. 35.4 lb. per sq. in.

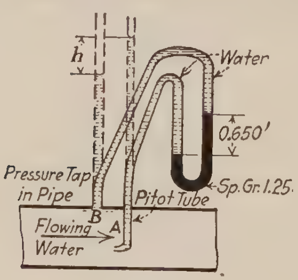


FIG. P.13.

30. The U-tube (Fig. P.13) gage contains a liquid of sp. gr. 1.25, non-miscible with water. The gage is used to find the difference h that would be shown by open water columns as per dotted lines in sketch. When the gage difference reads 0.650, what is the corresponding h? *Ans.* 0.1625 ft.

31. If crude oil of sp. gr. = 0.88 is flowing in the pipe of Fig. P.13 and the U-tube contains water in *the bottom* of the U, for a gage difference 6.50 ft. what is the difference in oil heads, $h_A - h_B$? (Oil replaces the water in the tubes of Fig. P.13.) *Ans.* 0.887 ft.

32. With a depth of 16.6 ft. of water in the tank A (Fig. P.14), the left-hand mercury column reads 3.0 and the right-hand mercury column reads 5.0. The specific gravity of the oil in tank B is 0.80. Compute the difference in elevation h between the water and the oil. *Ans.* 23.6 ft.

33. Pipe and left tube of gage (Fig. P.15) contain oil of sp. gr. of 0.88. What is the pressure in pounds per square inch at the center of the pipe? *Ans.* 29.1 lb. per sq. in.

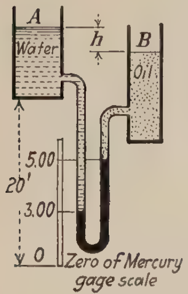


FIG. P.14.

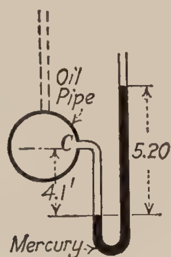


FIG. P.15.

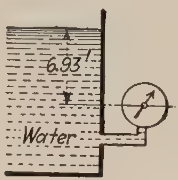


FIG. P.16.

34. When the barometer reads 25 in. what is the reading of the ordinary pressure gage shown in Fig. P.16? *Ans.* 3 lb. per sq. in.

35. Pipes A and B (Fig. 16) and upper parts of U-tube gage columns are full of water. Compute the difference of pressure-heads between A and B if gage difference is 2.00 ft. when a liquid of sp. gr. of 1.20 is in the lower part of the U-tube. *Ans.* 0.40 ft.

36. How high would the oil rise in an "open tube" above C (Fig. P.15) if the specific gravity of the oil is 0.80? *Ans.* 84.4 ft.

37. Manometer, or differential gage, of the "reservoir" type with single indicating glass column (Fig. P.17). In the lower part of cylindrical reservoir R, and in the lower part of the glass column C, there is a liquid of sp. gr. = 1.25, non-miscible with water. The other columns and interior spaces contain water. The inside diameter of reservoir R is 4 in., and that of the glass tube C is 0.4 in. It is desired to graduate a scale alongside C so it will indicate correctly *feet difference of water heads, i.e.,* difference of elevations of free water surfaces corresponding to the two different pressures communicated to A and B by rubber-hose connections. Compute the graduation *interval* on the scale (in feet) for a difference of 0.100 ft. of water.

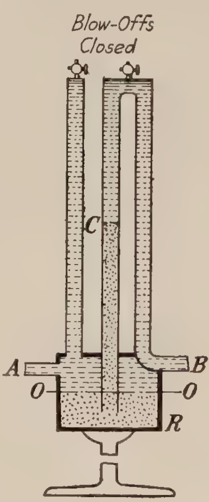


FIG. P.17.

Ans. 0.396 ft.

38. The gas holder (Fig. P.18), a large inverted cylindrical steel "cup," is made of metal $\frac{3}{8}$ in. thick, and is 50 ft. in diameter and 30 ft. high. Two per cent is to be added to the weight of the superficial steel for laps, rivets, and bracing. One square foot of the metal $\frac{1}{4}$ in. thick may be assumed to weigh 10 lb. The gas pressure in the holder is measured by the water difference $h' = 3$ in. On top of a hill at B, a U-tube connected to the gas main shows a water difference h''. The average weight of the gas in the main is 0.035 lb. per cu. ft., that of the atmosphere being 0.0782 (or 3 per cent less than for standard pressure and temperature). (*a*) For $h = 400$ ft. (as between "down town" in Ithaca and the Cornell campus), compute the gage difference h'' in inches. (*b*) With the loading on top of holder to maintain equilibrium as shown, compute the necessary counterweight in pounds. *Ans.* (*a*) 6.31 in.
(*b*) 71,600 lb.

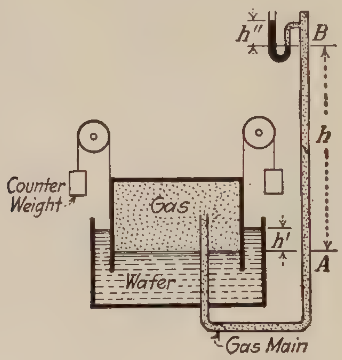


FIG. P.18.

CHAPTER II

TOTAL LIQUID PRESSURE AGAINST SUBMERGED SURFACES

CENTER OF PRESSURES

15. General Considerations.—In the design of structures that have to resist liquid pressure (such as pipes, cylinders, subaqueous tunnels, tanks—water, oil, gas, etc.— , reservoirs, dams, retaining walls, cofferdams, caissons, bulkheads, flashboards, water gates and valves, flumes, floats, buoys, barges, ships, etc.), the engineer commonly has to deal with and find values of loads caused by distributed and non-uniform liquid pressures. The problem confronting the engineer often involves finding the total resultant of the liquid pressures against part of a structure in *amount, position, and direction.* In many cases the *components* of the total pressure load must be calculated.

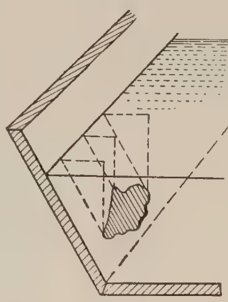


Fig. 20.

16. Plane Surfaces. *General.*—Any submerged plane surface other than a horizontal one involves a greater pressure at the bottom than at the top. Any such surface may be considered as subdivided from top to bottom into an infinite number of infinitesimally narrow rectangular strips (Fig. 20). On each of these strips the liquid pressure varies in intensity *uniformly* from top to bottom in accordance with the law $p = hw$.

Figures 21 to 25 illustrate the variation of pressure for a variety of vertical and inclined surfaces. These figures represent vertical sections through the liquid and show only the edge of the surface exposed to the liquid. The head (or height of liquid column) causing the pressure at any point is always a *vertical* distance, but for any surface in question the pressure is

perpendicular to the surface. Hence the dotted arrows in Figs. 21 to 25 may be considered to represent elemental prisms of liquid, as though the pull of gravity were perpendicular to the surface. In fact, this is merely the application of the statement in Pascal's law that pressure applied at any point of a mass of liquid is transmitted undiminished, hence equally, in all direc-

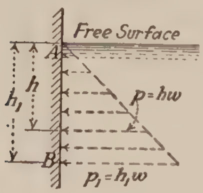


FIG. 21.

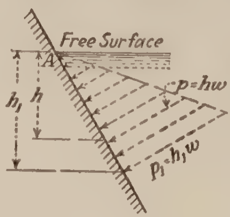


FIG. 22.

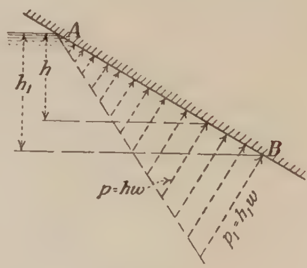


FIG. 23.

tions. On the basis of this fact the length of the dotted arrow is always made equal to the head on that portion of the surface to which the arrow points. On each elementary strip of Fig. 20 the point of *average* pressure is at the midpoint, *i.e.*, at the center of gravity of the strip (see lower parts of Figs. 24 and 25).

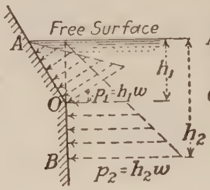


FIG. 24.

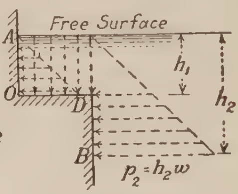


FIG. 25.

All the pressures on the series of strips form a system of parallel forces like the action of gravity, each force applied at the center of gravity of its portion of the area. For such a system of forces, from mechanics, **the point of average pressure for the whole plane area is at the center of gravity of the area,** because the total pressure $P = \int pdA = \int hwdA = h_{cg}wA$, and $h_{cg}w$ is recognized as p_{cg}, the pressure at the center of gravity.

Since, in general, for any distributed load, the total pressure against any plane surface equals the area times the average unit

pressure, or $P = Ap_{ave}$. the total hydrostatic pressure against a plane area may also be stated

$$P = A \times p_{cg}, \tag{8}$$

where p_{cg} denotes the intensity of pressure at the center of gravity of the area.

Another common way of stating the law is: *The total liquid pressure against a submerged plane surface equals the weight of a prism of the liquid, whose base equals the area subjected to pressure and whose height equals the depth (vertically) of the center of gravity of the area below the surface of the liquid.*

If the body of liquid is entirely confined and has no free surface, the rule still holds for the equivalent or imaginary free surface. Of course, this total resultant pressure must act in a direction perpendicular to the plane of the area. (As to the *point of application*, see p. 35, also the *Caution*, p. 26.)

17. Vertical Rectangular Planes.—For the pressure against a vertical rectangle whose top is in the surface of the liquid, with height h and width b (Fig. 26*a*) we have $P = A \times p_{cg} = bh \times \frac{h}{2}w = \frac{1}{2}wbh^2$.

For unity width,

$$P_1 = \frac{1}{2}wh^2, \tag{9}$$

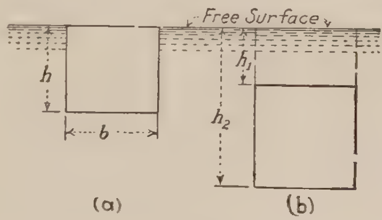


FIG. 26.

where w, as before, is the weight of a unit volume of the liquid,

If the *top* of the rectangle is *below the surface* (Fig. 26*b*).

$$P = Ap_{cg} = b(h_2 - h_1) \times \left[h_1 + \frac{(h_2 - h_1)}{2}\right]w.$$

Examples.—1. In Fig. 26*a*, let $b = 6$ ft. and $h = 8$ ft. Then $P = (6 \times 8) \times (8/2 \times 62.4) = 48 \times 250 = 12{,}000$ lb. *horizontally.*

2. Let the rectangle of example 1 be submerged 3 ft. (Fig. 26*b*), so that $h_1 = 3$ ft. and $h_2 = 3 + 8 = 11$ ft. Then

$$P = (6 \times 8) \times \left[\left(3 + \frac{8}{2}\right) \times 62.4\right] = 48 \times (7 \times 62.4) =$$
$$48 \times 437 = 20{,}970 \text{ lb.}$$

In this case the total pressure against the submerged rectangle may be regarded as the *difference* between the pressure on a rectangle 11 ft. deep and on one 3 ft. deep, both with tops at the surface, or $P = \frac{1}{2}wbh_2^2 - \frac{1}{2}wbh_1^2 = 22{,}650 - 1680 = 20{,}970$ lb., the same as above.

3. What are the total pressures against the upper and lower **triangles** formed by a diagonal on the rectangle of Fig. 26*b* (*an exercise for the student*). *Ans.* Upper triangle, 8470 lb.; lower triangle, 12,490 lb.

Caution.—The *points of application* of these total resultant pressures (if the effect of the distributed liquid pressures is imagined to be concentrated into a single force for convenience in designing engineering structures) *will always be below the center*

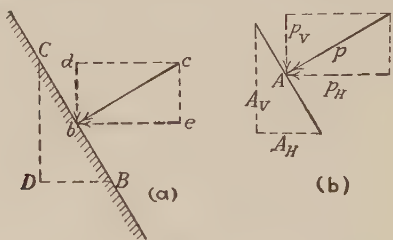


Fig. 27.

of gravity of the area except when its plane is horizontal. This is so because the larger unit pressures occur at the lower portion of the area and the smaller unit pressures occur at the upper portion. The exact location of the balancing point of the pressures, the "center of the pressure load," commonly called the "*pressure center,*" is discussed on page 35. The student should note, however, the distinction between the *point of average pressure* and the "*pressure center.*"

At present, the problem is to find the *amount* of the total pressure and of its components.

18. Components, in Any Direction of Liquid Pressure.—In many cases of stresses in structures having liquid pressure against inclined or curved surfaces, the solution of the problem is simpler

if the components (usually horizontal and vertical) of the total pressure rather than the total pressure itself are dealt with.

In Fig. 27*a* the length of the arrow *bc* represents the *average* intensity of pressure acting perpendicularly to a plane surface whose edge is *BC*. The horizontal and vertical components of both the pressure and the area are shown also in Fig. 27*b*. (If the length *BC* represents the area, *CD* will represent its vertical projection and *BD* its horizontal projection, because the mean width perpendicularly to the sketch is common to both the area and its projections.)

In Fig. 27*b* the projections of p and of A are designated by the subscripts V and H. From similar triangles, $\frac{p_V}{p} = \frac{A_H}{A}$, or $p_V \times A = p \times A_H$. So also $\frac{p_H}{p} = \frac{A_V}{A}$, or $p_H \times A = p \times A_V$. But $p_V \times A = P_V$, the total vertical component, and $p_H \times A = P_H$, the total horizontal component, hence

$$P_V = p \times A_H \tag{10}$$

and

$$P_H = p \times A_V \tag{11}$$

Hence the **Rule:** *The component, in any direction of the total liquid pressure against any plane surface equals the average unit pressure times the projection of the area on a plane perpendicular to the direction.* (For curved surfaces, see page 28.)

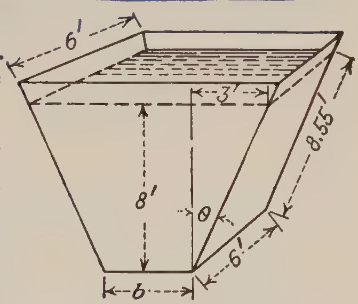


FIG. 28.

Example: What is the total pressure, and what are the horizontal and vertical components, against the inclined side of the tank shown in Fig. 28?

Solution: The total *normal* pressure $= P = Ap_{\text{ave.}} = Ap_{cg} = [6\sqrt{8^2 + 3^2}]\,[(8/2)\ 62.4] = 6 \times 8.55 \times 250 = 51.3 \times 250 =$ 12,820 lb., acting perpendicularly to the inclined side.

From this normal force the vertical and horizontal components can be computed, since $P_H = P \times \cos\theta = \left(\frac{8}{8.55}\right)P$, and $P_V = P \times \sin\theta = \left(\frac{3}{8.55}\right)P$.

However, if the problem is such that it is not necessary to find the value of the total normal force P, its components may be found directly quite simply, as follows (Eqs. (10) and (11)):

Horizontal component of P equals the product of the projection of the area A on a *vertical* plane and the average intensity of pressure, or $P_H = A_V \times p_{cg} = (8 \times 6) \times 250 = 12{,}000$ lb.

Vertical component of P equals the product of the projection of A on a *horizontal* plane and the average intensity of pressure, or $P_V = A_H \times p_{cg} = (3 \times 6) \times 250 = 4500$ lb. This latter is, of course, merely the weight of the wedge-shaped volume of water vertically above the inclined side (see p. 32).

Having found P_H and P_V, the *total normal pressure* may be found by combining the components, thus: $P = \sqrt{P_H^2 + P_V^2}$.

19. For curved surfaces the rule for finding components of *total pressure* against *plane* surfaces must be *limited to* the *horizontal component,* as the following discussion will show.

In Figs. 29 and 30 are shown the edges of two differently curved surfaces, but which have equal extents horizontally and

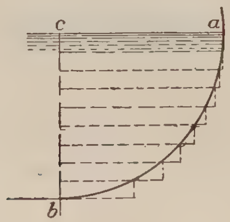


FIG. 29.

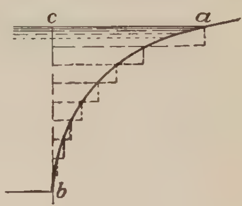


FIG. 30.

vertically. Simple inspection shows that there is more liquid vertically above the curved surface of Fig. 29 than above that of Fig. 30, and therefore, as will be shown on page 30, the total vertical pressure is greater for Fig. 29 than for Fig. 30.

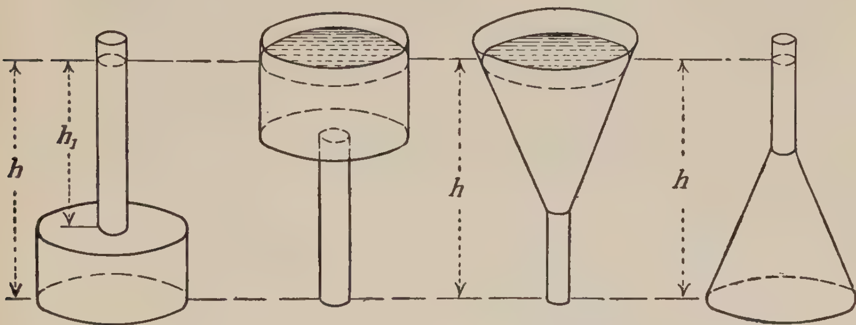
Or, from the viewpoint of actual distribution of pressures, if each of these curves is imagined to be composed of a series of short straight lines each extending through equal small intervals of depth, then inspection of the sketches shows that the *horizontal projections* of these subdivisions at the same level are *not equal* in the two curves.

Now, for each elemental portion considered as plane, it appears, since $P_V = p \times A_H$, that the curve which has the greater

horizontal extent near the bottom, where p is large, will have a greater summation of the elemental vertical components than the curve with greater horizontal extent near the surface of the liquid where p is small.

Considering the total horizontal components, it is noted that for equal intervals of depth, and therefore of pressure, the vertical projections of the curves are equal, no matter what the shape of the curve. Hence the total horizontal pressure P_H, being the same as for the vertical projection of the whole curved surface, has the same value for both.

Of course, if Eq. (11), $P_H = p \times A_V$ is used, p refers to the average intensity of pressure on the *vertical projection* of curved



(a) (b) (c) (d)

FIG. 31.

surface, and not to the average pressure on the curved surface itself. Or, in considering the curved surface as made up of a series of narrow plane surfaces, progress must be made by *equal vertical intervals.*

20. The hydrostatic paradox is the term applied to the fact that the *total* liquid pressure (force tending to burst) against the walls of a container may be many times greater, or may be much less, than the total weight of the contained liquid.

In cases a and b of Fig. 31, the weights of the liquid contents are equal, and the intensities of pressure at the bottoms are also equal, due to the same head h. However, the *total* bursting pressure near the bottom is much greater in case a than in case b. Also in case d of Fig. 31 the total bursting pressure is greater than in case c.

The paradox is cleared away by noting that in cases *a* and *d* there is a vertical *tension* in the sides of the lower parts of the containers (inclined in case *d* but having a vertical component), which tension is caused by the *upward* liquid pressure against the overhanging parts of the vessels. These vertically upward pulls around the circumference of the bottom are exactly enough (as will be shown presently) so that the weight supported by the floor on which the vessel rests is merely the weight of the liquid plus that of the container.

On the other hand, cases *b* and *c* of Fig. 31, although having a small total pressure at the bottom, transmit the weight of the overhanging liquid down to the foundation by a *compressive* stress in the sides or shell of the lower part of the container. Case *c* is further discussed on page 31.

21. Total Vertical Liquid Pressure against Overhanging and Underlying Portions of Structures and Containers.—The simple case of the total pressure against the bottom of a vertical prismatic tank has already been discussed on page 2 in connection with Fig. 2.

Likewise, for the total downward pressure against the *inside bottom* of the vessel shown in *a* (Fig. 31), $P = A \times p = Ahw$, where A is the area of the bottom. But this is the weight of a prism of liquid of area A for the full height h, *and exceeds the total weight of the liquid.*

Now, for the *upward* liquid pressure against the annular plate acting as a *cover* for the lower portion of vessel *a* (Fig. 31) let h_1 be the height of the smaller central prism and A_1 its cross-section. Then the total *upward* pressure against the bottom of this annular cover plate is $P_u = (A - A_1)h_1w$. This is seen to be the expression also for the weight of the imaginary liquid in the volume *vertically above* the annular area in question, extending from the area to the plane of the free water surface. As stated in discussing the hydrostatic paradox, this upward pressure represents the *excess* of the downward pressure against the bottom over the total weight of the liquid in the container.

22. For non-horizontal surfaces, such as those shown in Fig. 31, *c* and *d*, the principle governing *total vertical pressures* (downward or upward) appears directly if the liquid in a container is imagined

as being held in equilibrium *not by the reaction pressures from the walls of the container, but by liquid pressures,* just as if the container were submerged in a large mass of the same liquid to such a depth that the pressure heads outside and inside at the same level are equal (free surfaces in the same horizontal plane). Then the container may be imagined to disappear, and the contained liquid would still be in equilibrium within the enveloping mass of liquid. Obviously, the upward and downward *total* pressures against any imagined surface or shape within a mass of liquid at rest are equal and opposite. Otherwise the liquid could not be at rest.

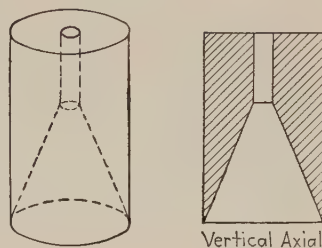


FIG. 32.

But the total downward pressure against an underlying surface must equal the weight of the liquid volume vertically above the surface and extending up to the free surface of liquid, and if there is no free surface, then up to the equivalent or imaginary free surface. Thus the total *upward* pressure of the liquid against the conical portion of vessel d (Fig. 31) must be equal and opposite to the weight of the imaginary volume vertically above the conical surface, as shown in perspective and in axial section in Fig. 32. Also it may be here again noted that the weight of this overlying volume is the excess of the total pressure on the inside bottom over the total weight of the water.

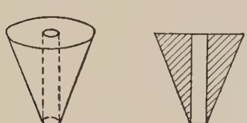


FIG. 32*A*.

For case c (Fig. 31) the liquid pressure against the bottom equals the weight of the central "core" of height h. The weight of the liquid outside of this "core," of shape shown in Fig. 32*A*, is transmitted down to the foundation through the metal, and not through the central liquid "core," since the liquid pressures against the core portion are all horizontal, being normal to the surface.

These considerations dispose of the *erroneous idea* that such a conical mass acts like a wedge and concentrates its weight at

the lower extremity. The truth, of course, is that the volume shown in Fig. 32*A* is supported all along its outside circumference by the vertical components of the normal pressures (or reactions) of the conical shell against the liquid.

23. Evidently then it is a **General Rule** that: *The total vertical component of liquid pressure against the submerged surface of any shape is equal to the weight of the real or imaginary volume of liquid vertically above the surface in question and extending up to the level of the free surface (real or imaginary) of the liquid.*

24. Deeply Submerged Areas of Small Vertical Extent.—Very common cases in hydraulic engineering involving curved or "stepped" surfaces, such as "dished" convex or concave ends of tanks and boilers; hollowed-out plunger pistons of pumps; circular pipes and cylinders for water, oil, steam, compressed

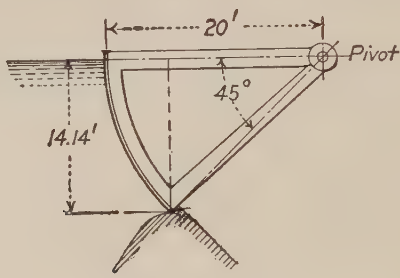


Fig. 33.—Tainter gate.

air, etc.; needle-type or Johnson valves; etc.—in which cases the vertical extent of the exposed area is so small, compared to the head, that the *average* pressure may be regarded as *uniformly distributed*—involve merely the simple direct use of Eqs. (10) and (11) to find the total force in a given direction.

25. Example: Find the horizontal and vertical components of the thrust against the pivot of the *Tainter gate* due to water pressure, per foot of length perpendicularly to the sketch (Fig. 33). (The surface exposed to water pressure is a portion of a circular cylinder.)

Solution: $P_H = A_V p_{cg} = \frac{1}{2} w \overline{14.14}^2 = 6240$ lb. per ft.

P_V = the weight of the imaginary volume of water vertically *above* the curved surface. The area of that portion of the

circular segment above the curve and below the water line = $\pi \times \frac{\overline{20}^2}{8} - \frac{\overline{14.14}^2}{2}$.

Hence $P_V = (157.08 - 100)62.4 = 57.08 \times 62.4 = 3560$ lb. per ft. of length perpendicular to the sketch.

The *line of action* of this total resultant P_V is in the vertical line through the center of gravity of the imaginary volume of liquid vertically above the curved surface.

The *resultant* of P_V and P_H must pass through the pivot because the water pressures are everywhere normal to the curve and hence directed towards the center.

As to the line of action of P_H, it will be found presently that it is $\frac{2}{3} \times 14.14 = 9.43$ ft. below the water surface (see p. 37).

26. Example: What is the "*hoop tension,*" due to water pressure, in the circular metal shell near the bottom of the riveted steel standpipe shown in Fig. 34, *i.e.*, the force tending to pull apart a vertical joint?

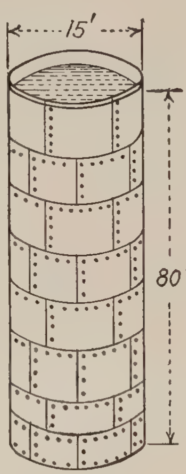


FIG. 34.

Solution: Take as a *free body* one-half of a ring or "hoop" of the standpipe of small height L so that the average pressure may be considered as uniformly distributed vertically (Fig. 35). The water pressure tends to separate the standpipe into such halves along innumerable vertical axial planes, and the "hoop tension" in the steel must resist this bursting effect. The water pressure acts radially outwards at all parts of the hoop with an intensity per unit area $p = hw$, but only the components of pressure perpendicular to the assumed plane of separation are effective in that direction. (Each elemental pressure around the ring may be assumed resolved into two components, one parallel to and the other perpendicular to the assumed plane of separation.)

If P_H represents the total water pressure at right angles to the plane of separation and T the total tension in the metal section of length L and thickness t (Fig. 35) then $P_H = 2T$.

Also, from Eq. (11), $P_H = p \times A_V = p \times Ld$. Hence $2T = pLd$, or $T = pLr$. If L is taken as unity length, then

$$\mathbf{T} = \mathbf{pr}, \text{ the "hoop-tension" formula.} \tag{12}$$

For the standpipe of Fig. 34, $T = \left[80 \times \frac{62.4}{144}\right]\left[\frac{15}{2} \times 12\right] = 34.6 \times 90 = 3120$ lb. *per in. of height.*

If the allowable tension in the steel is 16,000 lb. per sq. in., and the "efficiency" of the riveted joint is two-thirds, then the necessary thickness is $\frac{3120}{10{,}667} = 0.292$ *in.*

In practice, for permanent structures, some extra thickness, say from ⅛ to ¼ in., is usually allowed for rusting, electrolysis, etc. Cast-iron pipe must be thick enough to stand the shocks of

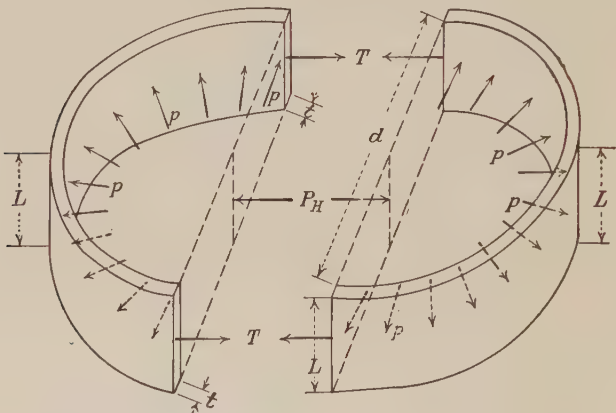


FIG. 35.

ordinary rough usage in loading, unloading, laying, and occasional cutting.

If the excess fluid pressure is on the *outside*, as for boiler flues, circular cofferdams, submarine boats, subaqueous tunnels, etc., the induced stress is *compression*, and $\mathbf{C} = \mathbf{pr}$.

With either inside or outside pressure, any portion of such a ring or hoop, less or greater than a half circumference, is held in equilibrium against the radial pressures by a pair of tangential tensions or compressions, likewise $= pr$ per unit of width of the ring. This statement assumes that the portion of the circular

arch or hoop, as the case may be, is *thin*, and that the influence of any internal moment due to fixed ends is negligible, or else is to be accounted for separately.

The hoop-tension formula may be derived by considering some other portion of the ring than the 180 deg. used in Fig. 35. The student should try, say, 60 deg., and place the components of T at the ends of the arc, one component on the prolongation of the chord and the other perpendicular to the chord. The value of T can be found from the values obtained for its components.

27. Liquid Pressure Loads against Plain Areas and Pressure Centers.—This subject will first be presented by graphical methods which are particularly adapted to plane areas of simple geometric shapes and afterwards in Par. 30 a general mathematical proof will be given.

28. Pressure Center.[1] *Simple Cases.*—Rectangular and circular plane areas are among the commonest cases arising in engineering, where it may be of importance to locate the pressure center. Triangular and trapezoidal areas are less common but may occur as elements of a cone or frustum of a cone as well as in the sides and ends of tanks, caissons, floats, etc. Other plane geometric shapes are rather rare in practice.

Figures 36 and 37 are perspective sketches showing how the variation of liquid pressures builds up the total pressure loads against the rectangular plane area $ABCD$ (Fig. 36); and the triangle ABC (Fig. 37) showing the shapes of what may be termed the "pressure prisms." (Side views of such "pressure prisms" have already been shown in Figs. 21 to 25.) The perpendicular length from any point of the area to the sloping face of the "prism" equals the head on that point. For vertical areas this gives a 45-deg. slope to the inclined base of the "pressure prism," as in Figs. 36 and 37.

Obviously,

1. The weights of these "pressure prisms" of liquid—*ABCD-EFGH* for the rectangle *ABCD* of Fig. 36, and *ABC-DEF* for

[1] The authors use the term "pressure center" as preferable to the term "center of pressure," which is often confused with the point of average pressure.

the triangle ABC of Fig. 37—are equal to the total pressures against the areas, acting perpendicularly.

2. The center of gravity of the "pressure prism" locates the resultant (or the balancing point) of the pressure load against the

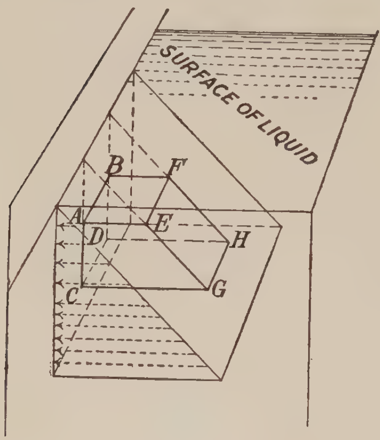


FIG. 36.

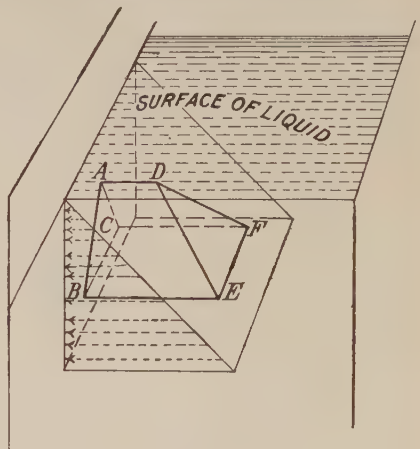


FIG. 37.

area. The point where a perpendicular through the center of gravity of this prism strikes the area is called the "pressure center" of the area subjected to liquid pressure.

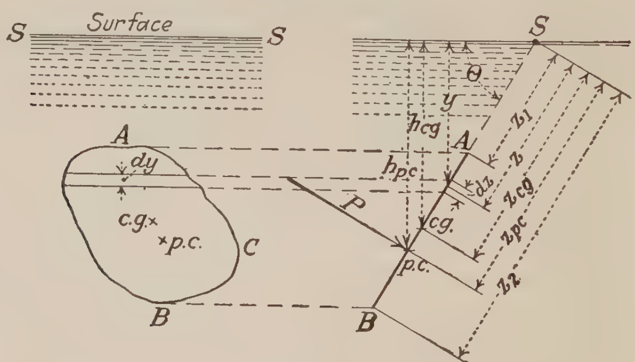


FIG. 37A.[1]

The student should carefully note the distinction between (*a*), *the point of average pressure on the area,* which is at the center of gravity of the plane area exposed to liquid pressure; and (*b*), the

[1] See Par. 30, p. 39, for general mathematical treatment of pressure center.

pressure center of the area, which is the point of application of the resultant of all the elemental pressures against the area, which resultant passes through the center of gravity of the "pressure prism," and which is *always below the center of gravity of the area* (except for horizontal areas).

For the cases shown in Figs. 38 to 41, where the *top* of the area is *at the water surface:*

Rectangle (Fig. 38) $h_{pc} = \frac{2}{3}h$ (center of gravity of triangular prism).

Triangle, vertex up (Fig. 39) $h_{pc} = \frac{3}{4}h$ (center of gravity of pyramid).

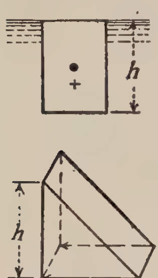


FIG. 38.

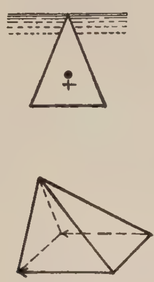


FIG. 39.

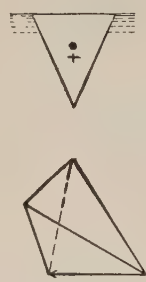


FIG. 40.

FIG. 41.

Triangle, vertex down (Fig. 40) $h_{pc} = \dfrac{h}{2}$ (center of gravity of prostrate pyramid (triangular base), being three-fourths of the length of the median line from vertex of pyramid to center of gravity of base, *i.e.*, $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$ of the height of the triangular base above the vertex of the triangle).

Circle (Fig. 41) $h_{pc} = \frac{5}{8}h = \frac{5}{8}d = \frac{5}{4}r$. (Center of gravity of ungula of cylinder with the oblique cut starting at the square-cut base, involving the same rule as the pyramid of Fig. 40, *i.e.*, $\frac{3}{4}$ of $\frac{1}{2} = \frac{3}{8}$ of the diameter above the bottom of the circle. This merely happens to be true; in this case the median is not a straight line.)

29. When the tops of the areas are submerged (such as shown in Figs. 42 to 45) the positions of the centers of gravity of the constituent parts of the pressure prism in any particular problem are usually known. Use may then be made of the fundamental law that the moment of a whole volume equals the sum

of the moments of its parts. Thus if $V =$ the whole volume of a pressure prism, V_1, V_2, etc. the volumes of its component parts, z_{pc} the distance (moment arm) from the surface to the centers of gravity of the whole prism, $(z_{pc})_1$, $(z_{pc})_2$, etc. the distances (moment arms) to the individual centers of gravity of the parts, then

$$Vwz_{pc} = V_1 w(z_{pc})_2 + V_2 w(z_{pc})_2 + \ldots$$

But $V = V_1 + V_2 + \ldots$

Therefore,

$$z_{pc} = \frac{V_1(z_{pc})_1 + V_2(z_{pc})_2 + \ldots}{V_1 + V_2 + \ldots}. \tag{13}$$

Where the area for which the pressure center is being found is vertical this reduces to

$$h_{pc} = \frac{V_1(h_{pc})_1 + V_2(h_{pc})_2 + \ldots}{V_1 + V_2 + \ldots}. \tag{13a}$$

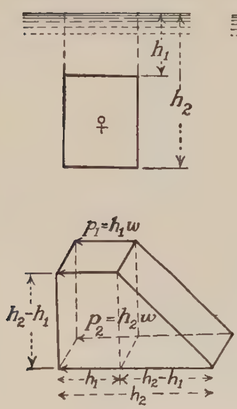


FIG. 42.

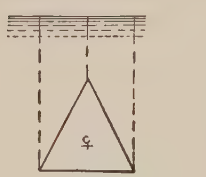


FIG. 43.

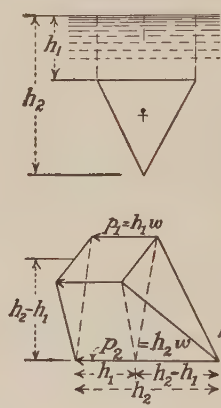


FIG. 44.

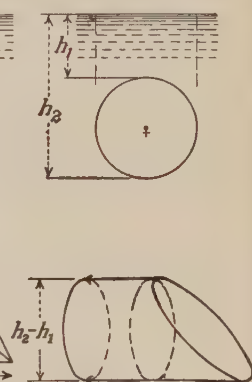


FIG. 45.

The following expressions for the h_{pc} for common plane areas of simple geometric shapes are stated as a matter of record for reference in checking computations made by Eq. (13a) (see example below). Such special equations need not be memorized.

Rectangle (Fig. 42) $h_{pc} = \frac{2}{3}\frac{h_2^3 - h_1^3}{h_2^2 - h_1^2}$. (14)

Triangle, vertex up (Fig. 43) $h_{pc} = \frac{1}{2}\frac{3h_2^2 + 2h_2h_1 + h_1^2}{2h_2 + h_1}$. (15)

Triangle, vertex down (Fig. 44) $h_{pc} = \frac{1}{2}\frac{3h_1^2 + 2h_1h_2 + h_2^2}{2h_1 + h_2}$. (16)

Circle (Fig. 45) $h_{pc} = h_{cg} + \frac{r^2}{4h_{cg}} = (h_1 + r) + \frac{r^2}{4(h_1 + r)}$. (17)

Example: Locate the pressure center of the vertical submerged triangle shown in Fig. 46.

Using Eq. (13*a*), referring to Fig. 44, for the shape of the "pressure prism," note that the length of the left part of the prism is h_1, or 5 ft., in this problem. Also the length of the right or beveled part of the "prism" is $h_2 - h_1 = 6$ ft. Calling V_1 the volume of the left part, and V_2 the volume of the right part, Eq. (13*a*) gives

$$h_{pc} = \frac{V_1(h_{pc})_1 + V_2(h_{pc})_2}{V_1 + V_2} = \frac{\left[5A\left(5 + \frac{6}{3}\right) + 6\frac{A}{3}\left(5 + \frac{6}{2}\right)\right]}{\left(5A + 6\frac{A}{3}\right)}.$$

Note that A, the area exposed to the pressure, cancels out, as will always be the case.

Therefore a *unit length of triangular prism* could be used for the *unit of volume.* Hence $h_{pc} = \dfrac{(5 \times 7 + 2 \times 8)}{(5 + 2)} = \dfrac{51}{7} =$ 7.29 ft. *Ans.*

This value for h_{pc} may be checked by substitution in Eq. (16).

Although this value is only 0.29 ft. below the center of gravity of the triangle, still the total pressure is 10,480 lb. and this causes a moment about the center of gravity of the triangle of 3000 ft.-lb., which is a considerable item.

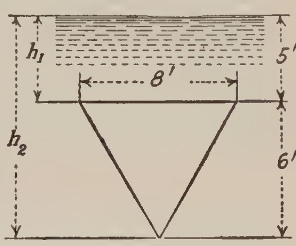


Fig. 46.

If the base of the triangle were at the surface, the center of pressure would be 1 ft. below the center of gravity of the triangle, or one-sixth of the whole height of the triangle, as compared to one twenty-first of the height of the triangle when submerged 5 ft.

30. Center of Pressure. General Mathematical Proof.—Let ABC (Fig. 37A) be any submerged plane area with water pressure acting on one side only. The line AB is a side view of the area which makes any angle θ with the horizontal. The area may be divided up into horizontal strips dz wide and area

dA at depth y below the surface, then the pressure on each strip $= dAyw$.

Let P represent the total pressure on the area and z_{pc} the distance along the plane (or plane produced) from the pressure center to $S—S$.

Then taking moments about $S—S$, the line formed by the intersection of the water surface and the plane of the area, the moment of the total pressure of the area equals the sum of the moments of the individual pressures on horizontal elements of the area.

Thus $$Pz_{pc} = \int_{z_1}^{z_2} z(dAyw).$$

But $$y = z \sin \theta.$$

Therefore, $$P \times z_{pc} = w \sin \theta \int_{z_1}^{z_2} (dA)z^2$$

and $\int_{z_1}^{z_2} (dA)z^2$ is the moment of inertia (I_s) of the area A about the line $S—S$. Therefore, $P \times z_{pc} = w \sin \theta I_s$. But P = total pressure $= A \times p_{cg} = A \times h_{cg} \times w$. Therefore $z_{pc} = \dfrac{w \sin \theta I_s}{Ah_{cg}w}$.

$$z_{pc} = \frac{I_s}{Ah_{cg}} \sin \theta. \tag{18}$$

If the area is perpendicular to the water surface $\theta = 90$ deg. and $z_{pc} = h_{pc}$.

Hence for perpendicular plane areas

$$h_{pc} = \frac{I_s}{Ah_{cg}}. \tag{18a}$$

In words, *the head on the center of pressure equals the moment of inertia divided by the static moment.*

Also since $h_{cg} = z_{cg} \sin \theta$, Eq. (18) may be written

$$z_{pc} = \frac{I_s}{Az_{cg}}. \tag{18b}$$

In Eqs. (18), (18*a*), and (18*b*), I_s is the moment of inertia of the area referred to the line where the plane of the area cuts the water surface, not the gravity axis. Since the moment of inertia, I, commonly given in handbooks is for a gravity axis, it is necessary in this case to find I_s about a parallel axis, *viz.*: $I_s = I_{cg} + A(z_{cg})^2$, where I_{cg} is the moment of inertia about the gravity

axis and z_{cg} the distance in the plane of the area from the free surface to the center of gravity of the area.

Example: Locate the pressure center of the vertical submerged triangle shown in Fig. 46.

$$h_{pc} = \frac{I_s}{A(h_{cg})} = \frac{(I_{cg}) + A(h_{cg})^2}{A(h_{cg})}.$$

$$\text{Therefore } h_{pc} = \frac{\frac{1}{36}bd^3 + \frac{bd}{2} \times (h_{cg})^2}{\frac{bd}{2} \times (h_{cg})} = \frac{\frac{1}{18} \times \overline{6}^2 + \left(5 + \frac{6}{3}\right)^2}{\left(5 + \frac{6}{3}\right)} =$$

$$7 + \frac{2}{7} = 7\frac{2}{7} \text{ ft.}$$

or $h_{pc} = 7.29$ ft. *Ans.*

(This value is also given in Par. 29.)

Problems

1. Compute total force tending to push plug out (Fig. P.19). (Total volume in upper part is 5.83 cu. ft. and in lower part 2.50 cu. ft.) *Ans.* 312 lb.

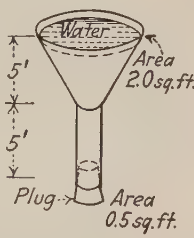


Fig. P.19.

2. Find total pounds pressure on the triangle (Fig. 43), width of base 2 ft., height 3 ft.; plane is vertical with water pressure on one side only. $h_1 = 2$ ft., $h_2 = 5$ ft. *Ans.* 749 lb.

3. Cylinder, 2 ft. diameter, on 45-deg. inclined plane (Fig. P.20). Take 1-ft. length perpendicular to sketch. Compute total *horizontal* push in pounds per foot, *i.e.*, the resultant towards the right. *Ans.* 62.4 lb.

4. Rectangle in vertical plane (Fig. 26(*b*)). Sketch side view of "pressure prism," and compute the distance from the surface to the center of pressure when $h_1 = 3$ ft., $h_2 = 6$ ft., and $b = 2$ ft. *Ans.* 4.67 ft.

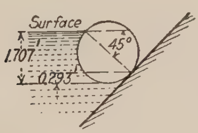


Fig. P.20.

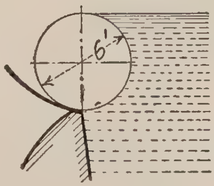


Fig. P.21.

5. Automatic crest for dam. The steel cylinder, resting on top of the dam, is arranged so as to roll up the two inclines at the ends, when the water level rises to the top of the cylinder as shown in Fig. P.21. This

allows the water to escape between the cylinder and the crest of dam. For 1-ft. length of cylinder, 6 ft. diameter, in position shown, find: (*a*) the net total lifting effort of the water pressure against the cylinder; also (*b*) the horizontal component of water pressure. *Ans.* (*a*) 884 lb.
(*b*) 1124 lb.

6. Allowing 60,000 lb. per sq. in. as the ultimate tensile stress in steel, what should be the minimum thickness of a steel pipe 30 in. in diameter under a pressure-head of 300 ft? Use a factor of safety of 6.
Ans. 0.195 in.

7. A circular concrete tank 30 ft. high with walls 12 in. thick and an internal diameter of 20 ft. was designed to be completely filled with water. In the construction of the tank the reinforcing rods were placed at 6-in. vertical intervals. These rods were square steel rods ¾ by ¾ in. Allowing no tension in the concrete, the ultimate strength of the steel equal to 64,000 lb. per sq. in., and a factor of safety equal to 4, compute the maximum depth to which the tank should be filled. *Ans.* 28.8 ft.

8. A concrete tank of 15-ft. internal diameter with wall 1 ft. thick is 30 ft. high. Compute the proper spacing of ½-in. square steel rods (cross-section equals ¼ sq. in.) at a depth of 25 ft., if the tank is to contain water. Safe tensile stress in steel = 16,000 lb. per sq. in. Safe tensile stress in concrete = 0 lb. per sq. in. *Ans.* 4.1 in.

9. The circular well curbing or ring pedestal of reinforced concrete is to be poured in position as shown in Fig. P.22. The wet concrete to be used is a liquid weighing 150 lb. per cu. ft. How many pounds of vertically upward pressure are exerted against the outside forms (to be resisted by loading, clamping down, or otherwise preventing the forms from rising)?
Ans. 19,100 lb.

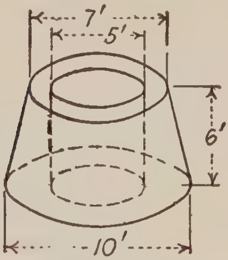


FIG. P.22.

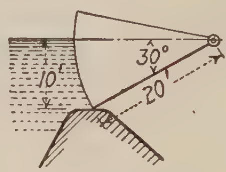


FIG. P.23.

10. Tainter gate (Fig. P.23). For 1-ft. length perpendicular to paper compute in pounds: (*a*) the total horizontal push of the water pressure against the gate; (*b*) the total vertical component of the water pressure against the gate; (*c*) noting that the resultant of the water pressures must pass through the hinge, show graphically how the line of action of the vertical component may be found. *Ans.* (*a*) 3120 lb.
(*b*) 1130 lb.

11. In a long V-trough, there are horizontal tie rods across the top at 6-ft. intervals. Find the tension in the ties due to the water pressure, depth $h = 4$ ft. Trough depth H is 5-ft., sides 60 deg. with horizontal. If one end of the trough be closed with a partition at right angles to the surface of the water, what is the total pressure and where is the center of pressure on this partition due to the water on one side? *Ans.* 1065 lb.

12. A-B is a water gate hinged at B (Fig. P.24). Compute what weight W, fastened to the center of the gate is necessary to keep the gate from opening. The weight of the gate itself is 4500 lb. and is of uniform thickness. The length of the gate perpendicular to the sketch is 5 ft. *Ans.* 16,320 lb.

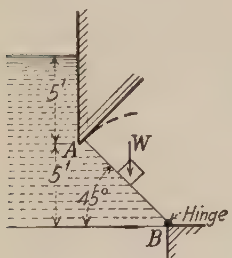


Fig. P.24.

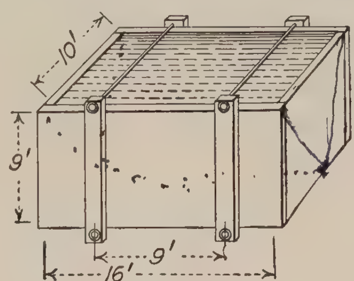


Fig. P.25.

13. (Problem arising from the failure of molasses tank of U. S. Industrial Alcohol Co. in Boston. See *Eng. News-Record*, Feb. 13, May 15, and Oct. 7, 1920, pp. 353, 974, and 691, respectively.) The tank was 90 ft. in diameter and contained 48-ft. 10-in. depth of molasses weighing 11¾ lb. per U. S. gal., or 88 lb. per cu. ft. With tensile strength of steel = 55,000 lb. per sq. in., and a factor of safety of 3, assuming that the 1-in. rivets in the vertical joints cut out just one-third of the metal (so that joint "efficiency" is 66⅔ per cent), calculate: (*a*) The proper thickness in inches of the metal for the side of the tank close to the bottom. (*b*) If the Boston building laws allow 10,000 lb. per sq. in. for rivet shear, how much (if any) were the 1-in. rivets overstressed in shear, there being 11 rivets per foot in the vertical joints near the bottom of the tank? *Ans.* (*a*) 1.32 in.
(*b*) 2.24 times.

Hint.—Imagine as a free body one-half of the metal ring and layer of liquid between two horizontal planes 1 in. apart (see sketch Fig. 35).

14. The sides of a rectangular tank are held together by two sets of tie rods as in Fig. P.25. Inside dimensions are as shown. Assuming that the tank is full of water and that the ties resist entire bursting pressure, calculate: (*a*) total water pressure against one side; (*b*) tension in each upper tie rod; (*c*) tension in each lower tie rod. *Ans.* (*a*) 40,400 lb.
(*b*) 6,730 lb.
(*c*) 13,470 lb.

15. The portion of a metal flume that is filled with water is semicircular in cross-section and has a 3-ft. radius. (*a*) What is the total horizontal water pressure against one-half of the flume for an 8-ft. length? (*b*) What is the total vertical water pressure against one-half of the flume?

Ans. (*a*) 2250 lb.
(*b*) 3530 lb.

16. If each horizontal tie rod, 1 ft. above the diameter of the flume of problem 15, holds together 8 ft. of flume length (4 ft. each side of tie), what is the tension in each tie?

Hint.—Make a sketch putting in all the forces necessary to make a "free body" of the half flume. Center of gravity of semicircle and quadrant of circle is distant $0.4244r$ from diameter. *Ans.* 1686 lb.

17. For a length of 10 ft. perpendicular to sketch (Fig. P.26), what is the compression load on the prop? *Ans.* 9430 lb.

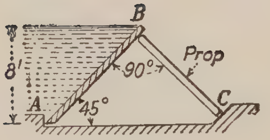
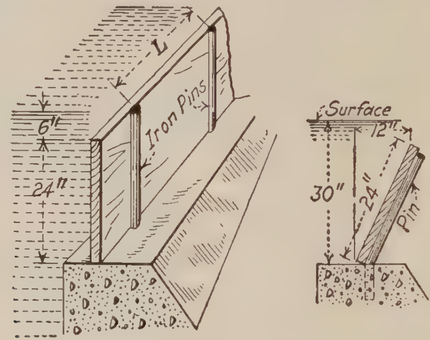


Fig. P.26.



(a) (b)

Fig. P.27.

18. Flashboards are supported by iron pins in crest of dam as in Fig. P.27,(*a*) Given: Resisting moment of a beam $= \frac{SI}{c}$. For a circle, $I = \frac{\pi d^4}{64} =$ $0.0491\, d^4$, where S is the allowable extreme fiber stress, I is the moment of inertia of the cross-section referred to the neutral axis, and c is the distance from neutral axis to extreme fiber. Tests show yield point of the iron to be at 45,000 lb. per sq. in. extreme fiber stress. What is the proper spacing (in inches) for iron pins of ¾-in. diameter, so that flashboards 24 in. high will yield when water flows 6 in. deep over the top? Dynamic effects of water on flashboards may be neglected, and static conditions assumed. As preliminary, express and calculate (inch-pounds): (*a*) total resultant pressure against L in. of flashboard length, (*b*) the moment arm of the resultant, (*c*) bending moment of water pressure on the pin. (*d*) Finally compute L.

(*e*) (A separate problem. Do not use any values from previous problem without reconsideration and recalculation.) If the pins and flashboards have yielded to the extent shown in sketch (Fig. P.27, (*b*)), what is now the

moment of the water pressure tending to bend the pins? Calculate for one pin. Here again neglect dynamic effects. Express moment in terms of L and compare with that found in (*c*). *Ans.* (*a*) $15.6L$ lb.
(*b*) 9.33 in. above bottom.
(*c*) $145.5L$ in.-lb.
(*d*) 12.76 in.
(*e*) $167L$ in.-lb.

19. New fire hose of $2\frac{1}{2}$-in. diameter is commonly tested up to 400 lb. per sq. in. pressure. Corresponding to this pressure, how many pounds weight can a strip of the hose fabric 1 ft. wide carry? *Ans.* 6000 lb.

20. In a flushing device the sides AB and DE (Fig. P.28) of the passageway between the two tanks are held apart at the top by struts AD spaced 5 ft. apart. As preliminary to finding the thrust in the strut, find the values of (*a*) the vertical and (*b*) the horizontal component of the total pressure against one side 5 ft. long perpendicular to sketch. *Ans.* (*a*) 1404 lb.
(*b*) 5620 lb.

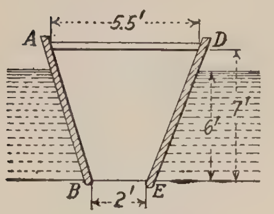


FIG. P.28.

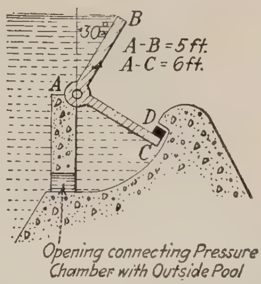


FIG. P.29.

21. *Automatic crest for dam* (Fig. P.29). BAC is braced so as to be rigid. It is to be in equilibrium for water surface as shown in sketch. Leaf AB weighs 500 lb. per ft., and leaf AC weighs 600 lb. per ft. perpendicularly to paper. Neglect friction at A and C, and consider a length 1 ft. perpendicular to paper. Calculate values as follows: (*a*) Water pressure against AB. (*b*) Center of pressure on AB, distance from A. (*c*) Water pressure against AC. (*d*) Center of pressure on AC, distance from A. (*e*) Proper weight of counterweight D, for equilibrium, center of gravity of D $5\frac{1}{2}$ ft. along AC from A. (*f*) Vertical component of *reaction* at hinge A. (*g*) Horizontal component of *reaction* at hinge A. (*h*) Reaction at hinge, amount, and direction referred to vertical. *Ans.* (*a*) 675 lb.
(*b*) 1.66 ft.
(*c*) 2180 lb.
(*d*) 3.26 ft.
(*e*) 801 lb.
(*f*) 346 lb.
(*g*) 1677 lb.
(*h*) 1713 lb.

22. Each of the six headgate openings sketched in Fig. P.30 at the Beebe Lake canal is a vertical rectangle $2\frac{1}{2}$ ft. wide and 10 ft. high, the tops being 8 ft. below the ordinary lake level, and 11 ft. below flood level. The downstream side is here assumed to be without backwater. (*a*) Calculate for flood level the total horizontal water pressure in pounds against the whole framework surrounding and including all the gates, a rectangle 22 ft. high and 20 ft. wide. (*b*) The framework is supported horizontally against the water pressure at levels 1 ft. above and 1 ft. below the openings. Calculate the value in pounds of the load on the *upper* support. (*c*) Calculate for ordinary water level the water pressure in pounds against the upstream face of *one gate*, overlap as shown in vertical section. (*d*) To raise the gate with ordinary lake level requires at least 800-lb. pull on the end of an operating lever 6 ft. long. This force moves 4 ft. for the gate to go up 4 in. Calculate the coefficient of friction (brass strips sliding on brass strips).

Ans. (*a*) 302,000 lb.
(*b*) 185,000 lb
(*c*) 23,800 lb.
(*d*) 0.403

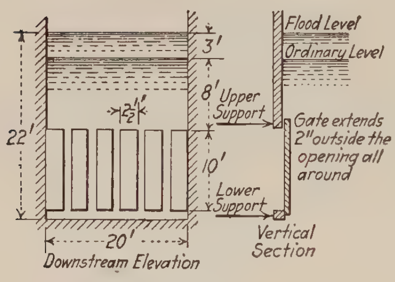


Fig. P.30.

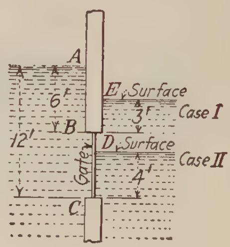


Fig. P.31.

23. Case I. Vertical rectangular sluice gate 6 ft. long (perpendicular to paper), supported entirely at top and bottom edges. Water on both sides. Diagrammatic sketch (Fig. P.31). Find in sequence: (*a*) total water pressure on left of gate; (*b*) total water pressure on right of gate; (*c*) total pressure of gate against top support at *B*; (*d*) same against bottom support at *C*.

Case II. Find (*a*), (*b*), (*c*), and (*d*) as above. Before making use of values from previous problem, check them carefully.

Ans.	Case I	Case II
(*a*)	20,200	20,200 lb.
(*b*)	13,480	3000 lb.
(*c*)	3370	8320 lb.
(*d*)	3370	8900 lb.

24. Automatic drop shutter on top of Betwa Weir, Paricha, India (Fig. P.32). (From Wilson's "Irrigation Engineering.") In all, there are 300 shutters, each 12 ft. long and 6 ft. high held by four tension rods. One-third

of the shutters are arranged to drop when the water has risen so as to flow 3 ft. deep over the top of the shutters. Experiments showed that the pivot on the shutter must be just six-sevenths of the distance from the bottom to the center of pressure. This allows for friction of the shoe and for departure from true static conditions. (*a*) Assuming static conditions (*i.e.*, a gate 6 ft. high, top submerged 3 ft.), compute the total pressure against one shutter. (*b*) Locate the center of pressure above bottom of shutter, for assumed static conditions, and then locate actual position of pivot.

Ans. (*a*) 27,000 lb.
(*b*) 2.14 ft. actual position.

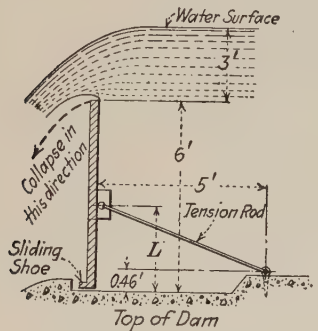


FIG. P.32.

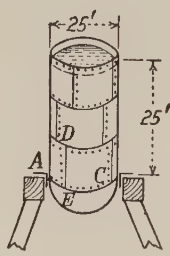


FIG. P.32*A*.

25. Elevated tank, hemispherical bottom, riveted steel plate (Fig. P.32*A*). (Surface of a sphere = area of four great circles. Volume of a sphere = two-thirds of volume of circumscribing cylinder.) (*a*) Due to water pressure, what is the stress (per inch length of joint) vertically across the circumferential joint *AEC*? (*b*) Due to water pressure, what is the stress (per inch length of joint) horizontally across the vertical joint *DE* at point *E*? In other words, what is the "hoop tension," in a "hoop" 1 in. wide vertically, due to the water pressure? *Ans.* (*a*) 1082 lb.
(*b*) 1630 lb.

CHAPTER III

STABILITY OF GRAVITY DAMS

31. Below is a brief statement of some of the engineering aspects of the subject as preliminary to the development of formulas based on fairly reasonable assumptions.

A **gravity dam** is a barrier of practically solid masonry that resists the pressure of the impounded water in much the same manner, roughly speaking, as any block of masonry by virtue of its weight and size resists overturning and sliding when subjected to a horizontal push against one side and also perhaps to an upward prying action under the bottom. The designation "gravity dam" distinguishes the structure from other types of dams, such as *arch, hollow, buttressed, buttressed multiple-arch, bear-trap, wicket, coffer,* etc. These latter types involve important structural and stress differences from a gravity dam.

32. Earth dams are of the same general type as gravity dams, but, being always somewhat porous as well as lacking in cohesive or monolithic strength, they are less subject to exact stress analysis, and are usually built in accord with safe precedent without going into details of stresses. Of late years the "hydraulic-fill" method of depositing the material has been much improved, so that an earth dam of the best construction is at present a structure far more reliable than the earth dikes or causeways of former years which were made to serve (as long as they would) as dams to hold back water.[1]

If a masonry core wall is used in an earth dam, a fairly substantial structure is obtained. But in any case the amount of material used is very greatly in excess of that for a masonry dam, and the permanency is less assured. Usually, earth dams are built where good rock foundation cannot be had, though several have been built on good bedrock. (Data of various earth

[1] For a good discussion of earth dams, see *Trans. Am. Soc. Civil Eng.*, vol. 87, pp. 1–141, 1924.

dams and many masonry dams are given in the civil engineering handbooks.)

33. The cross-section of a **typical high masonry dam** is a modification, usually slight, of a right-angled triangle with vertical face upstream and vertex at highest assumed water level. Figure 47 shows the *ideal* economical triangular section; Fig. 48, the same with addition of a topping mass; and Fig. 49, a development of the design practically necessary as well as theoretically desirable.

Figure 47 shows a triangle with base equal to about eight-tenths of the height. This value of the ratio is roughly an average of prevailing practice, but in any particular case it depends on the weight of the masonry, probability of pressure beneath dam, ice thrust at top, etc. (as hereinafter discussed), so that the ratio

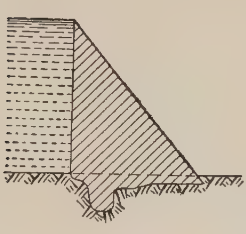


FIG. 47.

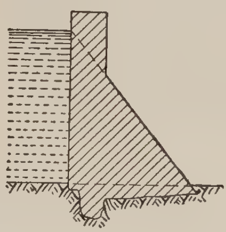


FIG. 48.

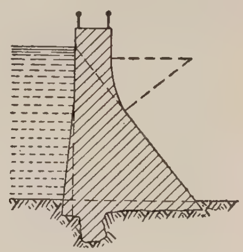


FIG. 49.

ranges from about 0.6 to more than 1.0 for the ideal triangular cross-section portion.

Figure 48 shows the addition to the top of Fig. 47 of a trapezoidal block, so as to raise the top a few feet above the highest water surface, and so as to secure the practically necessary thickness (often for a roadway across the dam). (Most *high dams* do not have water flowing over the top of the main dam. The excess water is discharged by a specially built spillway of some sort.)

In the cross-section shown in Fig. 48, however, the addition of the trapezoidal block at the top causes the center of gravity of the dam to move a little upstream from its position in the triangle of Fig. 47. This is undesirable, since it causes tension in the masonry,[1] unless counteracted, as is shown presently. Therefore, such a shift in the center of gravity is provided for

[1] Near bottom of downstream face of dam, when reservoir is empty.

by widening the base of the dam by adding a triangular portion to the lower part of the upstream face, as shown in Fig. 49, which shows also the addition made near the top of the downstream face so as to avoid the rather sudden change in cross-section illustrated in Fig. 48, thereby securing better distribution of stresses.

Figure 49 represents the general shape of the cross-section of a well-designed high masonry gravity dam (see also Figs. 56 and 57).

34. Forces Acting on a Dam.—Among the forces acting or that may act on a dam some may be exactly predicted and calculated; others are assumed as more or less probable as to action and magnitude, and the allowances made for them are largely matters of individual engineering judgment, influenced by experience with and records of failures and safe standings of dams previously built in various ways and under various conditions of service and exposure.[1]

A *summary of forces usually considered*, even if not all allowed for, follows: (1) the *static water pressure* against the upstream face of the dam; (2) *the weight of the masonry*. These two forces certainly exist in all cases and are quite definite in amount and as to the resultant lines of action. Other forces, less certain of action and less definite in magnitude, include: (3) uplift of water pressure beneath or within the dam; (4) ice thrust due to expansion (with rising temperature) of a continuous sheet of thick ice previously formed on the surface of the pond; (5) impact of waves and floating masses, including the effect of a rush of water due to the failure of a dam further upstream or to a landslide; (6) dynamic pressure of deviated flowing water (principally for overflow or spillway types of dams); (7) earthquake shocks.

A few comments on items 3 and 4 follow: Removal of the probability of *uplift pressure* beneath or within a solid masonry dam involves great practical difficulties in construction. It is far from easy to accomplish the following: (*a*) to make the *upstream face* of the dam impervious at all points, and permanently so; (*b*) to make a water-tight seal between the dam and the rock (or other) foundation; (*c*) to prevent access of water from the pool to the rock beneath the dam (prevention usually attempted

[1] See "Notable Dam Failures," *Engineering News-Record*, March 22, 1928, p. 472.

by means of a cutoff wall or by grouting and plugging fissures in the rock, or by both means); (*d*) to provide effective drainage within and beneath a dam so that no water can accumulate *under pressure;* (*e*) to assure to such drainage non-interference by choking because of freezing or of deposition of lime leached from the cement in the masonry.

Among means of securing good drainage may be mentioned: fair-sized drain pipes or a series of porous concrete blocks sometimes leading to or replaced by longitudinal galleries large enough for a man to enter for inspection; also deep earth fill at the toe of the dam (downstream base) to keep out the frost.

Ice thrust in mid- or far-northern and corresponding southern latitudes may be guarded against, if a maintenance force is available, by cutting a narrow lane through the ice near the dam as often as ice forms several inches in thickness. Conservative engineering, however, argues against dependence on such removal of ice thrust for the safety of a great structure through many years. It is better to make due allowance in the original design for any probable ice thrust.

The maximum possible ice thrust equals the force necessary to crush or crumple the ice, which in reservoirs is seldom more than 2 or 3 ft. thick over a large area. In most ponds upstream from dams the shape of the coverging hillsides is such that they, rather than the dam, bear the stress due to a considerable expansion of the ice sheet. If possible, the dam should be so located as to secure this advantage.

Recent allowances for ice thrust by engineers range from 20 to 25 tons per lineal foot of dam. For ice 3 ft. thick this means a compressive stress in the ice of about 100 lb. per sq. in.

(To crush 12-in. cubes of pure hard ice requires 21 to 64 tons per sq. ft., or about 300 to 900 lb. per sq. in.)

35. Details of Securing Stability for Gravity Dams.—A dam must be built safe against *sliding* downstream on its base, or on any construction joint above the base; or sliding together with a layer of rock beneath its base;[1] also safe against *shearing off* on any plane above the base. A dam must also be safe against *overturning*, and this involves, as shown in detail presently, not merely a matter of balance between the obvious moments of

[1] See Review of St. Francis Dam Catastrophe with Reports of Four Boards of Engineers and Geologists, *Engineering News-Record*, May 10, 1928, p. 727–736.

water pressure and of weight of masonry, but also the question of the *distribution of the stress* in the masonry so that both crushing by compression and crack opening by tension may be avoided.

36. Safety against sliding demands that the *weight* of the dam (or possibly the weight diminished by upward water pressure), multiplied by the *coefficient* of *friction,* shall exceed the total horizontal pressures which include the horizontal component of the total water pressure and, perhaps, ice thrust at the surface of the pond and any impact allowances or dynamic pressures.

The Undependability of Friction against Sliding.—Assume that a masonry dam is built on top of a flat rock surface, and that the permanency of the adhesive bond of the cement to the rock is questioned or else that this bond is regarded merely as a factor of safety to increase the frictional resistance to sliding. For such a case the approximate demands made on the resistance to sliding by the friction between the masonry of the dam and the bedrock will now be computed (see Fig. 50). Note that the assumption of a simple triangular cross-section is sufficient for the purpose, and that the investigation holds for any horizontal plane above the rock bottom.

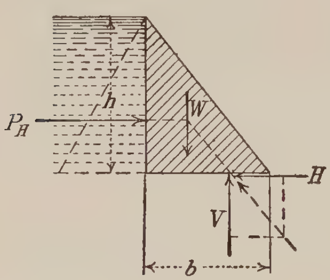


FIG. 50.

Assume that the masonry weighs 9/4 times as much as water, or $w' = \frac{9}{4}w$, = about 140 lb. per cu. ft., which is about the low limit for good rock or concrete. Assume that $b = 0.8h$, which is a reasonable value. Then, for stability against sliding, the available resistance H *must at least equal* P_H. But if H is provided entirely by *friction,* its maximum value is $H = f \times W$, where f is the coefficient of friction *between the dam and the bedrock.* But $P_H = \frac{1}{2}wh^2$ per unit length perpendicularly to the sketch. Then $fW = \frac{1}{2}wh^2$. But $W = \frac{1}{2}bhw'$, and if $b = 0.8h$, $W = 0.4h^2w' = 0.4h^2\frac{9}{4}w = 0.9h^2w$.

Then $0.9fh^2w = \frac{1}{2}wh^2$, or $f = 0.555$. (If a wider base and a heavier masonry had been assumed, *e.g.,* $b = 0.95h$ and $w' =$

$\frac{5}{2}w$, or 156 lb. per cu. ft., the necessary value of f would be 0.42.) This is too high a value to depend on for available friction against sliding, with leeway for a proper factor of safety. (See any text or handbook giving angles and coefficients of friction, *e.g.*, for dry rock on dry rock $f = 0.4$ to 0.7.)

Because of this fact, *in good construction* the base of a masonry gravity dam is not merely laid on top of a somewhat flat rock surface, but is set down in a channel excavated across the valley down to and into solid bedrock which is left quite rough and jagged at the bottom of the cross-channel. Thus dependence is placed primarily on the *shearing strength* of the masonry, which is purposely made so as *to avoid plane joints extending through the dam* at any place.

The use of "cyclopean" masonry is now quite general in the construction of large dams. (See *Eng. News-Record*, Sept. 21, 1922, p. 466, for method of placing large rock "plums" in the concrete of the Hetch Hetchy Dam, California.)

37. Safety of Dam against Shearing.—Referring to Fig. 50, let s = average shear per unit area of horizontal cross-section. Assuming $b = \frac{2}{3}h$, a fairly low value, $P_H = bs$, or $\frac{1}{2}wh^2 = \frac{2}{3}hs$, or $s = \frac{3}{4}hw$, and for foot-pound units, $s = 46.8h$. This is for static water pressure. (For ice thrust see below.)

For heights of dam, h = 100, 200, and 300 ft., respectively, s = 4680, 9360, and 14,040 lb. per sq. ft., or 32.5, 65, and 97.5 *lb. per sq. in.* Even for a dam 600 ft. high and with at least 400 ft. thickness at the base (like that proposed for Boulder Canyon of the Colorado River) the average horizontal shear would be only about 200 lb. per sq. in., which is a safe value for good concrete, particularly if of high-grade "cyclopean" variety.

Ice thrust causes shearing effect that must be resisted by sufficient thickness of masonry near the top of the dam. An ice thrust of 50,000 lb. per ft. demands a thickness of at least 5 ft. if the shearing stress is not to exceed 70 lb. per sq. in. in this portion of the dam. (Severe weathering effects occur at the water surface, and a liberal factor of safety should be allowed.) If the top of the dam carries a roadway, ample thickness is assured for ice thrust and wind and wave effects. For high dams the shear due to water pressure is much greater than that due to ice

thrust. Placing P_H, which $= \frac{1}{2}wh^2 = 50{,}000$ lb., then $h =$ 40 ft. for the height where shears due to these causes are equal. But if the dam is 80 ft. high the water pressure would be four times as great.

Problems for the Student: (*a*) Show that an ice thrust of 25 tons per lineal foot of dam causes an overturning moment about the bottom of the dam approximately one-half as great as the moment due to the water pressure against the upstream face of the dam 100 ft. high; (*b*) also that the ratio between these moments varies inversely as the square of the height of the dam, for a fixed ice-thrust value. (*c*) For what height of dam are the moments equal? *Ans.* 69.4 ft. (*d*) For what height is the water moment 10 times the ice moment? *Ans.* $\sqrt{10} \times 69.4 = 219$ ft.

To summarize the results of the discussion thus far: (*a*) a gravity dam of masonry, economically built, cannot be allowed safely to depend on friction against sliding; (*b*) such a dam is safe against shearing if properly constructed to approximate a monolith, and if of about the previously assumed base thickness (which is demanded by additional stability requirements now to be considered). (There are special cases where the mass of the dam is made so great that safety against actual sliding is amply assured. Earth-fill dams are of this type, if properly constructed for imperviousness and drainage.)

38. Safety against overturning might seem, at first thought, to demand simply that the resultant of the water pressures and the weight of the masonry should not fall outside the base on the downstream side. For dams of any considerable size, however, the resultant line of action cannot be allowed to approach near the downstream edge of the base. The reasons will be summarized after an examination into *how the masonry of the dam carries vertical compressive stress.*

Two main assumptions are made:

1. Navier's rule is assumed to apply, *viz.:* within the elastic limit, when there is a variation in pressure on a plane cross-section, "the pressure per unit area *varies uniformly* from the extremity under greatest compression to the place of least compression (or of no compression)."

2. Masonry is assumed to be *destitute of strength in tension.* Hence the distribution of vertical compressive stresses from the upstream edge to the downstream edge across a horizontal layer of the dam resembles the distribution of water pressures on vertical or inclined planes, *i.e.*, "straight line" or uniform variation, resulting in so-called *triangular* or *trapezoidal* loading. (In Fig. 51 the resultant R of all the supporting forces on the base of the dam is resolved into vertical and horizontal components V and H. The effect of the water pressure P_H is to move the point of application of the resultant of P_H and W from vertically beneath the center of gravity of the dam (where it is when $P_H = 0$) to a point further downstream.) The larger the value of P_H (and this implies increase in the head of water) the further downstream along the base does the resultant act. It is to be noted that the *horizontal component* of the resultant is the total *shear* to be resisted. The weight of the dam (together with any vertical components of water pressure that may exist on inclined or stepped upstream face) is assumed to cause the *compression* in the masonry. Obviously, the greater P_H becomes the greater is the unit compression in the masonry at the *toe B* and the less is it at the *heel A*.

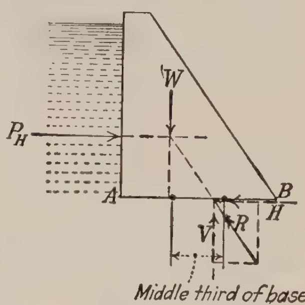


Fig. 51.

39. Confining attention now to the **compressive stresses vertically** in the masonry at the base AB or any horizontal section above it, the "pressure prism" representing the vertical reaction forces on the base takes the forms shown in Fig. 52. In this figure are shown several conditions that arise with different levels of water in the pond or for different designs of cross-section of the dam. The upper portion of the several sketches of Fig. 52 shows the dam considered as a free body with the resultant weight and the resultant total water pressure balanced by the reactions of the foundation, vertical and horizontal. Actually, of course, the horizontal shear is absorbed all along the base; and the support of the dam is spread over all (or most of)

the base, *not uniformly,* but as shown by the lower portions of the sketches *a* to *g* of Fig. 52.

Figure 52*a* shows a condition that might arise with *empty reservoir* with an improper design. The addition of the top portion to the triangle of masonry moves the center of gravity of the dam. If, as is usually the case, the center of gravity of the added masonry is upstream from the middle third of the base, this results in relieving the toe *B* from *compression* and causing *tension.* This is true because the resultant of the vertically upward reaction pressures must be colinear with the downward resultant. But the upward resultant acts through the center of

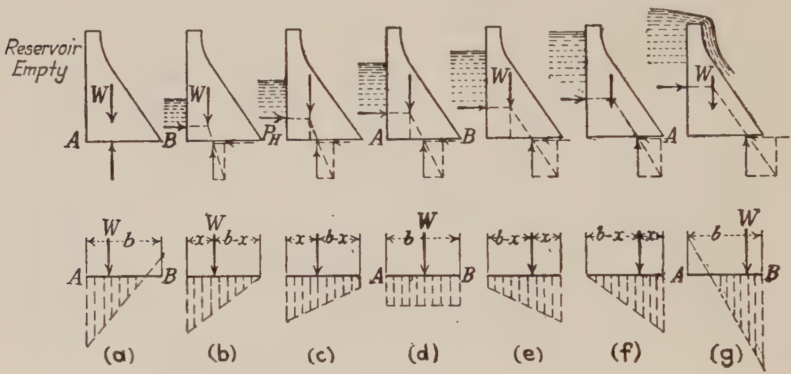


FIG. 52.

gravity of the reaction pressure prism, whose side is triangular. Only when the resultant comes within the *middle third* of the base as in Fig. 52*c*, *d*, and *e* is the whole base subjected to compressive stresses. But it has already been assumed (in accordance with the results of experience) that masonry cannot successfully and permanently resist tension. The remedy is discussed in Par. 33 in connection with Figs. 48 and 49.

Figure 52*b* shows the limiting position where the resultant of water pressure and weight may be allowed to cut the base if it is desired to keep the whole base under *compressive stress* but diminishing to zero at the toe *B*. A dam should be so designed as to keep the resultant a little *inside the middle third* of the base (and of any horizontal section above the base) when the reservoir is

entirely empty. Otherwise there will be tension at the toe *B* whenever the water level falls below such a level as sketched in (*b*).

For the usual high-water levels in the pond back of a properly designed dam, Figs. 52*c*, *d*, and *e* show how the distribution of vertical compressive stresses on the base changes as the water rises in the reservoir, shifting the resultant from nearer the heel *A* towards the toe *B* more and more. (The student should note that the distribution of compressive stresses is non-uniform except for the particular case where the resultant cuts the base at midpoint, as shown in *d*. Only for this case is the compression uniform.)

Figure 52*f* shows the resultant at the downstream edge of the middle third of the base, a condition with zero stress at the heel *A* of the dam.

Figure 52*g* shows a condition that may arise in an extreme flood, or owing to inadequate design, the resultant of *P* and *W* passing outside of the middle third on the downstream side, thus causing *tension* at the heel *A*.

Good design calls for condition *b* of Fig. 52 as the extreme limiting condition of the stresses for dam with empty reservoir. For economy of material in a high dam, this involves a batter to the lower portion of the upstream face, as shown in Fig. 49 (see also Figs. 56 and 57). Especially does good design call for *f* as the corresponding extreme limit of stress distribution for highest water level and greatest overturning forces.

40. Summarizing the reasons for not allowing the resultant to approach the downstream edge of the base:

1. *Tension* in the masonry *must be avoided.* This applies to both the upstream face of the dam with full reservoir and to the downstream face with empty reservoir. With the occurrence anywhere of tension, horizontal cemented joints are liable to open in the course of time, or cracks are liable to form in places of weak bond, even in the best of massive concrete. This would allow the pressure of the penetrating water to exert an upward force on the masonry above the crack, thus assisting the forces tending to overturn the dam. Such cracks would also partly destroy the monolithic character of the dam, which is assumed

in the calculations, and which is approximately attained only by great care in the course of construction.

Compressive stresses, on the other hand, tend to preserve the sealing effect and the bond of the masonry joints, thus keeping out not only water pressure, but also possible frost pressure and organic growths, as well as reducing leaching action, leakage, and undue deterioration generally.

2. (*a*) As large a cross-section as possible should be available to resist compressive stresses. If part of the horizontal cross-section is subjected to tension, the average unit compression on the rest will be larger than otherwise. (*b*) Also the *unavoidable variation* in unit compressive stress from point to point within the dam should have as great a distance as possible in which to change from minimum to maximum, so that the *rate of change of stress may be as small as possible.* Then changes in water level and in temperature will have less effect. (*c*) Finally, the *minimum* compressive stress should be kept *somewhat above zero* so that the maximum may be as small as practically feasible, and so that a certain small margin may prevail against the tendency towards tension.

Calculation of Vertical Compression Stresses in the Masonry of Gravity Dams

41. Resultant outside Middle Third.—If, according to Navier's rule, the compression varies *uniformly* from maximum to minimum (straight-line distribution when represented graphically), then if the resultant falls outside of the *middle third* of the base, or at the edge of the middle third (Fig. 52*a*, *b*, *f*, and *g*) the distribution of the stresses being then *triangular*, the *maximum unit compression is just twice the mean.* The width of base subjected to compression is, for these cases, $3x$, where x is the distance from the edge under greatest compression to the point where the resultant cuts the base. Only for the particular cases b and f is $3x = b$. For all cases with resultant in either *outside third*, such as a and g, $3x$ is less than b.

The mean unit compression is $\frac{V}{3x}$, where V is the vertically *downward* component of the total resultant per unit length of dam (perpendicular to the cross-section).

42. Resultant within the Middle Third.—If the resultant falls within the middle third of the base (c and e, Fig. 52), all parts of the base are subject to compressive stress with trapezoidal distribution as shown. The value of the maximum stress is of importance, since in no case should it be permitted to exceed a safe value.

Let the minimum value per unit area $= c_{\text{min.}}$ at one edge of the base and the maximum value $= c_{\text{max.}}$ at the other edge (Fig. 53), and let c_{mean} = the average value of the stress over the base. Consider a unit length of dam perpendicular to the cross-section. Then $c_{\text{mean}} = \frac{V}{b}$ where V = vertical component of the total resultant force downward and b the width of the base. There are several methods of calculating the values of $c_{\text{max.}}$ and $c_{\text{min.}}$ in terms of c_{mean} or in terms of the total vertical reaction. Two methods will be given by examples, first, by dividing the trapezoid representing the distribution of pressures on the base into a triangle and rectangle, and, second, by considering this stress diagram for the masonry to be made up of a combination of direct load and flexural stress.

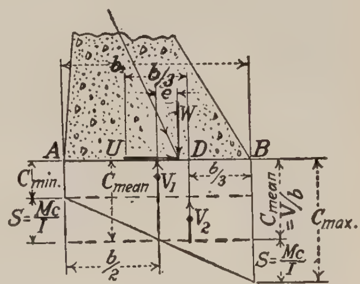


FIG. 53.

Example: Assume that the base of the section of a dam (any horizontal section) is 100 ft. wide and that the resultant of the vertically upward reaction forces V (1,080,000 lb.) acts at a point 40 ft. from the toe B of the dam (Fig. 53). It is required to find the maximum and minimum vertical compression stress in the masonry.

Dividing the trapezoid, representing the pressures on the base, into a rectangle and triangle and taking moments about any convenient point, say, in this case, the center of the base, the moment of the whole trapezoid = the moment of its parts.

$$\text{Therefore } V_1 \times 0 + V_2 \times \left(\frac{100}{2} - \frac{100}{3}\right) = V \times 10$$

$$V_2(50 - 33.3) = 10V \quad \therefore V_2 = 0.6V$$

But $V_2 = \left(\frac{c_{max.} - c_{min.}}{2}\right) \times 100$ and $V = \left(\frac{c_{max.} + c_{min.}}{2}\right) \times 100$

$$\therefore (c_{max.} - c_{min.}) \times \frac{100}{2} = 0.6 \times \left(\frac{c_{max.} + c_{min.}}{2}\right) \times 100$$

$$\therefore c_{max.} - c_{min.} = 0.6(c_{max.} + c_{min.})$$

$$0.4c_{max.} = 1.6c_{min.} \quad \therefore c_{min.} = \frac{1}{4}c_{max.}$$

Also $$\left(\frac{c_{max.} + c_{min.}}{2}\right) \times 100 = V = 1{,}080{,}000$$

$$\therefore c_{min.} = 1{,}080{,}000 \times \frac{2}{100} \times \frac{1}{5} = 4320 \text{ lb. per sq. ft.}$$
$$= 30 \text{ lb. per sq. in.} \quad \textit{Ans.}$$

And $c_{max.} = 120$ lb. per sq. in. *Ans.*

If it is desired to have c_{min} approach nearer to zero stress, say, one-eighth of a new $c_{max.}$, it is obvious the resultant would approach nearer to the downstream edge of the "middle third." For empty reservoir the variation of pressures is reversed, but the same principles apply.

The **second method** is to consider the stress as in the mechanics treatment for a short column. The maximum and minimum stresses may be divided into: (1) a stress due to direct load and (2) a stress due to a bending moment. The stress due to the total reaction, if the resultant acts at the center, $= \frac{V}{b}$, but as the resultant is eccentric a distance e, a flexural stress is induced $= \frac{Mc}{I}$ where $M = V \times e$, $c = \frac{b}{2}$ ft., and $I = \frac{1}{12}(1) \times b^3$.

$$\text{Therefore } c_{max.} = \frac{V}{b} + \frac{Mc}{I} = \frac{V}{b} + \frac{Ve \times \frac{b}{2}}{\frac{1}{12}b^3}.$$

Therefore

$$c_{max.} = \frac{V}{b}\left(1 + \frac{6e}{b}\right) \tag{19}$$

and

$$c_{min.} = \frac{V}{b}\left(1 - \frac{6e}{b}\right). \tag{20}$$

For the problem above $c_{\text{mean}} = \dfrac{1{,}080{,}000}{100} = 10{,}800$ lb. per sq. ft.

$$c_{\text{max.}} = \frac{1{,}080{,}000}{100} + \frac{1{,}080{,}000 \times (50 - 40) \times 50}{\frac{1}{12} \times (100)^3}$$

$c_{\text{max.}} = 10{,}800(1+.6) = 17{,}280$ lb. per sq. ft. $= 120$ lb. per sq. in. *Ans.*

$c_{\text{min.}} = 10{,}800(1 - 0.6) = 4320$ lb. per sq. ft. $= 30$ lb. per sq. in. *Ans.*

It should be noted that when the resultant cuts the downstream edge of the middle third $\left(e = \dfrac{b}{6}\right)$ the maximum stress is twice the average and the minimum equals zero, and that when e is less than $\dfrac{b}{6}$ a trapezoidal distribution of stresses exists. If the resultant falls outside the middle-third points this method is not applicable unless tension in the masonry be allowed, which is improper except for special cases of reinforced construction for the reasons previously stated.[1]

43. Maximum Compression as Due to Weight of a Column of Masonry of Same Height as the Dam.—In Fig. 54 are shown the triangular loadings and the opposing triangular reaction distri-

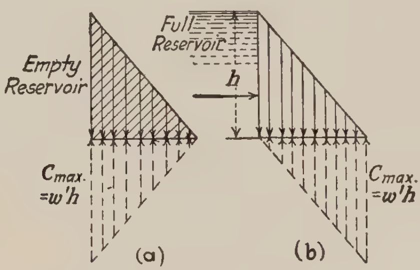


FIG. 54.

butions for the two cases of empty reservoir and full reservoir, the resultants being, respectively, at the upstream and the downstream edges of the middle third of the base.

It is seen that in each case the maximum unit compression is that due to a column of masonry of height h, or $c_{\text{max.}} = w'h$.

It therefore appears that, if the limiting position of the result-

[1] Another form for Eq. 19 is given in Problem 13, p. 68, where n is the ratio e/b.

ant be the edge of the middle third, and if the cross-section of the dam is only a slight modification of a right-angled triangle, then the maximum unit compression in the masonry, as found from $c_{max.} = w'h$, will be about 1 *lb. per sq. in. for each foot height of dam,* because w' for concrete and rock ranges ordinarily from about 135 to 156 lb. per cu. ft., causing a compression from 0.94 to 1.08 lb. per sq. in. per foot height. (Note that the thickness of the dam does not enter into this calculation. Of course, it is involved in the proper size to keep the resultant within the middle third of all horizontal sections of the dam.)

Limit of Height of Gravity Dams.—The crushing strength of granites, sandstones, and limestones ranges from 3000 to more than 15,000 lb. per sq. in. The ultimate strength of concrete made of these materials (and used alone in the dam or as mortar in "cyclopean" construction) averages from 2000 to 4000 lb. per sq. in. Hence it seems reasonable to conclude that a height of 600 ft. is practically feasible with ordinary good material and with ordinary assumptions as to factor of safety in design, provided good bedrock foundation and high-class construction methods are assured. For quite heavy masonry of specific gravity = 2.7, or $w' = 168.5$ lb. per cu. ft., this would mean a vertical compression of 700 lb. per sq. in. for the basal triangular cross-section and resultant at downstream edge of middle third, still allowing a good factor of safety. The actual combined stresses near the toe in the masonry are greater than the vertical stresses only, and for high dams a more complete mathematical investigation than outlined here is desirable.

The matter of the exact effect on the stresses within the dam of various possible distributions of uplift pressures beneath and within the dam requires a more elaborate analysis. In general, it is much better to take all possible precautions in design and construction than to deal with unknown forces. Most engineers, however, are agreed that uplift water pressure is likely to occur to some extent and should be allowed for. (See recent textbooks on masonry dams.)

44. Problem: What is the minimum thickness b of the base of a dam 200 ft. high (its preliminary basal triangular section), water at level of the top of the dam, if water pressure *beneath*

the dam is assumed to range uniformly from full value hw at heel to zero at toe; also assuming ice thrust of 50,000 lb. per lineal ft.? The resultant of the reacting pressures of the masonry or rock formation in this preliminary study is assumed to cut at the downstream edge of the middle third of the base.

Hints.—(See "free-body" sketch, Fig. 55.) Take length = 1 ft. perpendicular to sketch. Take moments about point O, so as to eliminate the moment action of the reaction H and V. Express all forces as multiples of w, in this case w' being $= 2.5w$. This will make numerical values in equations of the solution smaller, and will avoid necessity of multiplying terms by value of w because it cancels out. Also keep forces and lever arms in terms of b and h until the end, so as to have a result useful for other heights with same specifications. *Ans.* $b = 0.815\sqrt{h^2 + 4800}$. For $h = 200$ ft., $b = 173$ ft. or $b = 0.865h$. Without ice thrust, $b = 0.815\,h$. For lighter masonry, say $w' = \frac{9}{4}w$, or 140.5 lb. per cu. ft., and without ice thrust, $b = 0.895h$. This calls for $b = 179$ ft. for $h = 200$ ft., thus adding 9.8 per cent to the volume for the lighter masonry under the same conditions.

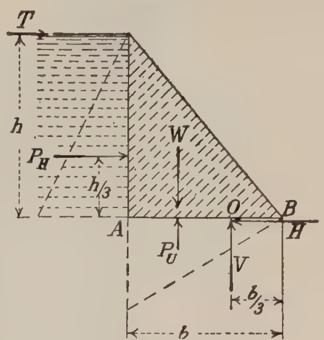


FIG. 55.

Another Problem.—Show that, with no uplift pressure beneath a triangular dam, assuming only water pressure on the upstream face, $b = \sqrt{\frac{1}{N}}\,h$, where N is the ratio $\frac{w'}{w}$.

45. Figure 56 shows the **maximum section** of the **Ashokan Dam** in the Catskills, New York, part of the water supply system of New York City. This dam has built within it a drainage system to intercept the water so as to prevent seepage through the dam or accumulation of water under pressure within the dam.

Taken as a whole, this dam is seen to depart only slightly from one with the ideal cross-section of a right-angled triangle, as remarked on page 49. The student should try to find the assumptions that will give the triangle of dimensions shown in

Fig. 56, h = 200 ft., b = 180 ft. Figure 57 shows the **maximum cross-section** of the **Hetch Hetchy Dam,** part of the new water supply system of San Francisco, Cal. The basal triangle for this dam has a base about seven-tenths of the height, as com-

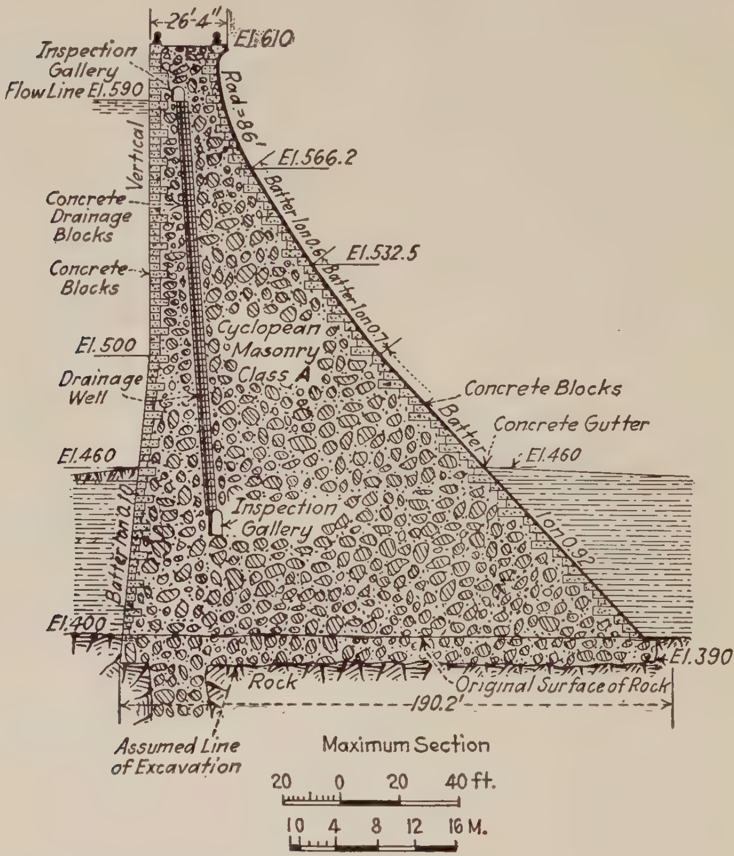


FIG. 56.—Ashokan Dam in the Catskills near Kingston, N. Y. (Eng. News, Vol. 57 (1907), p. 519.)

pared to a ratio 0.9 for the triangle of the Ashokan Dam.[1] Note, however, that this dam is curved in plan, which adds to the

[1] A common method of final design proceeds by aid of graphical methods, starting with the top block of masonry and taking some ten (more or less) intermediate horizontal sections between top and foundation. The width and disposition of masonry at each temporary base is made such as to keep the resultant lines of pressure always somewhat within the middle third, for both full and empty reservoir.

stability. Usually such curvature is taken as offering resistance to the extraordinary forces, such as earthquake shocks, impacts, etc. (see p. 50). Often the main section of an arched dam is still kept at full "gravity section." There are some notable

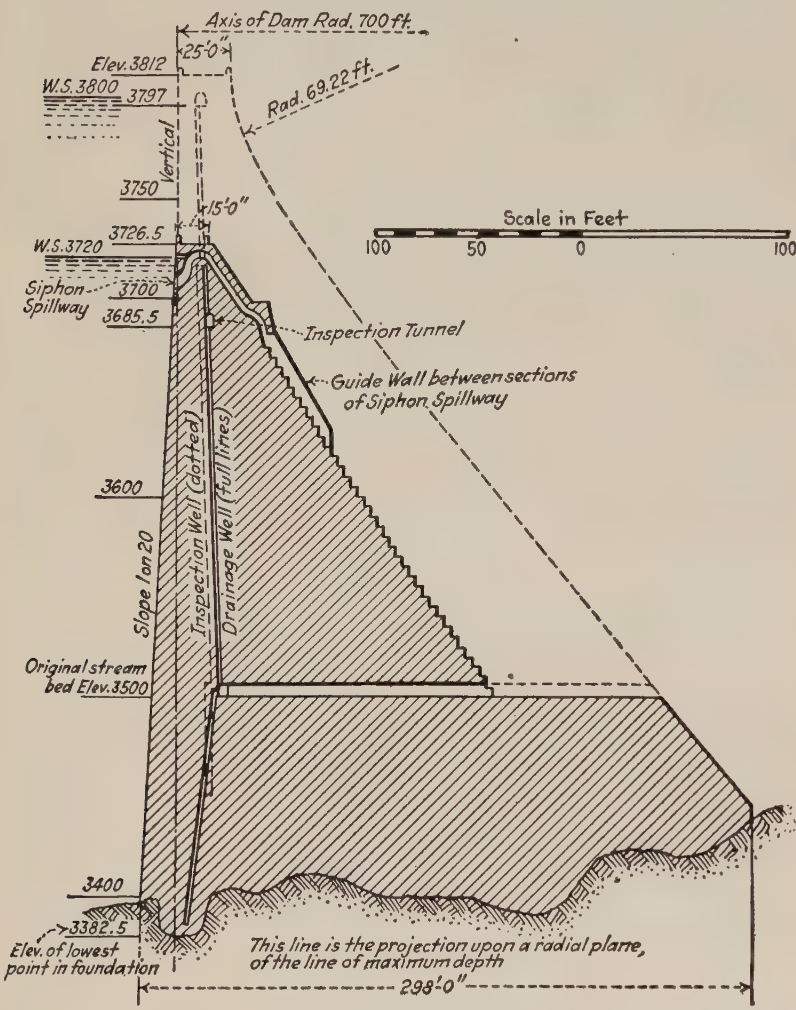


FIG. 57.—Hetch Hetchy Dam, California (Trans. Am. Soc. C. E., Vol. 85 (1922), p. 879.)

curved dams, however, that resist the ordinary water pressure only partly by gravity, the rest being by horizontal arch action with thrust against the walls of the gorge.

46. For circular arch dams built between gorge walls and depending for stability on the strength of the arch to resist a horizontal compressive stress, and for **multiple-arch dams** built as inclined spans between pier abutments, see Hoop Tension (p. 34) for ideal thin-arch action. For full analysis of stresses in thick arches, especially near the abutments and base, the elastic theory is involved and the student is referred to treatises on this subject.

For the extra *dynamic pressure* in cases where water flows over the top of a dam, see Pressure Due to Deviated Flow (p. 284). However, such a matter as the extra overturning effect due to partial vacuum sometimes formed between the overfalling sheet of water and the dam and the matter of vibrations due to pulsations in such a partial vacuum all require careful consideration and such provisions as experience dictates.[1]

Problems

1. If masonry weighs $w' = (5/2)w$ and $b = \frac{3}{4}h$, compute, for trial section shown in Fig. 50, the *factor of safety* against *sliding* if the coefficient of friction between dam and foundation is 0.6. *Ans.* 1.125.

2. If a shearing stress along a horizontal section of 100 lb. per sq. in. is not to be exceeded, compute the maximum value for h in feet for the dam shown in Fig. 50 when $w' = \frac{9}{4}w$ and $b = 0.7h$. *Ans.* 323 ft.

3. If the vertical compressive stress at A on the base of a dam (Fig. 52*e*) is just 5 per cent of that at B, compute where the resultant cuts the base. $x = ?$, first in terms of b, then in feet for $b = 50$ ft.

Ans. $0.349b = 17.5$ ft.

4. If resultant of P and W cuts the base at the midpoint, compute the height of the water h' in terms of h, and then in feet for $h = 100$ ft. (see Fig. P.33).

Ans. $0.928h = 92.8$ ft.

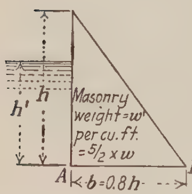


FIG. P.33.

5. The trial section of a triangular dam (Fig. 47) 90 ft. high has been designed so that when the pond is full of water, upward water pressure not considered, the resultant force on the base of dam is supposed to cut the base at the downstream edge of the middle third. Width of base $b = 60$ ft., $w' = \frac{9}{4}w$. Consider length perpendicular to sketch equal 1 ft. (*a*) Show that the design is correct or incorrect for the above assumption. (*b*) What is the maximum

[1] For excellent practical notes on forces acting on and construction of masonry dams, see paper by C. L. HARRISON and accompanying discussions by twenty engineers in *Trans. Am. Soc. Civil Eng.*, vol. 75, 1912, pp. 142–225.

compressive stress vertically in the masonry? (*c*) What are the amount and the nature of the stress in the concrete at the upstream corner of the base of the dam? *Ans.* (*b*) 87.7 lb. per sq. in.
(*c*) zero.

6. In the dam of problem 5, the ice thrust at the top of the dam, amounting to 50,000 lb., was neglected in designing. What effect will be produced on the following conditions: (*a*) Where will the resultant cut the base? (*b*) What is the maximum compressive stress vertically in the masonry? (*c*) What are the amount and the nature of the stress in the concrete at the upstream corner of the base of the dam?

Ans. (*a*) 8.1 ft. from toe.
(*b*) 217 lb. per sq. in.
(*c*) Joints open; masonry cannot take tension.

7. Canal retaining wall with river adjoining (Fig. P.34). (*a*) Find width of base ($b = 3' + x'$) so that resultant passes through downstream edge of middle third. (*b*) Find maximum compressive vertical stress in the masonry (pounds per square inch). (*c*) What is the unit shear at the base (pounds per square inch)? (*d*) If the canal be emptied, river as before, what is the maximum vertical compressive stress in the masonry (pounds per square inch)? *Ans.* (*a*) 6.06 ft.
(*b*) 22.4 lb. per sq. in.
(*c*) 3.86 lb. per sq. in.
(*d*) 20.4 lb. per sq. in.

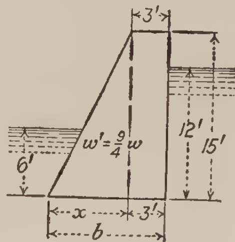


FIG. P.34.

8. For the dam shown in Fig. P.33, how high would be the water in the reservoir to cause a vertical compressive stress of 50 lb. per sq. in. in the masonry at *B*? Given $h = 60$ ft., $w' = \frac{9}{4}w$, and $b = 40$ ft.

Ans. 57.0 ft.

9. Considering shearing stress only, and a basic triangular gravity dam (Fig. P.35) with $b = \frac{3}{4}h$, (*a*) at what depth (in feet) is the shear 100 lb. per sq. in.? (*b*) For this same unit shearing stress if there is ice thrust = 50,000 lb. per ft., what additional thickness (in feet) is necessary on account of the extra shear? Does this depend on *h*?

Ans. (*a*) 346 ft.
(*b*) 3.47 ft.; does not depend on *h*.

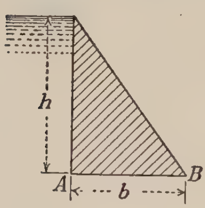


FIG. P.35.

10. Considering resistance to sliding, shape as in Fig. P.35, taking $b = (3/4)h$, and $w' = (9/4)w$ with water pressure beneath dam varying from full head at *A* to zero at *B* (thus reducing the effective weight), show (by numerical value) why there is danger.

Ans. Friction coefficient = $16/15$, which is greater than 1.

11. Sketch as in problem 9; find *b* in terms of *h* for $w' = (5/2)w$, no ice

thrust, water pressure beneath dam as in problem 10, for resultant $0.35b$ from B. *Ans.* $b = 0.838h$.

12. Why is a right-angled triangle theoretically the most economical basic cross-section (*i.e.*, to resist effect of water pressure)?

Ans. Center of gravity of dam should be as near upstream edge of middle third as possible.

13. Ideal triangular dam (Fig. P.35) $w' = \frac{5}{2}w$, ice thrust = 62,400 lb. per ft. (= $1000w$), water level at crest of dam. No water pressure beneath base. Find b in terms of h if resultant cuts base so that $n = 0.10$, n being the same as in $\frac{c_{max.}}{c_{mean}} = 6n + 1$. *Ans.* $b = \sqrt{\frac{h^2}{2} + 3000}$ ft.

14. Resistance to sliding is being considered for a dam, and the maximum safe coefficient of friction is taken = $\frac{1}{3}$. Dam has vertical upstream face of height h, water even with top, base width = $(\frac{3}{4})h$, $w' = (\frac{9}{4})w$. Buoyancy of water pressure beneath dam reduces effective weight of masonry, pressure varying from full at heel to zero at toe. *Shape* of cross-section, as to width of top of dam, depends on the necessary area A of cross-section for sliding resistance. Compute minimum safe value of A in terms of h, and then decide whether a triangular section, apex at water surface, would suffice, or, if not, the proper top width of a trapezoidal section in terms of h.

Ans. Top width demanded = $(\frac{11}{12})h$. This abnormal result (top wider than base) emphasizes the economic undependability on friction alone.

15. If only 25 per cent of horizontal sections through dam can be relied on as rock keys (Fig. P.36) to bring into play true shearing stress, how many feet high may a dam be built without exceeding a shearing stress on the effective keys or dowels of 200 lb. per sq. in.? Dam is assumed triangular with $b = (\frac{3}{4})h$.

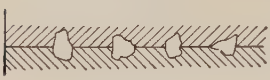


Fig. P.36.

Ans. 173 ft.

16. Explain clearly the circumstances under which tension in masonry of a gravity dam is related to eccentricity of the resultant force.

17. If the resultant cuts base of width b at a distance $\frac{b}{18}$ inside middle third, what is the ratio of $c_{max.}$ to $c_{min.}$? *Ans.* $c_{max.} = 5c_{min.}$

18. To insure good design for *empty reservoir* and dam as in sketch (Fig. P.37), at what height h_1 below level of apex of triangle should batter on upstream face begin? (h_1 (in feet) = ?) Give reasons for the assumption.

Ans. $33\frac{1}{3}$ ft.

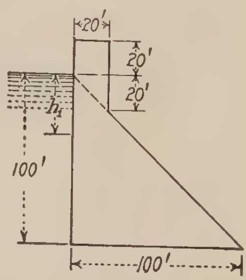


Fig. P.37.

19. For dam with $h = 100$ ft., $b = 0.6h$, and $w' = (\frac{9}{4})w$, find where the resultant cuts base (feet from B, the toe of dam). Water pressure on upstream face only.

Ans. 15.3 ft. from B.

20. Triangular dam, vertical upstream face, water up to the top. No water pressure beneath dam. Masonry weighs $w = \frac{5}{2}w$. For base $b = \frac{4}{5}h = 24$ ft., the height h being 30 ft. (*a*) Where does the resultant of the forces cut the base? (*b*) Find maximum and minimum vertical stresses in foundation.

Ans. (*a*) 11 ft. from toe.
(*b*) 20.3 lb. per sq. in.
12.2 lb. per sq. in.

21. Assume that there is water pressure beneath a dam, varying uniformly in intensity from full hydrostatic value at A to zero at B (Fig. P.38). Take w' (of masonry) $= \frac{9}{4}w$, or 140 lb. per cu. ft. Compute width of base b, so that the resultant of forces will cut base $0.38b$ from B. *Directions:* Sketch dam as a free body, indicating all resultant forces necessary to hold the dam in equilibrium, including the H and V components of the anti-resultant (or foundation reaction pressure against dam). In dealing with gravity forces, consider the cross-section of the dam as made up of two parts, a rectangle and a triangle (see sketch). Do not combine these. Take unity length perpendicularly to sketch. Express each force in terms of w, which will then cancel out in the moment equation. Reduce each expression for length of moment arm to simplest form before using it in moment equation. Sum moments about point where resultant cuts base.

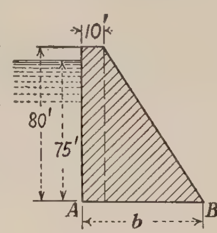


FIG. P.38.

Ans. $b = 61.6$ ft.

22. The triangular cross-section for a gravity dam (Fig. P.35), vertical upstream face, is known as the theoretically most economical section. Moreover, most high dams actually built approximate this shape. Compute the base width b in terms of h, *i.e.*, find the fraction or multiple of h that b is, for resultant at downstream edge of middle third, for the following cases: (*a*) water pressure aginst upstream vertical face only; $w' = \frac{9}{4}w$ (140 lb. per cu. ft.); (*b*) the same, but with $w' = \frac{5}{2}w$. Ans. (*a*) $\frac{2}{3}h$.
(*b*) $0.633h$.

23. (*a*) For cross-section as in problem 22, and case (*a*), *i.e.*, for water pressure against upstream face only and for resultant cutting base at downstream edge of middle third, how does the maximum vertical compressive stress in the masonry vary with the height of the dam? (Express $c_{max.}$ in terms of h and other values.) (*b*) For $h = 100$ ft., 200 ft., 300 ft., find maximum compressive stress in masonry in pounds per sq. in. for conditions as in problem 22.

Ans. (*a*) $p_{max.} = hw'$
(*b*) 97.5 lb. per sq. in.
195 lb. per sq. in.
292 lb. per sq. in.

24. Make a sketch showing the distribution of vertical compressive stress

in the masonry along the base of a dam when the resultant comes at any point in the downstream half of the middle third of the base.

25. Using the sketch called for in problem 24 as a basis, locate where the resultant cuts the base when the vertical compressive stress at toe is just nine times that at heel.

26. Compare two dams of triangular cross-section and of the same material. If the second has depth of water, height of dam, and width of base, each twice as great as the first, is the unit compressive stress at the toe in the second dam less than, greater than, or equal to, that in the first dam? Why (in a few words)?

27. Will a dam (gravity type) surely overturn if the water rises 10 per cent higher than the level which makes resultant cut the base at the downstream edge of middle third? *Ans.* No.

28. *Trapezoidal Dam with Collapsible Flashboards.*—Conditions shown in Fig. P.39 with added upstream slope 1 horizontal to 5 vertical. Water

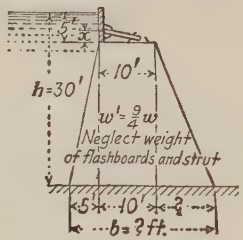


Fig. P.39.

level 5 ft. up on vertical flashboard. With 50 lb. friction between top of dam and bottom of flashboard, *i.e.*, per foot of length, find: (*a*) Distance x, up from bottom of flashboard, that hinge should be placed for impending overturning of flashboard. (*b*) Base width b such that resultant pressure will act at downstream edge of middle third of base. (*Hint:* For simplicity, sum moments about this point.) (*c*) Maximum vertical compressive stress per square inch of masonry. (*d*) Unit shear in concrete at base (pounds per square inch).

Ans. (*a*) 1.566 ft.
(*b*) 21.4 ft.
(*c*) 39.3 lb. per sq. in.
(*d*) 9.1 lb. per sq. in.

CHAPTER IV

AIR AND GASES

STATIC CONDITIONS

47. The subject of **air and gases** is here reviewed only in its relations to the necessities of general engineering practice, without going into such internal changes as occur, for instance, within the cylinder of an air compressor or of a gas engine during a stroke of the piston. The topics of adiabatic compression and expansion, and of the power involved, are taken up in texts on heat engines and auxiliaries. (The *flow* of air and gases through pipes is discussed on p. 281.)

For the general physical properties of gases the student should review a good physics textbook.

Considering a definite mass of gas (the same mass in the "two states"), let the subscript 1 refer to its first condition or "state" as to pressure, volume, and temperature; and the subscript 2 to its second condition or "state."

Boyle's or Mariotte's law is $p_1V_1 = p_2V_2$, isothermal, or with temperature constant, or

$$\frac{p_1}{p_2} = \frac{V_2}{V_1}. \tag{21}$$

In words, *absolute pressures vary inversely as the volumes* occupied.

$$\textit{Charles' or Gay-Lussac's law} \text{ is } \frac{V_1}{T_1} = \frac{V_2}{T_2} \tag{22}$$

isobaric, or with pressure constant, or $\dfrac{V_1}{V_2} = \dfrac{T_1}{T_2}$.

In words, *volumes vary directly* as the *absolute temperatures.*

Combining[1] Eqs. (21) and (22), the result is the general law,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}. \tag{23}$$

[1] Introducing an intermediate stage, subscript x, first let the pressure remain constant and the temperature change to T_x $(=T_2)$, then $\dfrac{V_1}{V_x} = \dfrac{T_1}{T_2}$. Now let the pressure change and the temperature remain constant, then $\dfrac{V_x}{V_2}\left(=\dfrac{p_2}{p_x}\right) = \dfrac{p_2}{p_1}$. Solving this for V_x and substituting in previous equation, the general equation (23) results.

Also, since the total weight of a given mass of gas is constant, no matter what change of pressure, volume, or temperature it undergoes, *i.e.*, since $W_1 = W_2$, or $V_1w_1 = V_2w_2$,

$$\frac{p_1}{w_1T_1} = \frac{p_2}{w_2T_2}. \tag{24}$$

Note especially that in using these formulas:

1. *Pressures* (p_1 and p_2 per unit area) must be numerically expressed in *absolute* units, *i.e.*, referred to complete vacuum as zero (see Par. 10).

2. *Volumes* (V_1 and V_2) must be those of the *same mass of gas in the two "states"* under comparison. This is particularly to be remembered when the "second state" of the gas is caused by adding or removing some gas to or from the mass present in the "first state" (see example 2, p. 74).

3. *Temperatures* (T_1 and T_2) must be *absolute,* and not referred to ordinary zero.

Absolute zero is 273°C. below zero Centigrade, which is also 273°C. below freezing. On the Fahrenheit scale absolute zero is 460° below zero Fahrenheit, and this is 492°F. below freezing, so that the standard freezing temperature is 273°*C. absolute* or 492°*F. absolute.*

48. Weight of Air and of Other Gases.—At standard atmospheric pressure of 760 mm. of mercury (absolute or barometer column), or 14.7 lb. per sq. in. *absolute,* and at freezing temperature (0°C., 32°F.) air weighs 0.0808 lb. per cu. ft. This is $\frac{1}{772}$ of the weight of water at its greatest density.

From this it may be inferred that, at ordinary temperatures, a difference of elevation of about 800 ft. near sea level changes the water barometric height roughly about 1 ft. and the mercury barometer height about 1 in. (see Fig. 14). However, the case of the "free" atmosphere, of very great vertical extent, is exceptional as far as most of the air and gas problems confronting the engineer are concerned. Almost all masses of air and gas in engineering are not "free," but are isolated and confined. The pressures they exert are due *not to the weight* of the gas (except to a very slight and usually negligible extent), but principally to the "tension" or "springiness" of the gas. The case is somewhat like that of a coiled spring, that, without weighing much itself,

nevertheless does exert relatively great pressure against the surfaces that confine it.

TABLE I.—WEIGHTS OF GASES AT FREEZING TEMPERATURE AND 14.7 LB. PER SQ. IN. ABSOLUTE PRESSURE

	Lb. per Cu. Ft.		Lb. per Cu. Ft.
Hydrogen	0.0056	Carbon monoxide	0.0780
Helium	0.0112	Nitrogen	0.0784
Coal gas (average)	0.032 ±	Air	0.0808
Marsh gas (methane)	0.0447	Oxygen	0.0893
Ammonia	0.0476	Sulphureted hydrogen	0.0962
Natural gas	0.046–0.081	Carbon dioxide	0.1227
Acetylene	0.0725	Chlorine	0.2780

49. Hints on Gas Problems.—*Pressures* may be stated in any convenient units that give a measure of the intensity of pressure, provided, of course, that the same unit is used on both sides of the equation.

Instead of using pounds per square inch or per square foot, it may at times be more convenient to use barometer heights in water or in mercury, or number of atmospheres, or total pressure against a piston or a cylinder head, etc.

Volumes may sometimes be expressed more conveniently in such relative measures as, for instance, "so many inches length" of cylinder; or in fractions or multiples referring to a temporary unit of some total volume or total weight, rather than in the conventional cubic inch or cubic foot units. Of course, as with pressures, the same unit must be used on both sides of the equation.

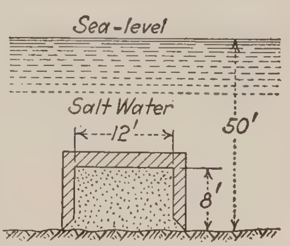


FIG. 58.

Example 1: It is deemed necessary, for the health and efficiency of the workmen, to renew the air in the circular caisson, submerged as shown in Fig. 58, every 20 min. The outside air temperature is 85°F., and inside the caisson it is 45°F. If 20 per cent of the free air drawn in by an air compressor is lost by leakage at pistons, clearance, valves, pipe line, and caisson, what must be the least allowable diameter of each of two pistons of the compressor for 100 strokes each per minute, 16-in. stroke?

Solution: The pressure of the air in the caisson, balanced by the water column, is $\frac{50 \times 64}{144} = 22.2$ lb. per sq. in. above atmospheric, or 36.9 lb. per sq. in. *absolute.* (The salt water of the ocean weighs about 64 lb. per cu. ft.) The volume of air in the caisson is 905 cu. ft.

Considering the volume of air filling the submerged caisson as the second "state" of a mass of gas initially part of the outside "free air," then, for $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$, or $\frac{14.7V_1}{545} = \frac{36.9 \times 905}{505}$, or

$$V_1 = \frac{545}{505} \times \frac{36.9}{14.7} \times 905 = 10.8 \times 2.51 \times 905 = 2450 \text{ cu. ft.}$$

Note how this arrangement of the computation shows the effects of temperature and of pressure on the resulting volume as a multiple of the known volume.

For the pressure of the "free air" 34 ft. (of fresh water), could have been used in which case the pressure of the submerged air would be $34 + \frac{50 \times 64}{62.4} = 34 + 51.3 = 85.3$ ft. Of course, the ratio of 85.3 to 34 is the same as 36.9 to 14.7.

This volume, 2450 cu. ft. in 20 min., requires of the compressors $\frac{2450}{20 \times 0.80} = 153$ cu. ft. per min. displacement, or $\frac{153}{200} = 0.765$ cu. ft. per piston stroke, or a total piston area $\frac{0.765}{16/12} = 0.574$ sq. ft., corresponding to a diameter of $10\frac{1}{4}$ in. *Ans.*

Example 2: Case where gas is added to (or subtracted from) another definite mass of gas.

A portable gas tank (1025 cu. in.) is "down" to 40 lb. per sq. in. pressure, and is to be pumped up to 80 lb. by a pump receiving gas under atmospheric conditions.

Question: How many cubic feet of gas must be pumped in if leakage and effects of "clearance" be neglected, the barometer reading 29 in.? The temperature inside the tank averages 90°F., with outside air temperature at 80°F., there being a little warming up of the tank temporarily, due to the pumping.

Solution: Evidently, there is a first and a second "state" to each of two masses of gas, *viz.*, the initial mass of gas in the tank

when at 40-lb. pressure, and, second, the added mass of gas pumped into the tank in order to bring the *combined* mass up to 80 lb. When dealing, as in this case, with the added mass, it must be noted that this added mass of gas does not occupy, in the final state, the whole tank, but only that part of the tank not occupied by the initial gas (now compressed into a smaller space).

To compute the volume within the tank occupied by the initial gas when further compressed from 40 to 80 lb. by the crowding in of new outside gas, the general Eq. (23) is applied to the mass of gas initially in the tank, as follows: $\frac{p_1' \times V_1'}{T_1'} = \frac{p_2' \times V_2'}{T_2'}$, whence $V_2' = \frac{p_1'}{p_2'} \times \frac{T_2'}{T_1'} V_1'$. Now V_1' (the initial volume of gas in the tank) = 1025 cu. in. Also p_1 is measured by $\frac{40}{0.49} + 29 = 110.6$ in. of mercury column absolute, and p_2' by $\frac{80}{0.49} + 29 = 192$ in. of mercury. Hence $V_2' = \frac{110.6}{192} \times \frac{550}{540} \times 1025 = 0.576 \times 1.0185 \times 1025 = 0.586 \times 1025$ cu. in. = 601 cu. in.

Then V_2, the added volume inside the tank, must = $(1 - 0.586)1025 = 0.414 \times 1025 = 424$ cu. in. Or, $1025 - 601 = 424$ cu. in.

Dealing next with the mass of gas pumped in, $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$, where p_1 is measured by 29 in. of mercury barometer height (*absolute*). V_1 is the volume of free air to be found; $T_1 = 540$°F. absolute; p_2 is measured by 192 in. of mercury column absolute; $V_2 = 424$ cu. in. as found above; $T_2 = 550$°F. absolute. Hence

$$V_1 = \frac{192}{29} \times \frac{540}{550} \times 424 = 6.62 \times 0.982 \times 424 = 6.50 \times 424 = 2750 \text{ cu. in.} \quad \textit{Ans.}$$

Comments: Note that in the above example, although the nominal or gage pressure has been exactly doubled (40 to 80 lb. per sq. in.), the absolute pressure is increased only 74 per cent. Hence it is incorrect to state: "Since we double the pressure we must force in just again as much gas." This would call for 512 cu. in. as compared with $\left(1 - \frac{110.6}{192}\right) \times 1025 =$

(1 − 0.576) × 1025 = 433 cu. in. if the small effect of temperature be neglected.

In this case the error in computed volume would be 18 per cent. In other cases the error might be much greater.

In short, when solving air and gas problems, computations must be made in terms of absolute pressures and temperatures. But in the *statement* of the problem the data are usually in customary units, and the final result should again be stated in customary units before the solution may be regarded as complete.

Problems

1. A cylinder containing helium gas shows gage pressure = 20 lb. per sq. in., barometer 25 in. When shipped to a place where barometer is 30 in., the gage shows gas pressure = 8 lb. per sq. in. Compute what fraction of the original mass of gas has escaped. Temperature constant.

Ans. 0.296.

2. Compressed-air surge tank (Fig. P.39*A*). In the C. U. Power Plant the pipe at *A* connects with and is 140 ft. below Beebe Lake. Assume static conditions. What thickness of steel is necessary merely to resist bursting at *D*? Allow 10,000 lb. per sq. in. for the steel.

Ans. 0.21 in.

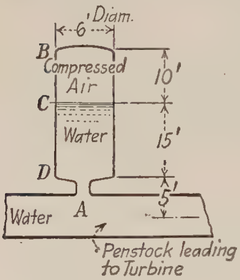


FIG. P.39*A*.

3. (See problem 2.) How many cubic feet of atmospheric air must be pumped into the upper part of the tank to lower the water level 10 ft. below *C*? Temperature inside and outside is 60°F.; barometer is 29 in. (assumed same at Beebe Lake and at power plant).

Ans. 1485 cu. ft.

4. A pressure chamber has an inside volume of 1000 cu. ft. The working pressure is 20 lb. per sq. in. above atmospheric temperature 50°F. What is the capacity of an air pump which changes the air in the chamber once in every 20 min., assuming that 40 per cent is lost by leakage? Atmosphere at 30 in. mercury pressure and temperature at 80°F.

Ans. 208 cu. ft. per min.

5. An oxygen cylinder contains gas at 3 atmospheres pressure at 60°F. If subjected to a heat of 1800°F. (red hot), what is the gas pressure (bursting pressure in pounds per square inch) if the volume increases 2 per cent?

Ans. 173.5 lb. per sq. in.

6. How many cubic feet per minute of free air at freezing temperature, barometer 20 in. (at high altitude), must be supplied by air compressors to a tunnel near the compressors, the temperature in the tunnel being 80°F. and the pressure just enough to balance a 60-ft. head of water (above atmos-

phere) coming down through the earth and rocks above the tunnel? There is to be one full replacement per hour of the air in the heading which is circular, averages 10 ft. in diameter, and is 1000 ft. long.

Ans. 4360 cu. ft. per min.

7. A balloon when filled is approximately a sphere 40 ft. in diameter. At sea level the confined gas is at standard atmospheric pressure. If no gas were allowed to escape, the balloon having been *completely* filled at the start, and the balloon rises to a height of 2 miles where the barometer reads 20 in., what would be the bursting stress per inch width of the fabric of which the balloon bag is made? Assume that balloon does not stretch.

Ans. 588 lb. per in.

8. A cubic foot of air at sea level, with barometer at 30 in. of mercury and temperature at 32°F., weighs 0.0808 lb. What does a cubic foot of air in Ithaca weigh when the temperature is 67°F. and the barometer 29 in. of mercury? *Ans.* 0.073 lb. per cu. ft.

9. A cylindrical tank 10 ft. in diameter and 10 ft. high contains air under a gage pressure of 125 lb. per sq. in. and at a temperature of 45°F. The valve is opened, allowing some air to escape, until the gage reads 100 lb. per sq. in. How much space will the escaped air occupy in the atmosphere where the pressure is standard and the temperature is 60°F.?

Ans. 1378 cu. ft.

10. A cylindrical tank 2 ft. in diameter and 10 ft. high contains air at 55°F. and a gage pressure of 30 lb. per sq. in. How many cubic feet of atmospheric air under standard pressure and a temperature of 35°F. will need to be forced into the tank so that the gage will read 40 lb. per sq. in. when the air in the tank is at a temperature of 45°F.? *Ans.* 22.8 cu. ft.

11. A spherical balloon (d = 40 ft.) is filled from a gas main in which the pressure-head is "6 in. of water" and the temperature is 40°F. The bottom of the bag is open to the atmosphere, the gas being introduced by a hose sticking up into the balloon. The gas is shut off as soon as the bag is "plump" full. Temperature of air (equals that of gas in balloon) is 80°F., and barometer reads 28 in. What volume of gas as in the main should a correct gas meter show delivered to the balloon? Give volume in cubic feet

Ans. 30,600 cu. ft.

12. A tank containing 100 cu. ft. of air at 2 atmospheres pressure and a temperature of 70°F. becomes heated during a fire and bursts. It is known that its bursting pressure is 250 lb. per sq. in. gage pressure. Neglecting the fact that the tank expanded, what was its temperature in degrees Fahrenheit when it burst? *Ans.* 4310°F.

13. A cubic foot of air at 80°F. and 80 lb. per sq. in. gage pressure weighs 0.475 lb. What does 1 cu. ft. weigh at freezing temperature and standard atmospheric pressure? *Ans.* 0.0808 lb. per cu. ft.

14. Assuming volume constant, what is the new pressure in an automobile tire which was 80 lb. per sq. in. in the garage at 65°F. and is then exposed to a temperature 10° below zero Fahrenheit? *Ans.* 66.5 lb. per sq. in.

15. If the pressure of 100 lb. per sq. in. of a volume V of gas is to be increased to 200 lb. per sq. in. by pumping additional gas into the tank, how many times V of additional gas at atmospheric pressure must be pumped into the tank, temperature remaining constant? *Ans.* 6.79V.

16. Compute how many per cent lighter air is at 100°F. than at 0°F., both at same pressure. *Ans.* 17.9 per cent.

17. How many cubic feet of free air at 30 in. barometer pressure and 100°F. must be pumped to fill a collapsed bag of 5-cu. ft. capacity, same being submerged 60 ft. below the surface of the water where the temperature is 40°F.? *Ans.* 15.5 cu. ft.

18. One cubic foot of air weighs 0.0808 lb. at standard pressure and freezing temperature. What is the weight of 3 cu. ft. of air at gage pressure of 80 lb. per sq. in. and at 80°F.? *Ans.* 1.425 lb.

19. How many cubic feet per minute of free air should be specified as the capacity of an air-compressor plant, allowing 50 per cent over capacity, *i.e.*, above the theoretical capacity (to take care of leakage, pipe friction, etc.), to supply 1000 cu. ft. per min. at 80 lb. per sq. in. gage pressure (to operate rock drills), at an altitude of 7500 ft. above sea level where the barometer reads 20.5 in.? *Ans.* 13,430 cu. ft. per min.

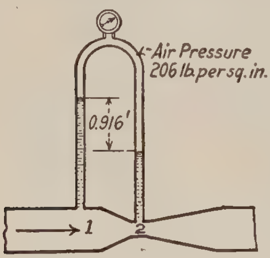


FIG. P.40.

20. The upper portion of a differential manometer (or water-column gage) shown in Fig. P.40 contains compressed air at 206 lb. per sq. in. (by common gage). Under so high a pressure the weight of the air is appreciable and is to be taken into account. Temperature is 70°F. For the gage difference 0.916 ft. shown, find the pressure-head difference between 1 and 2 in the Venturi meter. Air weighs 0.0808 lb. per cu. ft. at freezing temperature and 30 in. barometer. Water at 70° weighs 62.3 lb. per cu. ft. *Ans.* 0.9 ft. water

21. Diving bell or caisson, shape that of frustum of a circular cone, closed flat top, open bottom (Fig. P.41). Volume of cone = base × one-third height. Caisson is lowered from the outside air into salt water in upright position shown in sketch. Outside air at 90°F. and 30 in. barometer. Salt water at 40°F. to which submerged bodies cool. Neglect weight (not pressure), of air; also neglect thickness of shell. (*a*) How deep below the water surface is the bottom of the caisson when the water level has risen 4 ft. inside? (*b*) What is the necessary weight of the caisson for equilibrium in (*a*) *i.e.*, no lifting tendency and no downward pull on chains? *Next*, if air is pumped in so as to lower water level inside to the bottom edge of caisson, what is the necessary weight for equilibrium? The caisson does not move. (*c*) How many cubic feet of

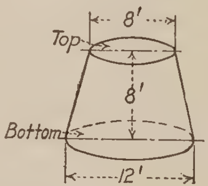


FIG. P.41.

"free air" (outside atmospheric air) must be pumped in to lower the inside water level down to the bottom edge of caisson?

Ans. (*a*) 46 ft. below surface.
(*b*) 16,350 lb., 40,850 lb.
(*c*) 1040 cu. ft.

22. Problems arising in driving tunnel in soft material by method of compressed air and shield (Fig. P.42). The air pressure in tunnel heading is maintained at the average pressure on shield face, due to overlying mud and water. The temperature in heading is 50°F. Outside air is at 20°F. and 30 in. barometer. For ventilation requirements and because of leakage out

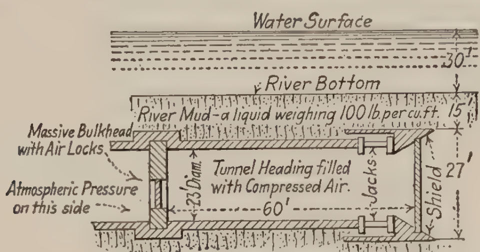


FIG. P.42.

of the air, it is necessary to pump in compressed air at the rate of one replacement per hour, exactly, for the air in heading between the lock bulkhead and shield face. (1 cu. ft. outside air = ? air inside.) (*a*) Find how many cubic feet of "free air" (*i.e.*, outside air) *per minute* must be supplied by air compressors. (*b*) Find total pressure against outside (right) of shield. (Should the bulkhead break or should there be an air "blow out" the shield and jacks alone would have to sustain this enormous pressure.)

Ans. (*a*) 1263 cu. ft. per min.
(*b*) 1355 tons.

23. A sunken submarine, weighing with its solid contents 250 tons (of 2240 lb. each), lies on bottom at an average depth of 200 ft. with all spaces and compartments filled with salt water weighing 64.0 lb. per cu. ft. Temperature at that depth is 38°F. The average specific gravity of boat and solid contents is 6.25 referred to fresh-water standard. Temperatures of air at surface is 30°C., and barometer indicates 29.6 in. of mercury. It is proposed to raise the boat by pumping in air to displace sufficient water from tight bulkheaded compartments. Taking into account the weight of air pumped in, find, for impending rising: (*a*) number of cubic feet of water that must be displaced by air; (*b*) number of cubic feet of free (surface) air that must be pumped in.

Ans. (*a*) 7370 cu. ft.
(*b*) 57,800 cu. ft.

24. (*a*) The temperature of the atmosphere is 65°F. and barometer reads 30 in. What space will 100 cu. ft. of surface or "free" air occupy when placed

in a collapsible air-tight bag and submerged in the sea to 20 fathoms depth where the temperature is 40°F.? (*b*) How many cubic feet of free air must be pumped into a canvas bag taken empty to this depth, so that it will be just "plump" full, the shape being a cylinder 4 ft. in diameter and 16 ft. long plus hemispherical ends? (*c*) The bag is provided with automatic air-escape valves that release air when the inside pressure is 5 lb. per sq. in. greater than the outside. If pumping be continued until air escapes through the relief valves, what is the tension per inch of bag fabric: (1) the hoop tension and (2) the tension in the ends? (*d*) In the last-mentioned state (inside air pressure 5 lb. per sq. in. greater than the sea-water pressure), what is the buoyant force of one such submerged bag? (Neglect weight of bag.) Is the buoyant effort the same near the surface? (*e*) How many cubic feet of submerged steel (structural) can the bag support?

Ans. (*a*) 20.6 cu. ft.
(*b*) 1143 cu. ft.
(*c*) 120 lb. per in., 60 lb. per in.
(*d*) 15,050 lb.
(*e*) 35.3 cu. ft.

25. A diving bell, square in cross-section, inside dimensions 15 by 15 ft. and 8 ft. high, together with air-lock shafts leading to the surface of the water, displaces 6500 cu. ft. salt water. It is resting on the bottom of the harbor and air is pumped in to the chamber so that the water is kept out level with the cutting edge. (*a*) What is the minimum total weight of the structure? (*b*) If the diving bell is resting on the bottom at 50-ft. depth with the working chamber completely filled with water how many cubic feet of air under atmospheric conditions are required to unwater it? Atmospheric temperature 60°F. and pressure of 30 in. mercury. Temperature sea water 40°F., weight 64 lb. per cu. ft.

Ans. (*a*) 417,000 lb.
(*b*) 4690 cu. ft.

CHAPTER V

BUOYANCY AND FLOTATION

50. Buoyancy is the tendency of a submerged body to rise in the liquid or gas because of the effect of upward fluid pressure in opposing the action of gravitation on the body. The word "buoyancy" may also refer to the upward pressures holding a floating body in equilibrium against its weight.

It has already been noted (p. 32) that the vertical component of liquid pressure against a submerged surface of any shape equals the weight of the volume of liquid vertically above such surface and extending up to the free surface (real or imaginary) of the liquid. The submerged surface of any completely immersed body may be regarded as subdivided into a lower and an upper surface.

For a solid body or a completely enclosed hollow space entirely under water (Fig. 59) the buoyant effect of the liquid is the *difference* between the *total upward pressure* against the "bottom" portion ADC and the *total downward pressure* against the "top" portion ABC. By the principle stated on page 32 this difference of pressures equals the weight of a volume of liquid with the exact shape and size of the solid.

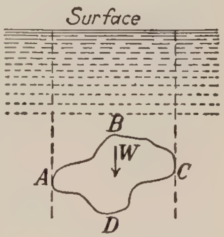


Fig. 59.

The same conclusion is reached by imagining the submerged solid to be replaced by liquid. As there is then merely a mass of liquid at rest, general equilibrium prevails. The pressures from the surrounding liquid on the now imaginary bounding surfaces of the internal body of liquid (having the shape of the solid) are balanced vertically. Hence the total upward pressure exceeds the total downward pressure by exactly the weight of liquid (imaginary) within the space actually occupied by the solid. The liquid pressures must act with the same intensity against the solid, because the backing

(or substance behind the surface) cannot make any difference in the magnitude of the liquid pressure against the surface.

51. The Principle of Archimedes (287–212 B.C.).—Since, for solids heavier than water, the weight carried by a hanger above the solid, or by a support beneath it, is less in water than in air, the popular term "loss of weight" is frequently applied to this buoyant force. If V represents the submerged or "displaced" volume, then

$$\text{Vertical buoyant force} = Vw. \tag{25}$$

If the weight of the completely submerged body is less than water the body will rise to the surface unless restrained by an outside force, *i.e.*, unless held down from above or tied down from below. If free to rise, the body will float, *i.e.*, it will settle with part of its volume above and part below the surface of the liquid, so that

$$W = Vw, \tag{26}$$

where W is the weight of the body, V is the volume *under water* or the "displacement," and w is the weight per unit volume of the liquid.

52. Specific Gravity.—The weight of fresh water is commonly taken as the standard in comparing heavinesses of solids and liquids. By weighing any insoluble solid first "in air" and then "in water" by suspension from a spring balance or a beam balance, the difference is known to be the weight of an equal volume of water. Hence the specific gravity =

$$\frac{\text{Weight}}{\text{Buoyant force}} = \frac{W}{Vw}. \tag{27}$$

If w' is the heaviness of the solid and V' its volume, then $W = V'w' = Vw'$, (since $V' = V$ for total submergence), and the specific gravity $= \frac{Vw'}{Vw} = \frac{w'}{w}$, the more familiar form.

If the *solid* substance is *lighter than water*, it will require a downward force, say W'', to keep it submerged and the buoyant force will be $W + W''$, whence the specific gravity $= \frac{W}{W+W''}$ which is less than 1.

The *volume* of *irregularly shaped solids*, or of the solid portions of sand, gravel, cement, etc., may be found by "weighing in water." (For cement a liquid that will not combine with it

must be used.) In all cases care must be taken to get rid of entrapped and adhering air. From Eq. (25), the volume $V = \frac{(\text{Buoyant force})}{w}$. Also $V = \frac{W}{w'}$.

Examples: 1. A bell buoy in sea water is to be assisted in floating upright by a submerged weight made of concrete and attached to the bottom of the buoy. How many cubic feet of concrete weighing 145 lb. per cu. ft. must be provided so as to get a net downward pull of 500 lb. from the weight?

Solution: Each cubic foot of submerged concrete is buoyed up by a force of 64 lb. (the weight of 1 cu. ft. of sea water). Hence the net downward force is 145 − 64 = 81 lb. per cu. ft. of concrete. It will require $^{500}/_{81} = 6.2$ cu. ft. of concrete. *Ans.*

2. What is the volume and what is the specific gravity of a sample of granite that weighs 238.0 lb. in air and has an apparent weight of 144.4 lb. in fresh water?

Solution: The buoyant force, 238.0 − 144.4 = 93.6 lb., is the weight of a mass of water with same volume as the stone. Hence the required volume of the piece of granite $= \frac{93.6}{62.4} = 1.50$ cu. ft. The specific gravity $= \frac{238.0}{93.6} = 2.54$. *Ans.*

53. The Principle of the Hydrometer.—A floating object rides higher in a heavy liquid than in a light liquid. The law, $W = Vw$, shows that, for a floating body of constant weight W, the submerged volume or *displacement* V varies inversely as w, the product being constant, because if $W = V'w' = V''w''$, then $\frac{V'}{V''} = \frac{w''}{w'}$.

Example: A ship of 20,000 tons (of 2240 lb., the usual marine unit) displacement, and having a draft of 30 ft. in the ocean, and with a water-line (horizontal) section of 30,000 sq. ft., enters the fresh-water portion of the Panama Canal. What depth of fresh water is necessary barely to float the ship?

Solution: Each ton requires $^{2240}/_{64} = 35.0$ cu. ft. of sea-water displacement to support it, or $^{2240}/_{62.2} = 36.0$ cu. ft. of fresh water. (The value 62.2 is the weight in pounds of 1 cu. ft. of fresh water at 80°F.) Hence in the fresh water there must be an increased

displacement of just 1 cu. ft. for each ton, or a total of 20,000 cu. ft. The extra draft required $= \frac{20,000}{30,000} = \frac{2}{3}$ ft. $= 8$ in. Hence the depth required in the canal is 30 ft. + 8 in. *Ans.*

The ordinary *hydrometer* float used in finding the specific gravity of liquids is of the shape shown in Figs. 60 and 61. It is made hollow, usually of glass, and is loaded in the bottom to make it float upright. A small difference in the density of the liquid causes a correspondingly small (but opposite) change in the submerged volume.

If the stem of the instrument is made quite slender, however, it will require considerable vertical motion to give even a small increment or decrement of submerged volume. Thus any desired degree of sensitiveness may be obtained by choosing the size of the cross-section of the stem relatively to the total submerged volume. If the stem is of uniform cross-section, as is commonly the case, the graduations for equal specific-gravity differences will not be uniformly spaced but the intervals will regularly increase towards the top. *Question:* Why?

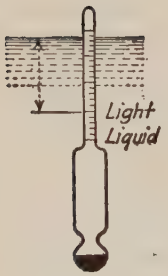


FIG. 60.

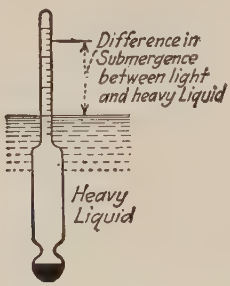


FIG. 61.

Example: Find the relation that must exist between the cross-section of stem and weight of hydrometer so that an increase of specific gravity of water of 1 part in 10,000 will cause the float to ride 1/10 in. higher. (See Figs. 60 and 61.)

Solution: Note that, since $W = Vw$, or $V = \frac{W}{w}$ (where W is constant), any small percentage change in w demands an opposite and practically equal small percentage change in V. In this case the volume of 0.1-in. length of stem $= 0.0001 \times V =$

$0.0001\frac{W}{w}$, or, since 1 cu. in. of water weighs $\frac{62.4}{1728} = 0.0361$ lb., $\frac{1}{10} \times A = 0.0001 \frac{W}{0.0361}$, or A (in sq. in.) $= \frac{W \text{ (in lb.)}}{36.1}$. Thus if $W = 0.1$ lb., $A = 0.00277$ sq. in., or the diameter of stem $= 0.0594$ in. ($=$ about $\frac{1}{16}$ in.). *Ans.* *Strictly*, $W = V'w' = V''w''$. But $V'' = V' - hA$, where h is the reduction in submerged depth; also $w'' = 1.0001w'$.

Hence $W = V'w' = (V' - hA)(1.0001w')$.

Therefore $V'w' = 1.0001V'w' - 1.0001hAw'$

or $0.0001V'w' = 1.0001hAw'$

$$\text{or } A = \frac{0.0001W}{1.0001 \times 0.1 \times 0.0361} = \frac{W}{1.0001 \times 36.1}$$

Thus the approximate method causes an error of only 1 in 10,000.

Problem: An iron float, a cylinder of 4-in. diameter, floats on mercury, axis of cylinder vertical, being used to actuate a recording mechanism, as in, for instance, a Venturi meter water flow recorder. If the iron cylinder weighs 5 lb., how deep will it float, and what rise or fall is required, to exert a push up or a pull down of $\frac{1}{4}$ lb. on the mechanism attached to the float? *Ans.* 0.81 in. deep, and a rise or fall of 0.04 in.

54. Stability of Flotation.—Stability for a freely floating body implies a tendency to return to the upright position after removal of any outside force that has temporarily caused a tipping or "heeling." Among the commonest of such outside forces are: wind and wave action; pressures due to tidal or river currents; pressures due to maneuvering a boat in a curved path by rudder or propeller action, or due to dynamic forces in launching; tow-line or anchor-line pulls, etc. Also, practically, stability implies that no inconvenient angle of "heel," independently of danger of incipient overturning, shall be caused by any probable shifting about of the cargo or of passengers; by any probable variation in weight and distribution of cargo (due, for instance, to consumption of fuel and supplies); by any probable manipulation of equipment (for instance, on barges in dredging or salvage or bridge-moving operation); etc.

55. Center of Buoyancy.—From the principles discussed on pages 81 and 82 it follows that the vertical line of action of the resultant of all buoyant, *i.e.*, vertically upward, liquid pressures

on a submerged portion of a body passes through the center of gravity of the displaced liquid. This point is called the *center of buoyancy.*

For totally submerged bodies of homogeneous composition the center of gravity of the solid and the center of buoyancy are coincident. The same is true for such masses as a submerged bag of air, a submerged barrel of oil, etc. For most boats and ships, however, the center of gravity is not coincident with the center of buoyancy.

Equilibrium of flotation implies that the center of gravity of the floating body lies in the same vertical line as does the center of buoyancy. Otherwise the buoyant and gravity forces would form a couple whose moment would either right or upset the boat, as the case might be.

56. Righting and Upsetting Moments.—It might seem at first thought that instability necessarily accompanies "top-heaviness," *i.e.*, when the center of gravity of a floating body is *above* the center of buoyancy. *But this is not true.* In fact, for almost all ships there is stability notwithstanding the usual

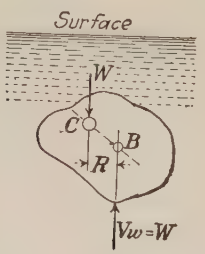


Fig. 62.

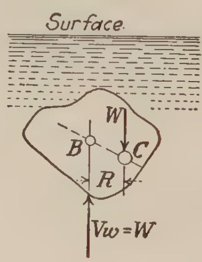


Fig. 63.

position of the center of gravity higher than the center of buoyancy. How this is possible will now be explained. As a preliminary, two opposite extreme conditions of flotation will be considered briefly.

1. In Figs. 62 and 63, C represents the center of gravity, B the center of buoyancy, and R the horizontal arm of the couple. For the particular case of a body floating so deeply that it is entirely submerged, *e.g.*, a submerged submarine, a slight tipping will cause an upsetting moment $= WR = VwR$ (Fig. 62) when

C is above B; and will cause a righting moment expressible in the same terms when C is below B (Fig. 63). *For such a case,* then, it is seen that "topheaviness" does mean instability, and that a condition with low center of gravity means stability.

But note carefully that this is a peculiar case of flotation, the center of buoyancy B remains fixed relatively to the body, the *submerged shape remaining unaltered* by the tipping.

2. Consider next a floating body of broad and shallow type but with the center of gravity high up above the water line, hence still further above the center of buoyancy (Fig. 64). In this case a slight tipping changes the *shape* of the originally rectangular submerged cross-section to either a trapezoid or a triangle, depending on the extent of tipping (Fig. 65). Note that the arm of the couple in this case is U.

Accompanying this *change of shape* of the displaced liquid there is a *shift sidewise of the center of buoyancy* in the direction of

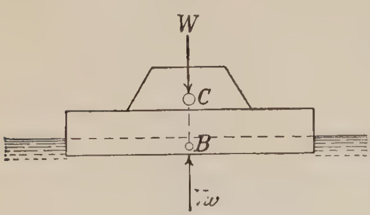


FIG. 64.

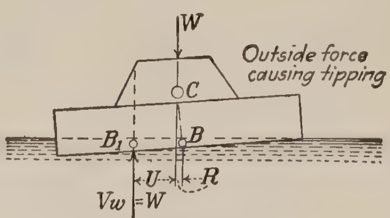


FIG. 65.

the tipping. Also, although C is higher than B (*i.e.*, although there is topheaviness), still it is seen in Fig. 65 that the moment (of the weight and buoyancy couple) is *righting* and *not upsetting.* This is so because the gravity line through C, although now falling a distance R outside of the former center of buoyancy B, falls well *inside of the new center of buoyancy* B_1 which has shifted a distance $U + R$. It is thus clear that what would be a small *overturning* moment ($= WR = VwR$) if B did not shift, actually is a larger *righting* moment $= WU = VwU$ because B has shifted much more than C relatively to the original position of B.

57. Boats and Water-craft of Usual Shapes. *Righting and Upsetting Moments Further Considered.*—It may now be inferred that stability for a floating vessel having a shape intermediate between the two extreme types above discussed (the submarine

and the scow) depends very much on the shape and the size of the boat near the water-line section, as well as on the relative positions of the centers of gravity and of buoyancy.

In Fig. 66, as in Fig. 65, the moment of the couple (a righting moment in this case) is seen to be $WU = VwU$. The calculation of the distance U for a given angle of heel involves the position of B_1, the new center of buoyancy. Now B_1 will always be at the center of gravity of the displaced liquid in the tipped position. Also it is known that the new submerged volume V is the same (although of different shape) as in the upright position.

With the guidance of these facts B_1 could be located and U computed and the moment $WU = VwU$. There is, however, a

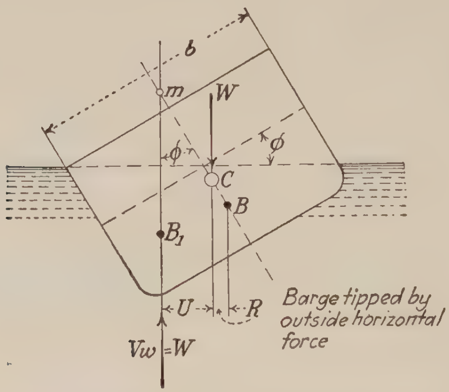


FIG. 66.—Barge tipped by outside horizontal force.

certain amount of complication in finding the position of B_1, and in dealing with a distance U inclined to the original horizontal direction, except in favorable cases.

The value of the righting moment $WU = VwU$ may be found, however, in most cases without finding U first, or at all, directly, as will now appear.

The Wedges of Immersion and of Emersion.—Figures 66 and 67 show that the original *shape* of the volume below the water line is altered for the tipped position by the addition of a triangular "wedge of immersion" on the low side, and by the removal of an equal "wedge of emersion" on the high side. These

"wedges" are of equal volume because the displacement volume is constant, no new vertical forces having been introduced.

By mechanics, if a portion FEG (Fig. 67) is removed from the original submerged mass of water $AEGH$, and placed on the other side in the position DFA, the whole weight times the resulting shift in its center of gravity equals the shifted portion of the weight times the distance moved, or

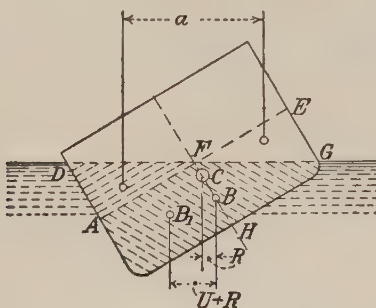


FIG. 67.

$$W(U + R), \text{ or } Vw(U + R), = V'w \times a, \text{ or } V'a \doteq V(U + R), \quad (28)$$

where a is the *horizontal* distance between centers of gravity of the "wedges," and V' is the volume of one of the wedges. Note that the distances U and R are also measured *horizontally*.

From Fig. 66 it is seen that $R = \overline{CB} \sin \phi$, so that the righting moment WU, or VwU, $= V'wa - VwR = (V'wa - Vw\overline{CB} \sin \phi)$, where ϕ is the angle of heel. If C were *below* B a plus sign would be used.

Instead of dealing with the angle ϕ, known dimensions of similar right-angled triangles may often be used, giving the equivalent of the trignometric functions directly (see example on p. 90).

For a boat with pointed bow and stern, $V'wa$ may be found as a summation of the moments of short lengths of the boat, for each of which the average value of a may be taken.

If both wedges are truly triangular (and this is a common case), then, from the trigonometry of the figure, for uniform or average width b,

$$\textit{Horizontal distance } a = \frac{2}{3}b\left[\frac{1}{\cos \phi} - \frac{1}{2}\tan \phi \sin \phi\right].$$

Practically, however, $a = \frac{2}{3}b$, since the expression in brackets is very close to unity for small angles of heel, and is only one-fifth of 1 per cent greater for 20 deg., 1 per cent greater for 30 deg., and 6 per cent for 45 deg.

Example: A barge, when empty and upright, with dimensions as in Fig. 68, has a draft of 1 ft., with center of gravity 1.5 ft. above the bottom, or 5.5 ft. below the plane of the gunwales. A cargo of steel beams is symmetrically loaded on top of the barge with the center of gravity of load 0.5 ft. above gunwales, or 7.5 ft. above the bottom. The barge now draws 4 ft. of water.

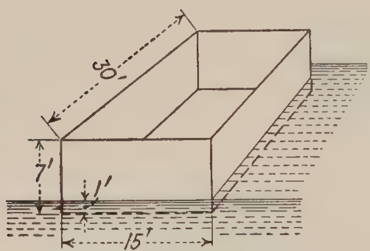


FIG. 68.—Empty barge

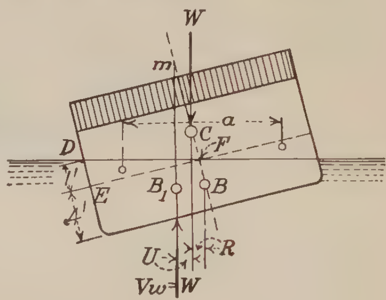


FIG. 69.—Cross-section through loaded and tipped barge. (Tipping caused by an outside horizontal force, not by shifting cargo, *i.e.*, C remains fixed with reference to the barge.)

If the barge is tipped as shown in Fig. 69 by an outside horizontal force, what is the moment of the couple brought into action, and is it positive or negative, *i.e.*, righting or upsetting?

Solution: By inspection of the added submergence, 3 ft., due to the load, the cargo is seen to weigh three times as much as the empty barge. Hence the distance of the point C, the center of gravity of combined barge and load, above the bottom is $\frac{3 \times 7.5 + 1 \times 1.5}{3 + 1} = \frac{24}{4} = 6.00$ ft. Hence the $\overline{BC}$ distance for the loaded barge is $(6.00 - \frac{4}{2}) = 4.00$ ft.

By similar triangles (see Figs. 66 and 69) $\frac{R}{\overline{BC}} = \frac{\overline{DE}}{\overline{DF}}$, or $\frac{R}{4.00} = \frac{1}{\sqrt{7.5^2 + 1^2}}$,

whence $$R = \frac{4.00}{7.57} = 0.528 \text{ ft.}$$

Now since $V'a = V(U + R)$, $(0.5 \times 7.5)10 = (4 \times 15) \times (U + R)$, and $(U + R) = \dfrac{37.5}{60} = 0.625$ ft.

Hence $U = 0.625 - R = 0.625 - 0.528 = 0.097$ ft. Since this is positive, the moment is positive, or *righting*, and $WU = VwU = (4 \times 15 \times 30)\ 62.4 \times 0.097 = 112{,}000 \times 0.097 =$ 10,900 *ft.-lb. Ans.*

The cargo weighs $(3 \times 15 \times 30)\ 62.4 = 84{,}200$ lb. The empty barge weighs one-fourth of $112{,}000 = 28{,}100$ lb.

Assuming greater angles of tipping (the cargo assumed to be fastened to prevent shifting), calculations may be made as above for 2, 3, and 4 ft., submergence of the low side instead of the 1 ft. of Fig. 69, finding $U = 0.74$, 1.38, and 2.03 ft., respectively, showing an increasing righting moment for these extents of tipping. But when the side has gone down 3 ft. the low gunwale would be "awash" at the water surface. When the side has gone down 4 ft., the low gunwale is submerged, and the opposite edge is out of the water. Under these conditions the arm of the wedge couple "a" no longer $= \frac{2}{3}b$.

58. Stability or Instability of Flotation.—At times it may be desired to determine whether or not a body will float stably in a desired "upright" position with C and B in the same vertical. (Note that it has already been found that the mere fact that C is above B does not of itself mean instability.) It is always possible to proceed as in the example on page 90 to find whether the moment arm U is positive or negative when the boat is assumed tipped through a desired angle, or to use the results of a general analysis on the same basis, *viz.:*

Alternative Method: For a portion of the boat of length dL (Fig. 70), the buoyant effort of one "wedge" =

$$V'w = \left[\frac{b}{2}\tan\phi dL\right]\frac{1}{2}\frac{b}{2}w = \frac{b^2}{8}\tan\phi dLw$$

The moment
$$V'wa = \left(\frac{b^2}{8}\tan\phi\, dLw\right)\frac{2}{3}b$$

or
$$V'wa = \left(\frac{1}{12}dLb^3\right)w\tan\phi.$$

But the value $\frac{1}{12}dLb^3$ is recognized as the *moment of inertia* of a rectangle of height b and width dL about a middle axis parallel to dL.

Hence for the whole boat, the summation of all such differential moments is $I_{WL}\ w \tan \phi$, where I_{WL} stands for the moment of inertia of the original water line section of the boat about the longitudinal, or "fore-and-aft," axis. Therefore $V'wa = I_{WL}w \tan \phi$.

From Eq. (28), $Vw(U + R) = V'wa$, or $VwU + Vw\overline{CB} \sin \phi = V'wa$. But, from Fig. 66, $U = \overline{mC} \sin \phi$. Hence, by substitution, $Vw\overline{mC} \sin \phi = I_{WL}w \tan \phi - Vw\overline{CB} \sin \phi$. When ϕ is a very small angle, $\sin \phi = \tan \phi$ and therefore $V\overline{mC} = I_{WL} - V\overline{CB}$, or

$$\overline{mC} = \frac{I_{WL}}{V} \pm \overline{CB}. \tag{29}$$

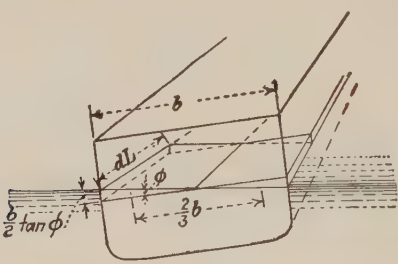


FIG. 70.

The minus sign is used when C is above B (as is the case for most loaded water craft).

Equation (29) could be written

$$\overline{mC} \pm \overline{CB} = \frac{I_{WL}}{V}, \text{ or } \overline{mB} = \frac{I_{WL}}{V}. \tag{30}$$

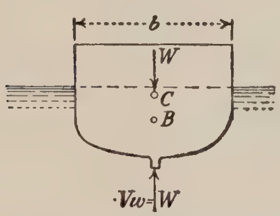
Problem for the Student.—Find whether or not a long square stick of wood (homogeneous) of specific gravity 0.70 will float with sides of square vertical. Will it float upright if specific gravity is 0.95? How about a square iron bar floating on mercury?

Position of Metacenter as a Criterion of Stability.—If the distance mC, calculated from Eq. (29), is found to be positive, the body is floating in stable equilibrium, the point m then being *above* C. This point m where the vertical through the new center

of buoyancy (B_1 in Fig. 66) cuts the original vertical is called the *metacenter*, from the Greek words meaning "beyond the center." If the "metacenter distance" $\overline{mC}$ is found to be negative, then the metacenter m is *below* C and there is instability with overturning moment.

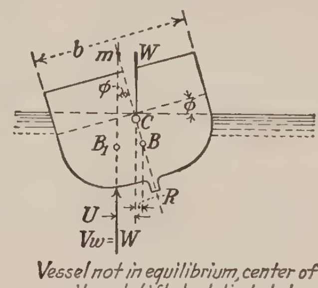
In Eq. (29), I_{WL} for other boats than barges with simple rectangular shapes may be found by subdividing the water-line section into a number of near rectangles and triangles, and summing the several moments of inertia. V is known when the total weight of the boat and the cargo is known, for $V = \frac{W}{w}$. The distance $\overline{CB}$ is known when the shape and the size of the submerged portion of the boat are known, these data being obtainable from the plans and observed draft, B being at the center of gravity of the displaced water, and C being located from the known distribution of weight in hull and cargo.

59. Effect of Shifting Cargo, and Experimental Location of the Metacenter and of the Center of Gravity of a Floating Body. As stated on page 86, if a body is floating freely and is in



Vessel in Equilibrium

FIG. 71.



Vessel not in equilibrium, center of gravity not shifted relatively to boat

Righting moment = WU = VwU

FIG. 72.

equilibrium, this implies that there is no tendency for it either to tip or to move up or down. Consequently, the resultant of the buoyant pressures of the water is equal and opposite to, and acts along the same line as, the vertical resultant of the gravity forces or weight of the floating body (Fig. 71).

If such a floating body is tipped (Fig. 72, same as Fig. 66), and is held in that position, the moment of the holding forces

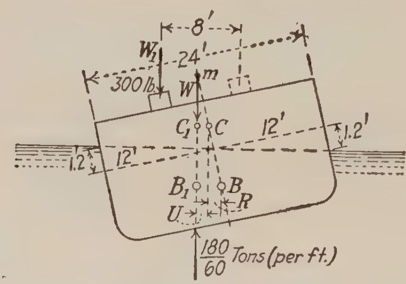
must be equal and opposite to the righting moment WU, or VwU. It is here assumed that no new outside *vertical* forces are introduced.

If the tipping is caused by the shifting of a portion of the cargo, as, for instance, by moving a weight W_1 a distance s (see Fig. 73), then $W \times U = W_1 \times s$, or

$$U = \frac{W_1}{Vw}s. \tag{31}$$

B is located from the shape and dimensions of the boat below the water line, *i.e.*, from the size and shape of the displaced liquid. Also the angle of heel, ϕ, may be measured, or else the extra depth of submergence on the low side of the boat or the amount of emergence on the high side.

Example: Assume for simplicity a boat or barge of 180 short tons total displacement (boat and cargo), with vertical sides and



Barge is in equilibrium in the tipped position, the center of gravity having shifted from C to C_1 due to weight W_1 being shifted

FIG. 73.

a rectangular water line (Fig. 73). $b = 24$ ft., $L = 60$ ft. If the effect of shifting a weight of 18,000 lb. (=300 lb. per foot length of barge perpendicular to sketch) a distance $s = 8$ ft. to one side horizontally from the center line is to cause one side of the boat to go down $y = 1.20$ ft., compute the CB distance, that is, the distance of the original center of gravity above the original center of buoyancy. The barge floats vertically (on an even keel) before the weight is shifted.

Solution: From the mechanics of the boat alone

$$\frac{180 \times 2000}{60} \times U = 300 \times 8.$$

Therefore $U = 0.40$ ft. Also, since the moment of the shift of the center of buoyancy equals the moment of the wedge shift, *i.e.*, $Vw(U + R) = V'wa$ from Eq. (28), where $Vw = W$.

Then $\left(\frac{180 \times 2000}{60}\right)(U + R) = \left(\frac{1.20}{2} \times 12\right)(62.4)\left(\frac{2}{3} \times 24\right)$.

Therefore $U + R = 1.20$ ft., and $R = 1.20 - 0.40 = 0.80$ ft.

By similar triangles, $\frac{CB}{R} = \frac{\sqrt{12^2 + 1.2^2}}{1.2}$.

$$CB = 8.03 \text{ ft.} \quad \textit{Ans.}$$

This computation shows that the center of gravity of the loaded barge is about 6 ft. above the water line.

It is sometimes more convenient to measure the "angle of heel" by a plumb bob, thus obtaining the angle of the wedge.

By corresponding experimental measurements and calculations the position of the center of gravity of any boat may be determined. In practice it is desirable to check the office computations in this way after any boat is launched and fitted.

Problems

1. A brick is 4 in. thick, 4 in. wide, and 8 in. long. When suspended from a spring balance and lowered in water to a depth of 12 in. it weighs $5\frac{1}{2}$ lb. (*a*) What is its true weight? (*b*) What is its specific gravity?

2. An irregular-shaped stone weighs 45 lb. When submerged in water it weighs 25 lb. (*a*) Compute its specific gravity. (*b*) Compute its volume in cubic inches.

3. A rectangular barge 5 ft. deep, 10 ft. wide, and 50 ft. long loaded uniformly with a cargo of 32,000 lb. draws 3 ft. of water when in a salt-water bay. How deep will it float when taken up to fresh water of a tidal river? (Sp. gr. salt water = 1.025.) *Ans.* 3.075 ft.

4. A ship of 76,000-sq. ft. horizontal sectional area at the water line has a draft of 40.5 ft. in sea water when loaded to capacity. (Sea water has average weight of 64.1 lb. per cu. ft.) In the fresh water (weight 62.4 lb. per cu. ft.), at the entrance of the Panama Canal, it is observed that the ship draws 41.4 ft. Find: (*a*) Weight of ship and cargo in long tons of 2240 lb. each (the customary unit for ships). (*b*) Ship's displacement in cubic feet for both fresh and sea water. (*c*) With an available water depth in the canal of 41 ft. above the lock sills, how many long tons of the cargo must be lightered so that the ship can barely pass into locks, say with 0.1-ft. clearance below keel? *Ans.* (*a*) 71,800 long tons.
(*b*) 2,580,000 cu. ft.; 2,510,000.
(*c*) 1058 long tons.

5. A piece of iron weighs 50 lb. When suspended from a spring balance and entirely submerged in water the spring balance registers 43.06 lb. What is the weight of 1 cu. ft. of the iron? *Ans.* 460 lb. per cu. ft.

6. A hollow steel cylinder 4 ft. in diameter and 6 ft. long weighs 1030 lb. How many pounds of lead weighing 700 lb. per cu. ft. must be fastened to the outside bottom to make the cylinder float on its long axis (diameter) in salt water? *Ans.* 1450 lb.

7. A stone weighs 36 lb. When it is lowered into a cylinder 2 ft. in diameter containing 2 ft. of water, the water level rises 0.0735 ft. It is noted that when the stone is submerged its "apparent weight" or "weight in water" is 21.6 lb. Find the specific gravity of the stone. *Ans.* 2.5.

8. A water-soaked pile 50 ft. long floats upright with top 12 in. above the river water. What is the condition of flotation in sea water? (Sea water weighs 64 lb. per cu. ft.) *Ans.* 2.2 ft. above sea water.

9. A hydrometer float (See Fig. 60) weighs 0.00465 lb. It has a stem at the upper end, cylindrical and exactly 0.1000 in. in diameter. How many inches higher will it float in dilute alcohol at sp. gr. 0.83 than in kerosene at sp. gr. 0.79? *Ans.* 1.0 in.

10. A hydrometer weighs 0.01 lb. and its stem is 0.01 sq. in. in cross-section. How many inches between correct graduations for sp. gr. = 1.00 and 0.90? *Ans.* 3.08 in.

11. (*a*) A tub of water rests on platform scales, and shows a total weight of 150.0 lb. A piece of trap rock that weighs 96.0 lb. "in air" is now suspended, completely immersed but not touching the tub. The scales now read 180.0 lb. *What is the specific gravity* of the piece of rock? (*b*) What is the volume of the rock in cubic feet? *Ans.* (*a*) 3.2.
(*b*) 0.48 cu. ft.

12. A float hydrometer (see Fig. 61) weighs 0.0361 lb. (= also the weight of 1 cu. in. of fresh water), and has a stem 0.100-in. diameter. (A of cross-section = 0.007854 sq. in.) In sea water of sp. gr. 1.025, how many inches higher than in fresh water will the hydrometer float? *Ans.* 3.11 in.

13. When a piece of granite of specific gravity equal to 2.54 is submerged in water the apparent weight is 144.4 lb. What is its true weight?
Ans. 237.5 lb.

14. A stick of wood of specific gravity equal to 0.7, 2 in. square and 6 ft. long, is to be used as a float in stream measurements. How many pounds of iron washers weighing 480 lb. per cu. ft. should be fastened to one end to make the stick float with 6 in. out of water? *Ans.* 2.6 lb.

15. A 4-in. by 4-in. waterproofed wooden timber, 18 ft. long, weighs 90 lb. This is to be loaded at one end so it will float upright with 2 ft. out of water (fresh). How many pounds of cast iron (at 450 lb. per cu. ft.) are required?
Ans. 24.4 lb.

16. If the above-mentioned loaded stick absorbs 5 lb. of water (without perceptible swelling) after floating for several weeks, how much length will show above water? *Ans.* 1.279 ft.

17. A timber 12 by 12 in. by 10 ft. long having a specific gravity of 0.6 is to be used as a buoy in salt water weighing 64 lb. per cu. ft. How many cubic feet of concrete weighing 150 lb. per cu. ft. should be fastened to one end so that 2 ft. of the timber will float above the surface of the water?

Ans. 1.6 cu. ft.

18. Signal buoy float (Fig. P.43). The cylinder, 2 ft. in diameter and 4 ft. long, floats with top just awash as per sketch. What is the proper total weight in pounds for the float and flagstaff? *Ans.* 785 lb.

19. A hydrometer (Fig. 60) having a stem of uniform diameter = 0.125 in., when placed in water, floats with 1 in. of its stem above the surface. Weight of hydrometer = 2.37 oz. At what point on the stem should a mark be placed corresponding to a specific gravity of 1.01?

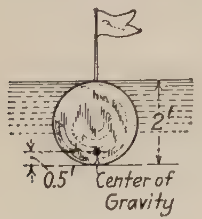


Fig. P.43.

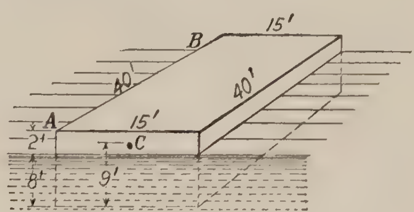


Fig. P.44.

20. The center of gravity of the barge shown (Fig. P.44) is on the mid-section 9 ft. above the bottom of the barge. If an outside force (wind or wave action) heels the barge until the gunwale AB is just at the water surface (fresh water): (*a*) compute the "moment of the wedge shift" in foot-pounds. (*b*) What is the "moment of the center of buoyancy shift" in foot-pounds? Righting or upsetting? (*c*) Compute the value of the righting or upsetting moment. (*d*) Is barge stable for small angles of heel?

Ans. (*a*) 187,200 ft.-lb.
(*c*) Upsetting moment 199,500 ft.-lb.
(*d*) No.

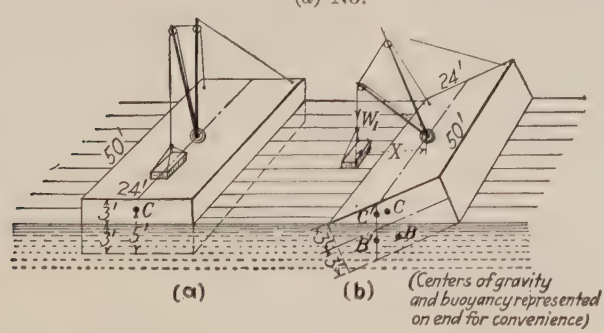


Fig. P.45.

21. A barge, with a crane for loading and unloading, floats together with its load in the position shown in Fig. P.45(*a*). The maximum lift of the

crane is 20 short tons. The center of gravity of the loaded barge (barge and W_1) in the initial position in fresh water is 5 ft. above the bottom of the barge. What is the horizontal distance out to one side from the center line of the barge (deck level) the crane could swing the 20 tons which it had lifted from the middle of the deck and tip the barge in the position shown in Fig. P. 45(*b*)? The barge floats stably in the second position.

Ans. 17.69 − 0.24 = 17.45 ft.

22. Where is the center of buoyancy of the loaded barge in problem 21 when in salt water?

23. The loaded barge in problem 3 when in the salt-water bay encounters a wind blowing against one side which heels the barge until the top edge of the opposite side is just at the water surface. The cargo does not shift. What is the highest permissible location of the center of gravity so that the barge will not turn over? Why? Show by sketch.

24. A submerged submarine of circular section amidships (diameter = 20 ft.) has its center of gravity 5 ft. above the bottom in salt water. If heeled or tipped 30 deg. what is the moment in foot-pounds, per foot of length of submarine? Is this a righting, a neutralized, or an upsetting moment?

Ans. 50,300 ft.-lb. righting.

25. (*a*) The lifting effort of a prismatic flat-bottom float regulator (*e.g.*, in operating a valve for a tank supply or discharge), in fresh water, is required to be 130 lb. when the water rises 1 in. around above the float above normal water line. How large must the float be? (*b*) For 150-lb. downward pull when the water has fallen 1 in. below normal water line, what is the minimum allowable displacement and weight for the same float as in (*a*)?

Ans. (*a*) 28.8 sq. ft.
(*b*) 2.40 cu. ft., 150 lb.

26. Find righting moment in foot-pounds, if buoy in problem 18 is tipped (by wind or wave) through angle of 30 deg. *Ans.* 196 ft.-lb.

27. It is proposed to move a bridge span by floating on a pair of scows, one under each end, crosswise. The bridge and scows are firmly fastened together so they act as a single unit in tipping (Fig. P.46). The bridge, 250 ft. long, weighs (including falsework) 500 tons. Scows are practically rectangular in shape, outside dimensions 80 ft. long, 25 ft. wide, and 7 ft. deep. When without load, the scows float 1 ft. deep. The center of gravity of the empty scows is 2 ft. above the bottom at the middle. (*a*) Compute the depth of flotation of the loaded scows. Find the distance of the center of buoyancy above the bottom. (*b*) Derive an expression (finally as simplified as possible) for $\bar{x}$, the distance of the center of gravity of the loaded scows above the bottom, in terms of the height h of the center of gravity of the bridge above the bottom of the scows. Then eliminate $\bar{x}$ in favor of the

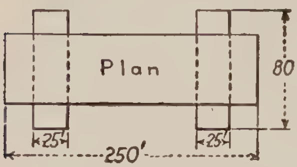


FIG. P.46.

$\overline{CB}$ distance, and finally state h in terms of $\overline{CB}$. (*c*) Compute the height above the bottom of the scows for the center of gravity of the bridge so that, for that height, there will be a zero righting moment, and for any greater height there will be an upsetting moment at the slightest tipping. (*d*) If the bridge is blocked up on the scows so that the center of gravity of the bridge is 50 ft. above the bottoms of the scows, and an outside force acting broadside on the bridge or endwise on the scows causes the scows to heel until the low ends of the gunwales are 6 in. above the water surface (measured along the inclined end), compute the value of the actual righting moment in foot-pounds (or is it an upsetting moment?).

Ans. (*a*) 5 ft., 2.5 ft.
(*b*) $h = 1.25\overline{CB} + 2.625$.
(*c*) 136 ft.
(*d*) 3,220,000 ft.-lb. righting.

28. (*a*) If the center of gravity of a boat in the upright position is 10 ft. above the center of gravity of the part under water, the total displacement being 1,000,000 lb. (= 500 short tons), and the boat is tipped 30 deg. causing the center of buoyancy to shift sidewise 8 ft., is the moment righting or upsetting and why? (*b*) In above problem, what is the value in foot-pounds of this moment? (*c*) What is the value in foot-pounds of the "moment of the wedge couple?" (*d*) How much average wind pressure in pounds per square foot will it take to hold the boat in the 30-deg. tipped position if 10,000 sq. ft. are exposed to the wind, averaging 15 ft. above the water line?

Ans. (*a*) Righting.
(*b*) 3,000,000 ft.-lb.
(*c*) 8,000,000 ft.-lb.
(*d*) 20 lb. per sq. ft.

29. Ice has a sp. gr. = 0.92, the sp. gr. of sea water = 1.025 (*i.e.*, unit weights respectively are 57.4 and 64 lb. per cu. ft.). What percentage of volume of an iceberg floats *above water?* *Ans.* 10 per cent.

CHAPTER VI

LOGARITHMIC PLOTTING

The widespread use of logarithmic plotting in general engineering practice has made it essential that engineers should understand the principles underlying this convenient method of representation and calculation.

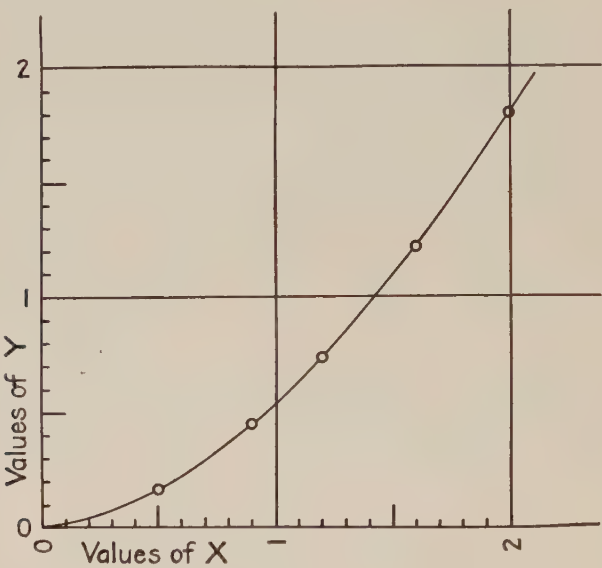


Fig. 74.

60. In **ordinary plotting** the coordinates or distances from the axes represent values of the variables. In **logarithmic plotting,** the coordinates represent values of the *logarithms of the variables.*

Thus Fig. 74 is the result of plotting directly the following simultaneous values of X and Y, the plotted points having been joined by a smooth curve.

X	*Y*
0.50	0.162
0.90	0.45
1.20	0.74
1.60	1.22
2.00	1.80

Below are tabulated the logarithms of these values.

Log *X*	Log *Y*
9.6990 (−10)	9.2095 (−10)
9.9542 (−10)	9.6532 (−10)
0.0792	9.8692 (−10)
0.2041	0.0864
0.3010	0.2553

It should be noted that the logarithms of numbers less than 1 are *negative*. Instead, however, of writing the logarithms of 0.50 as −0.3010 (*i.e.*, log. $\frac{5}{10} = 0.6990 - 1.0000 = -0.3010$), it is changed to a whole negative number plus a positive decimal, *i.e.*, either to $-1 + 0.6990$, written $\bar{1}.6990$, or to $9.6990 - 10$ (the −10 being usually omitted in writing, but always understood when adding, multiplying, or dividing logarithms).

In Fig. 74*A* these simultaneous values of the logarithms are plotted. The numbers marked along the axes to the left of and below the origin are in accordance with the usual scheme, just mentioned, of writing the negative logarithms. The student should study this carefully on Fig. 74*A*.

In Fig. 74*A* the plotted points give a straight line,[1] while in Fig. 74 with the direct plotting of the values there is obtained a curve resembling a parabola. Herein appears one advantage of logarithmic plotting. In Fig. 74 there are no ready means of determining the equation of the curve, but in Fig. 74*A*, with a *straight line*, the equation can be found readily as follows:

The equation of a straight line is of the form

$$y = ax + b \qquad (A)$$

where a is the slope of the line and b is the intercept on the Y-axis, *i.e.*, when $x = 0$, $y = b$. So, by measuring the slope (the tangent of the angle made with the X-axis) and the intercept, the correct equation of any straight line may be written.

[1] As emphasized later, only such values as follow an *exponential law* yield a straight line when their logarithms are plotted.

It is to be noted that the slope may be negative as well as positive. If the line is in the second and fourth quadrants the slope is negative. In general, when y increases with an increase in x the slope is positive, and when y decreases with an increase in x the slope is negative.

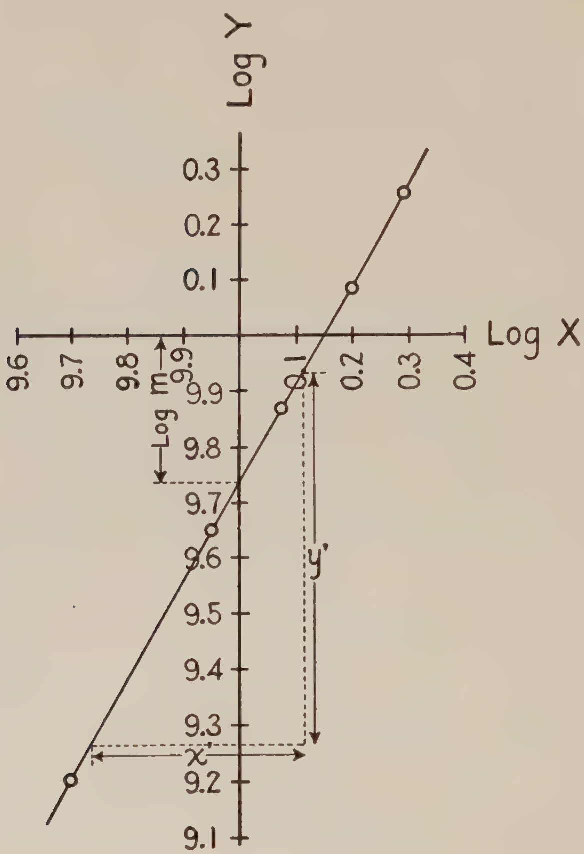


Fig. 74A.

In general, in dealing with an equation of the form

$$Y = mX^n \tag{B}$$

it may also be written

$$(\log Y) = n(\log X) + (\log m) \tag{C}$$

because if quantities are equal, their logarithms are also equal.

Equation (C) is of the same form as Eq. (A), *i.e.*, a straight-line equation. In Eq. (C) the slope of the straight line is n and the intercept on the (log Y) axis is (log m), *i.e.*, when (log X) = 0, (log Y) = (log m). Hence an equation like (B), which gives a parabola-like curve when corresponding values of X and Y are plotted, gives a straight line when the *logarithms* of X and Y are plotted. Conversely, when the logarithms of X and Y have been plotted, and the points found to lie on a straight line, it is known that the equation is of the form $Y = mX^n$, the slope of the line being equal to the exponent n and the intercept on the (log Y) axis being equal to (log m). Of course, to find m when (log m) is known a table of logarithms is used.

The above reasoning holds good for all real values of the exponent n, whether positive or negative, whole number or fraction. The value of m is assumed to be positive, as it usually is in equations that occur in engineering. Only positive values of the original variables, X and Y, are dealt with, since the logarithm of a negative number is an imaginary quantity.

With the above facts demonstrated it is possible to proceed to write the equation of the line in Fig. 74A. The slope is $\frac{y'}{x'} =$ 1.75 (see Fig. 74A). The intercept on the (log Y) axis is negative and by the chosen scale the distance below the origin equals -0.277, or, by the system of representing negative logarithms, it equals $9.733(-10)$, as may be read directly on Fig. 74A. Therefore the equation of the straight line is (log Y) = 1.75 (log X) + $(0.733 - 10)$. Taking the antilogarithms of both sides, $Y = 0.54X^{1.75}$, which is the desired equation in terms of X and Y, the original variables.

61. Use will now be made of **logarithmic scales** placed along the axes of Fig. 74A (see Fig. 74B, and the results noted).

On a logarithmic scale the divisions and marking are such that a division with some particular number represents (by its distance from the starting point) the logarithm of that number, *just as on the common slide rule.* Figure 74C shows an equal division scale and a logarithmic scale side by side. A careful study of these scales in their relation to each other will fix in mind the principle involved.

Thus by using the logarithmic scales it is not necessary, for instance, to scale off the intercept, as was done with Fig. 74*A* and then to look up the corresponding number in a table of logarithms.

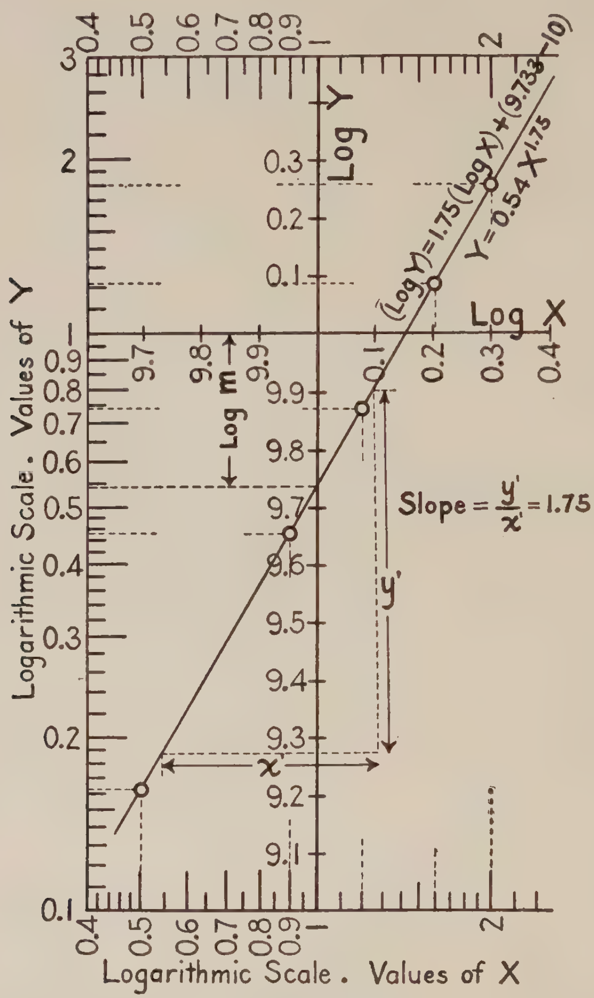


Fig. 74*B*.

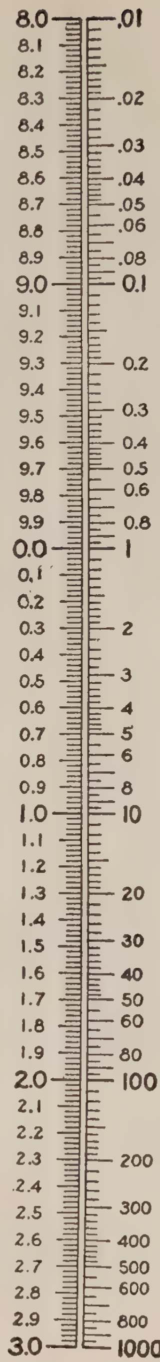
For example, in Fig. 74*B* on the logarithmic scale on the left horizontally opposite the intersection of the sloping line with the (log Y) axis there is a division marked 0.54. But this is the very

same value of m previously obtained by the longer roundabout method.

So also in Fig. 74B opposite each plotted point there is seen on the left and bottom logarithmic scales the divisions representing the very values of X and Y given at the beginning of this discussion. It thus appears that the logarithmic scales make it possible both to plot in proper position the logarithms of the given numbers without using a table and also to read off directly the number whose logarithm is represented by a given distance, *e.g.*, an intercept.

Hence for purposes of logarithmic plotting there is no need for the equal divisions along the axes, as in Figs. 74A and 74B, but the logarithmic scales can be advantageously substituted. This indicates the method of ruling *logarithmic cross-section paper* or a *logarithmic diagram.* Just as ordinary cross-section paper is made by drawing two sets of lines, equally spaced, perpendicular to each other, so logarithmic paper is made by drawing two sets of lines spaced according to a logarithmic scale.

62. Additional Notes on Logarithmic Paper and Logarithmic Plotting.— The "base" of such paper, and of a logarithmic scale in general, is the distance representing 1.0 in logarithms. Thus the "base" of the lower scales on the common 10-in. slide rule is about 10 in. (usually 25 cm.). The "base" of the upper scales of the 10-in. slide rule is about 5 in., and that of



Equal Division Scale, marked with logarithms of numbers on the upper Scale.

Fig. 74C.—Logarithmic Scale, marked with numbers whose logarithms are on the lower Scale.

the third scale on a "polyphase" slide rule is about 3.33 in., exactly one-half and one-third of the main scale.

A logarithmic scale has the same salient features as a common logarithmic table. Thus a table of logarithms contains the logarithms of all numbers between 1 and 10, advancing by intervals of, *e.g.*, 0.01, 0.001, or 0.0001, etc. Such a table is considered complete, but really it is not so, because the tabular logarithms are modified by adding or subtracting one or more whole units (*the characteristic*) whenever the number is more than 10 or less than 1.

Under these same conditions a logarithmic scale may be considered to be complete when its divisions extend from 1 to 10. The position of the decimal point can be provided for by shifting one "base" length for each place that the decimal point is moved, because changing the decimal point one place on a number changes its logarithm by 1.0. Then on a logarithmic scale the position of the division representing the number would be moved just one "base."

But in plotting it is inconvenient to use a scale that requires to be shifted about over the paper The paper should be ruled so it will furnish its own scale at all points. Evidently then, logarithmic scale cross-sections consist of a succession of panels, one base square extending in both the x and y directions. All panels are ruled alike, just as on the upper scales of a slide rule the right half is a repetition of the left half. In Fig. 74C the logarithmic scale illustrates a succession of five base distances each divided alike, and giving a range of values from 0.01 to 1000, or from 1 to 100,000, or 0.00001 to 1, etc.

It appears from Fig. 74B that the axes are situated where the logarithmic scales are marked 1 (unity), because $(\log 1) = 0$. So on logarithmic paper the line for $(\log X) = 0$ is marked with the value of X itself, *viz.*, 1. Therefore on logarithmic paper, in plotting lines from equations or in finding equations from plotted lines, the origin always is at the intersection of the lines marked 1, and the intercept is to be taken on the Y-axis, which is the line marked $X = 1$.

Usually on the unused sheet of logarithmic paper there is a division marked 1 every base distance along all four edges. Any

one of these may be chosen for the unity value, depending on convenience and the range of values it is desired to represent. After this the decimal points must be placed in proper sequence on the printed numbers of the sheet. The lines one or more "bases" to the left or right of the Y-axis represent ($\log X$) $= -1$, -2, etc., or $+1$, $+2$, etc., and the markings are to be changed

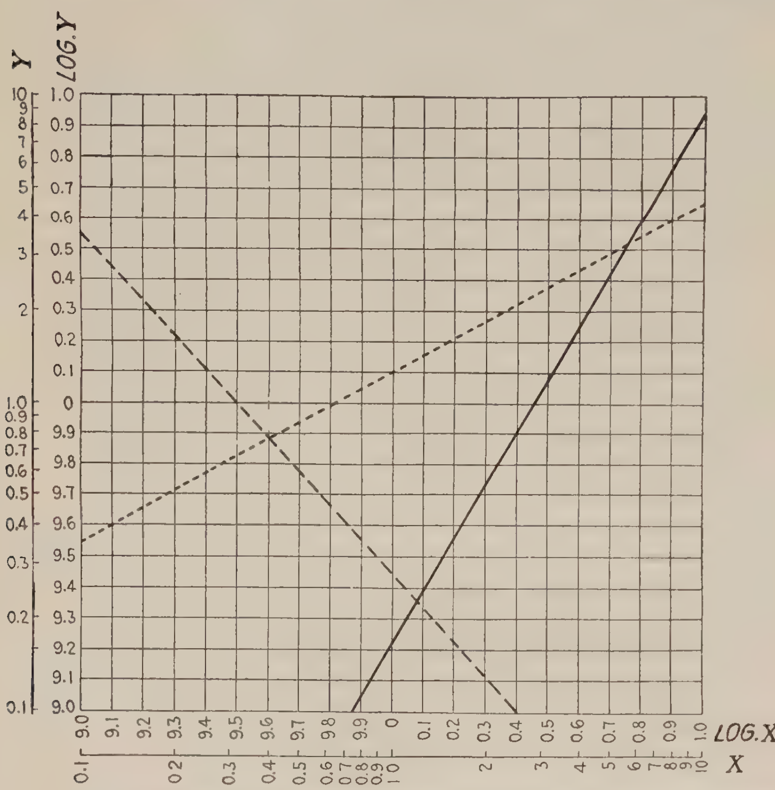


FIG. 74*D*.—Logarithmic plotting on background of ordinary cross-section paper. Compare this with Fig. 74*E*.

from 1, respectively, to 0.1, 0.01, etc., 10, 100, etc. In short, having selected an origin, it and no other must be used during the whole calculation. Decimal points are not to be disregarded any more than on the slide rule.

On Figs. 74*D* and 74*E* are shown logarithmic plottings of three equations. The only difference between the figures is that the *background* in Fig. 74*D* is composed of equal-division cross-section

lines as on ordinary cross-section paper, while the background in Fig. 74*E* is composed of lines ruled according to a logarithmic scale as on logarithmic paper. But note that each plotting is a *logarithmic plotting*.

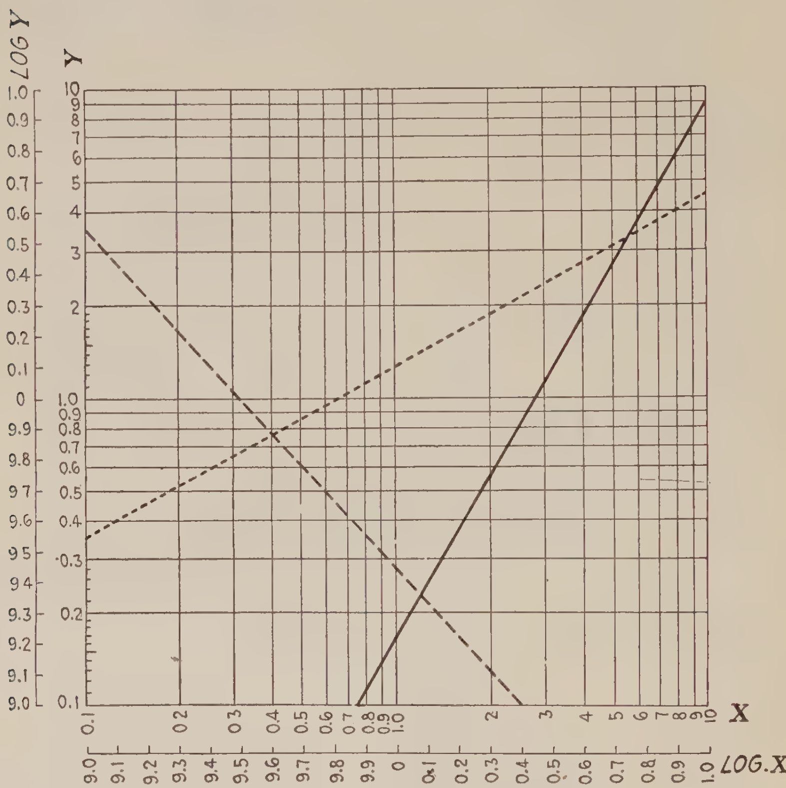


FIG. 74*E*.—Logarithmic plotting on background of logarithmic paper. Compare this with Fig. 74*D*.

The equations of these three lines in terms of X and Y are, respectively:

Full line, $Y = 0.166X^{1.72}$

Dotted line, $Y = 1.26X^{0.55}$

Dashed line, $Y = 0.282X^{-1.11} = \dfrac{0.282}{X^{1.11}}$

These equations are read directly from Fig. 74*E* or indirectly from Fig. 74*D* by first writing out the straight-line equations of the logarithms, namely:

$(\log Y) = 1.72(\log X) + (9.22 - 10)$
$(\log Y) = 0.55(\log X) + 0.1$
$(\log Y) = -1.11(\log X) + (9.45 - 10)$

By taking the antilogarithms of each side of these equations the previous ones are obtained. The advantage of using the logarithmic paper is obvious.

N. B.—It is to be noted that the slope of a line on logarithmic paper, as on common paper, is a ratio of two *distances*, and that these distances must be measured with an ordinary scale, *not with the logarithmic scale of the paper.*

Following are a few typical relations involving powers and roots, in the representation of which logarithmic plotting is useful:

Mechanics:

Freely falling body: $v = \sqrt{2gh}$, $s = \frac{1}{2} gt^2$.

Moment of inertia, *e.g.*, a rectangle: $I = \dfrac{bh^3}{12}$.

Deflection of a beam, *e.g.*: $d = \dfrac{(PL^3)}{(48EI)}$.

Shearing strength of rivets: $S = \dfrac{\pi}{4} d^2 s$.

Volumes and areas involving cubes and squares of linear dimensions.

Hydraulics:

Flow in pipe: $h_F = f \dfrac{L}{d} \dfrac{v^2}{2g}$, $h_F = K \dfrac{v^{1.86}}{d^{1.25}}$, $v = Cs^{.54}d^{.67}$.

Flow in open channel: $v = C\sqrt{Rs}$, $v = \dfrac{1.486}{n} R^{2/3} s^{1/2}$.

Velocity of jet: $v = C\sqrt{2gh}$.

Head corresponding to velocity: $h = \dfrac{v^2}{2g}$.

Power in a nozzle stream: Horsepower $= \dfrac{Awv^3}{(2g \times 550)}$.

Flow over a weir, *e.g.*: $Q = 3.33Lh^{3/2}$.

Time of emptying reservoirs, *e.g.*, a hemispherical bowl through orifice in bottom: $\dfrac{14}{15} \dfrac{\pi R^{5/2}}{A\sqrt{2g}} \dfrac{1}{C}$.

Current meter rating curves showing relation between velocity of water and time for meter to revolve a fixed number of revolutions.

Adiabatic expansion of gases: $P = \dfrac{C}{V^{1.41}}$, $P = Cw^{1.41}$. $V = \dfrac{C}{T^{2.44}}$, $P = CT^{3.44}$.

Power of steamships in terms of speed and displacement: Horsepower $= \dfrac{D^{2/3}S^3}{C}$.

Census and statistical data in engineering, industry, finance, and economics.

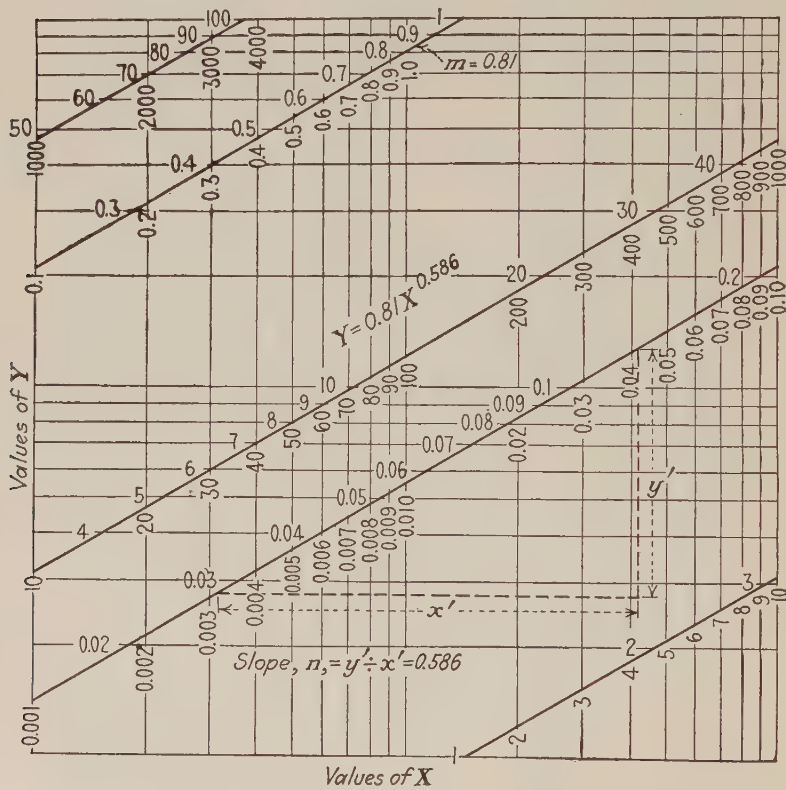


FIG. 74F.

Special Problems in Logarithmic Plotting

Instructions for problems 1 to 8. (See pp. 112, 113 for problems.)

Using ordinary cross-section paper and a table of logarithms, for part (*a*) the logarithms of the given simultaneous values are to be plotted, the line drawn, and the equation of the original variables found. See Fig. 74*B*, p. 104.

For part (*b*) the given equation is to be changed into the logarithmic form and represented as a straight line on ordinary cross-section paper. This line is to be used to find values of y corresponding to given values of x, indicating clearly how they are obtained. For example, let $x = 3.80$. What is y?

Using logarithmic paper for parts (*c*) and (*d*), the given simultaneous values are to be plotted directly, lines drawn, and the equation found without using logarithmic tables. For problems (*e*) and (*f*) the equations are to be written in the form $y = mx^n$ and then represented as straight lines covering a desired range of values. From the plotted line typical well-separated values of one variable are to be found for assumed values of the other variable. To establish confidence these should be checked by calculations.

For a method of extending lines to cover a wide range of values on a limited size of logarithmic paper, see Fig. 74*F*.

9. Represent the equation $y = \dfrac{26}{x^{0.46}}$ by a line on logarithmic paper and from the line pick off the value of y for $x = 50$. *Ans.* 4.3.

10. Given

x	y
0.023	35.0
0.285	2.40

, find the equation satisfying these simultaneous values by plotting on logarithmic paper.

11. Make a sketch showing how data on Q and h for a sharp-edged orifice would plot up on logarithmic paper, and explain the connection between the equation and the law for a freely falling body.

PROBLEMS 1–8.—EACH PROBLEM HAS SIX PARTS AND IS INTENDED TO BE A COMPLETE EXERCISE IN LOGARITHMIC PLOTTING FOR A STUDENT

(See instructions on p. 111)

Problem no.	(a)		(b)	(c)		(d)		(e)	(f)
	x	y		x	y	x	y		
1	0.66 1.89 5.50	0.080 0.375 1.81	$y = 0.04x^{0.39}$	0.165 0.435 0.845 1.31 1.82	0.207 0.75 1.82 3.27 5.08	0.0075 0.0155 0.0571	9.40 3.17 0.464	Relation between area (A) and diameter (d) of a circle (use foot as unit). $A = \frac{\pi}{4}d^2$. (d, 1 to 100.)	Relation between pressure (p) and volume (V) of a gas expanding adiabatically. $p = (p_o V_o^{1.41})V^{-1.41}$ p_o = 14.7 lb. per sq. in. (absol.) V_o = 2 cu. ft. (V, 0.1 to 100.)
2	0.462 0.95 4.40	0.0195 0.069 1.05	$y = 0.034x^{0.44}$	0.306 1.040 1.640 3.540 7.720	0.217 0.644 0.970 1.890 3.770	0.0069 0.0204 0.0620	7.90 1.575 0.310	Relation between shearing strength of steel pins (S) and diameter (d). Take unit strength = 12,000 lb. per sq. in. = s. $S = \frac{\pi}{4}sd^2$. (d, 0.1 to 10 in.)	Relation between diameter (d) and loss of head (h_F) for flow of water in a pipe of 1000-ft. length, the velocity (v) being constant at 8 ft. per sec. $d = 0.38^{0.8}v^{3/2}h_F^{-0.8}$. ($d$, 0.1 to 15 ft.)
3	0.093 0.495 1.43	2.56 5.95 10.2	$y = 0.50x^{1.87}$	0.350 0.631 1.030 2.320 4.400	0.206 0.452 0.870 2.600 6.100	12.5 41.5 147.	5.30 0.91 0.138	Relation between maximum theoretical power of a Pelton wheel ($L_{max.}$) and jet velocity (v). Take nozzle-tip diameter = 3 in. $L_{max.} = Qwh = (Awv^3) \div 2g$ A = area of nozzle tip. (v), 1 to 100.	Relation between absolute temperature (T) and volume (V) of a gas expanding adiabatically. T_o = 273°C. $T = (V_o^{0.41}T_o)V^{-0.41}$ V_o = 2 cu. ft. (V, 0.1 to 100.
4	0.093 0.52 1.50	0.256 0.61 1.05	$y = 0.35x^{2.10}$	0.280 0.510 1.020 2.230 5.200	0.105 0.232 0.590 1.680 5.200	18.6 116. 624.	3.46 0.688 0.1575	Relation between loss of head (h_F) and velocity (v) of water flowing in a pipe. Assume diameter and f constant. Diameter = 0.5 ft. f = 0.024. Length (L) = 1000 ft. $h_F = f\frac{L}{d}\frac{v^2}{2g}$. ($v$, 0.1 to 10.)	Relation between absolute pressure (p) and volume (V) for expanding steam. p_o = 300 lb. per sq. in. $p = (p_o V_o^{10/9})V^{-10/9}$ V_o = 0.8 cu. ft. (V, 0.8 to 30.

5	0.78 3.24 10.9	0.0587 0.260 0.940	$y = 0.16x^{1.59}$	0.432 0.800 1.420 2.080 7.540	0.123 0.281 0.610 1.020 5.780	13.5 34.2 125.0	3.50 0.891 0.131	Relation between slope (s) and velocity of flow (v) in a rectangular open flume, 6 ft. wide, 3 ft. deep. Assume Kutter's coefficient (C) constant at 105. $v = C\sqrt{Rs}$. (s, 0.0001 to 1) R = Area ÷ Wetted perimeter	Relation between loss of head (h_F) and diameter of pipe (d) if Q is constant at 1.0 c.f.s., length of pipe being 1000 ft. $h_F = 0.38\left(\frac{4Q}{\pi}\right)^{1.86} d^{-5}$ (d, 0.1 to 10 ft.)
6	0.81 3.05 10.0	0.61 2.45 8.55	$y = 0.13x^{1.68}$	0.207 0.572 0.970 1.820 5.480	0.345 0.850 1.350 2.350 6.120	18.8 82.9 385.	2.25 0.610 0.1575	Relation between discharge (Q) and head (h) for flow of water over a sharp-edged rectangular weir without end contractions. Assume length of weir constant at 5.0 ft. $Q_{c.f.s.} = 3.33\,Lh^{3/2}$ (h, 0.1 to 10 ft.)	Relation between diameter of pipe (d) and loss of head (h_F) with discharge, Q constant at 1.0 c.f.s. and length of pipe = 1000 ft. $d = \left[0.38^{0.20}\left(\frac{4Q}{\pi}\right)^{0.37}\right]h_F^{-0.20}$ (d, 0.1 to 10 ft.)
7	0.095 0.57 1.45	1.21 3.60 6.40	$y = 0.30x^{2.20}$	0.223 0.552 0.968 2.440 6.280	0.285 0.630 1.040 2.340 5.340	14.5 30.5 122.	2.44 0.820 0.105	Relation between discharge (Q) and head (h) for flow of water over an equilateral triangular weir. (h, 0.1 to 10 ft.) $Q = \mu\frac{4}{15}Lh\sqrt{2gh}$. $\mu = 0.62$ Note: Express L in terms of h.	Relation between volume (V) and absolute pressure (p) of expanding steam. p_o = 300 lb. per sq. in. (absol.) $V = (p_o^{9/10}V_o)p^{-9/10}$ V_o = 0.8 cu. ft. (p, 1 to 300.)
8	0.74 1.58 7.70	0.265 0.90 12.2	$y = 0.017x^{0.70}$	0.145 0.480 1.22 3.05 8.20	0.175 0.415 0.825 1.60 3.28	0.00205 0.0092 0.0710	15.8 6.90 2.25	Relation between absolute temperature (T) and pressure (p) of a gas expanding adiabatically. p_o = 14.7 lb. per sq. in. (absol.) $T = \left(\frac{T_o}{p_o^{0.29}}\right)p^{0.29}$ T_o = 273°C. (p, 1 to 1000 lb. per sq. in.)	Relation between loss of head (h_F) and diameter (d) for flow of water in a pipe of 1000-ft. length with velocity (v) constant at 8.0 ft. per sec. $h_F = 0.38v^{1.86}d^{-1.25}$. ($d$, 0.1 to 15 ft.)

CHAPTER VII

THE FLOW OF LIQUIDS THROUGH ORIFICES

63. Steady, Unsteady, Uniform, and Non-uniform Flows Defined. *Steady Flow.*[1]—Nearly all cases in this text, unless otherwise stated, deal with *steady flow,* in which the same volume of liquid flows past any given point *in each small unit of time, i.e.* **the discharge is constant.**

No flow is steady or uniform when started. A short time elapses, of necessity, before steady conditions can be established. *No ordinary flow is ever perfectly steady,* there being always some pulsations of longer or shorter duration, or some tremblings, in velocity and pressure. Practically, however, fluctuations of a few per cent. in velocity or pressure or size of stream are not regarded as inconsistent with the idea of steady flow. Many flows that appear quite steady to a casual observer will be found to be fluctuating considerably when closely examined. Still they are called steady flows. For example, the flow through a long pipe line carrying the discharge from a double-acting plunger or piston pump with air chambers in good order is considered practically steady as far as flow calculations are concerned, although a pressure gage shows fluctuations of several per cent.

Unsteady Flow.—For unsteady flow the discharge is not constant. It is exemplified by the changing conditions in rate of flow and in pressure during opening and closing of a faucet or valve in a pipe line; also by surging flow in a canal or pipe line due perhaps to governor action in a hydroelectric power plant retarding or accelerating the rate of flow, and by such cases of intermittently started and stopped flow as in the drive pipe of a hydraulic ram or ocean waves on a beach. In these cases conditions at any point undergo considerable change from instant to instant.

[1] The difference between turbulent and viscous flows is considered in Chap. XVI.

Non-uniform, Steady Flow.—The cases of flow through orifices, nozzles, Venturi meters, the flow over weirs, etc., deal with changing velocity and size of stream in a short distance. Such flows are called *non-uniform, i.e.*, with respect to velocity and size of stream from point to point along the line of flow.

Uniform Steady Flow.—The above-mentioned cases may be contrasted to the ordinary flow in long conduits of all classes, such as pipes, hose lines, aqueducts, flumes, sewers, canals, and rivers. In these latter cases there is little or no change in velocity or of size of stream from point to point along it, *i.e.*, the flow is properly called *uniform.*

Different laws control the two types of steady flow. Non-uniform flow is controlled by modifications of the law for a freely falling body, and uniform flow is controlled by the laws of fluid friction. Some important cases, such as short pipes and culverts, lie between these extreme types, having characteristics of both.

64. Physics Law of Free Fall.—Because of its extensive use in hydraulics the meaning of the physics law for a body falling freely in vacuo will be briefly reviewed; *viz.:*

$$v = \sqrt{2gh} \tag{32}$$

and its converse

$$h = \frac{v^2}{2g}, \tag{33}$$

where v stands for velocity, h for vertical height, and g for the acceleration constant of gravity commonly taken at 32.2 ft. per sec. per sec., $\sqrt{2g}$ being $\sqrt{64.4} = 8.02$ (foot-second units).

N. B.—If this value is used, h must be in feet and v in feet per second.

FIG. 75.

If a body from "at rest" at point 1 (Fig. 75) is allowed to fall freely without friction, it will pass point 2, a distance h vertically below point 1, with a velocity v equal to $\sqrt{2gh}$. For example, for $h = 0.01$ ft. (= about ⅛ in.), $v = 0.8$ ft. per sec.; for $h = 1$ ft., $v = 8.02$ ft. per sec.; for $h = 100$ ft., $v = 80.2$ ft. per sec.

Conversely, if from a point 2 a body is projected vertically upwards with a known initial velocity v, it will rise (if there is

no friction) to a height h, which $= \frac{v^2}{2g}$, before coming to rest. Or, for the falling body, if the downward velocity at point 2 is known, the height h can be computed. For example, for values of $v = 1$, 10, and 100 ft. per sec., respectively, the values of h are 0.0155, 1.55, and 155 ft.[1]

65. Flow through Orifices. *Introductory Considerations.*—Experiments show that a clear stream or jet of water (or of any other freely flowing liquid) escapes from an orifice with a velocity $v = \sqrt{2gh}$, nearly, where h is the *head* back of the orifice, either an actual overlying height of water in a tank or reservoir (Fig. 76) *or an equivalent pressure-head.* The actual velocity is some 1 to 3 per cent less than by this formula because of friction in

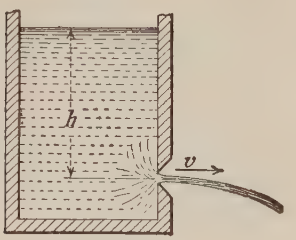


Fig. 76.

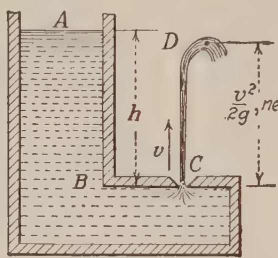


Fig. 77.

passing through the orifice, so that for closely approximate estimates we may state

$$v = 0.98\sqrt{2gh}, \tag{34}$$

the factor 0.98 being called the *velocity coefficient.*

In words, the velocity coefficient is the ratio of the actual issuing[2] velocity to the ideal, $\sqrt{2gh}$.

If a clear jet under a moderate head issues nearly *vertically upwards* (Fig. 77), it rises very nearly to the level of the water surface, thus showing that each particle of water in the issuing jet at C must have a velocity nearly equal to $\sqrt{2gh}$.

[1] The student of hydraulics should try to get a quantitative idea of the correspondence between a few such typical velocities and heights (or heads) so as to assist his judgment and to act as a general check against "wild" results in computations.

[2] See Fig. 79.

If the orifice pictured in Fig. 77 be closed and then suddenly opened, the very first particles of liquid coming out are seen to travel up and around the summit in the same path as thereafter traced by the continuous jet. Also, when the flow is suddenly interrupted at the orifice, the same path is followed by the last particles coming out. This is equally true for the orifice shown in Fig. 76—in fact, no matter in what direction the jet issues from the orifice.

This proves that each particle of water received its full impulse *at the orifice*, and that the particles in the jet are not pushed forward by those behind. The jet is *continuous* merely because of the slight cohesion among its particles. For great heights this cohesion is broken by air resistance.

Example: If a *clear jet* issues horizontally (Fig. 76) under a head of 6 ft., the velocity coefficient being 0.99, and the air resistance neglected, what is the issuing velocity, and how far vertically below the orifice level is the jet at a point 8 ft. out horizontally? *Given* that the vertical space passed over by a freely falling body is $s = \frac{1}{2}gt^2$. *Ans.* v = 19.44 ft. per sec. and s = 2.72 ft.

Another Example: If a jet issues with a velocity of 75 ft. per sec., what is the pressure in the water back of the orifice, the velocity coefficient being 0.98? *Ans.* 39.4 lb. per sq. in.

On page 122 more detailed consideration is given to various types of orifices and to the discharge through them, involving a variety of *velocity coefficients.*

66. Velocity-head.—In Fig. 77 the kinetic energy of the stream of water at C is made to reproduce the corresponding head by directing the stream vertically upwards. *But in any stream of liquid or gas flowing in any direction there is a corresponding kinetic energy that is capable of lifting the flowing liquid to a height that may be calculated from the velocity of the stream, viz.*, $h = \frac{v^2}{2g}$. The name velocity-head is applied to this function of the velocity because, aside from small losses by friction, the head that caused v is now in the form of kinetic energy, capable of again elevating the water to the original height.

Note that while v is a velocity, $\frac{v^2}{2g}$ is a height or *head*, a linear dimension, just as is $\frac{p}{w}$ (see p. 3).

67. Potential Head, Pressure-head, and Velocity-head.—If, in Fig. 77, the level of the orifice is taken as a reference level, it is customary to state that the water at A has a *potential head* (by virtue of its actual height, or *position in elevation* above datum) $= h$.

At point B, the water has a *pressure-head* (*i.e.*, a pressure corresponding to a head or height) $= \frac{p}{w}$.

At C the *flowing water* has a *velocity-head* (or head corresponding to its velocity) $= \frac{v^2}{2g}$.

In this special case (Fig. 77) where friction is very small and where full and free action is permitted, approximately $\frac{v^2}{2g} = \frac{p}{w} = h$.

Caution.—Unless it is perfectly clear that substantially all the pressure-head at any point is converted into velocity-head near that point, $\frac{v^2}{2g}$ should not be placed equal to $\frac{p}{w}$ (approximately).

Interconvertibility of Potential, Pressure-, and Velocity-heads.—In Fig. 77, again taking the datum level at B—C, the water at level A (in the tank) has elevation but has neither pressure nor velocity. At level B the water in the tank has lost elevation but has gained pressure, still without velocity. In going from B to C the water has gained velocity, no change in elevation. In going from C to D the pressure remains the same (atmospheric), but the water has lost its velocity and has again gained nearly the original elevation, thus completing the cycle of conversions.

Velocity-head Further Illustrated.—In Fig. 78 is shown an open stream flowing with velocity v. Against the current of the stream is pointed the opening of a *Pitot tube.* The water rises in the tube to a height h above the surface of the stream, such that $h = \frac{v^2}{2g}$ or $v = \sqrt{2gh}$, *i.e.*, the Pitot-tube water column balances and measures the *velocity-head* of the flowing water.

If a tube of this sort is inserted in a pipe which contains water flowing under pressure (Fig. 78*A*) the water column in the tube rises to a height h_v above the height shown by an adjoining open water column attached to a hole in the wall of the pipe; *i.e.*, the Pitot-tube column in this case shows the sum of the pressure-head and the velocity-head of the water at the "point opening" of the tube, thus incidentally demonstrating that both these sorts of head coexist.[1]

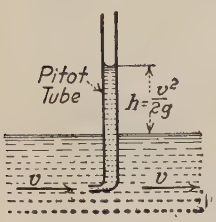


FIG. 78.

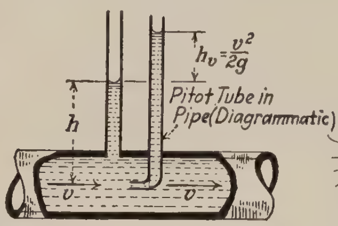


FIG. 78*A*.

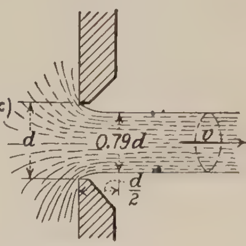


FIG. 79.

68. Discharge through Orifices.—The discharge Q, or volume of flow per unit of time, is given by

$$Q = Av, \tag{35}$$

where A is the cross-sectional area, and v the average velocity of the *issuing stream* of water. The issuing stream is not in all cases of the same size as the "clear opening" of the orifice because of *contraction effects* in certain cases and *Venturi,* or *flaring, effects* in other cases. (Read Par. 72, p. 126.)

Contraction Effects.—For a "square-edged" orifice (also called "sharp-edged" and "orifice in thin plate") the issuing stream does not attain the velocity given by Eq. (34), $v = 0.98\sqrt{2gh}$, as it passes through the plane of the opening at the upstream face. The actual conditions of flow are as shown in Fig. 79.

The stream touches only the upstream edge of the orifice, and at that cross-section the "stream lines" are not parallel but converging. In the plane of the opening all the pressure-head has not yet been converted into velocity-head.

At a distance of about one-half the diameter of the orifice out from the plane of the upstream edge, the convergence ceases,

[1] The velocity varies across the pipe (see p. 336).

the stream lines are parallel, and all of the pressure-head has been converted into velocity-head. It is the velocity at this downstream point to which Eq. (34) applies.

For a strictly *square-edged* orifice the contracted stream has a cross-section about 0.62 of the area of the orifice. Hence

$$Q = A'v = 0.62A \times 0.98\sqrt{2gh}, \text{ or } Q = 0.61A\sqrt{2gh}. \quad (36)$$

The ratio 0.62 is known as the "contraction coefficient" and 0.61 is called the combined or "discharge coefficient."

In words, the **discharge coefficient** is the ratio of the actual discharge to that computed from the full area of opening and

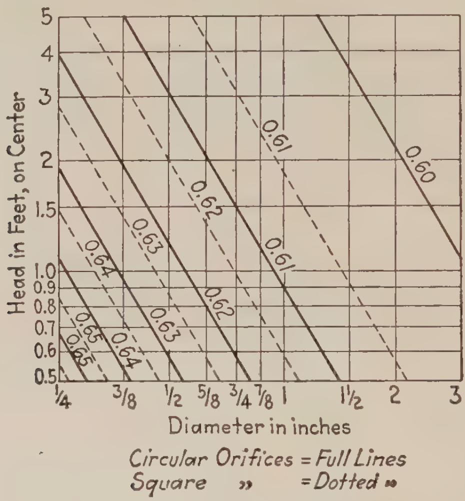


FIG. 80.—Coefficients for square sharp-edged orifices.

the ideal velocity. The **contraction coefficient** is the ratio of the cross-sectional area of the issuing stream (Fig. 79) to that of the orifice.

For ordinary purposes of design the value 0.60 for the discharge coefficient of a square-edged orifice is sufficiently precise. For small orifices less than 1 in. in diameter the discharge coefficient increases somewhat as the diameter decreases. Figure 80 shows Hamilton Smith's coefficients for square-edged orifices, free discharge for low heads. These are *discharge coefficients*, and the diagram shows the approximate range of the heads and diameters in the experiments analyzed by Smith. The data for this

diagram are from the experiments of Poncelet and Lesbros, 1827–1835, and those of Hamilton Smith, 1885, and earlier. These sets of experiments do not agree exactly, the diagram showing average results approximately. Many more recent experiments establish the reliability of these values within about 1 per cent.

69. Effects of Rough Upstream Face.—If the *upstream face* of the metal or material in which the hole is cut is *rough;* the flow approaching along the face at right angles to the issuing jet is retarded, thus *lessening the contraction* and *increasing the discharge.* The increase in discharge coefficient may be as much as 2 per cent for a roughness like coarse sandpaper as compared to smoothly finished metal.

Rounding the upstream edge of a square-edged orifice, ever so slightly, decreases the contraction and increases the discharge. The size of the issuing stream is chiefly affected, *not its velocity,* which remains practically the same. Figure 83 shows results of tests. There is an increase of about 3 per cent in discharge for a rounding of the upstream edge with radius of rounding only 1 per cent of the diameter of the orifice. When the radius of rounding of the upstream corner is 10 per cent of the orifice diameter, the discharge is about 30 per cent greater than for the same sized opening with square edges. The law of increase is Q per cent $= 3.1 \times R$ per cent, where R per cent means the percentage ratio of the radius of rounding to the diameter of the orifice.

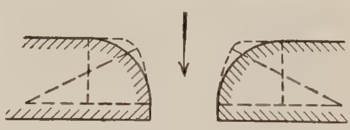


FIG. 81.

The results of tests shown on Fig. 83 indicate, and other experiments agree, that the quadrant of a circle is not the best curve for fully rounded-entry orifices. To secure a discharge coefficient $= 0.98$ or higher, which is 60.7 per cent increase over 0.61, it is necessary to have the convergence less rapid than is given by quadrant roundings of radius equal to the orifice diameter. The region where the flow has been accelerated to more than half the exit velocity is the critical portion of the orifice where the convergence must not be too rapid. In Fig. 81 the dotted lines represent a better discharging orifice than the solid lines.

Submerged Discharge.—When the stream issues under water, the contraction effect is reduced and the discharge is increased, whenever the bore of the orifice has any length, even when as short as one-quarter the diameter. But for the *orifices strictly "in thin plate,"* or beveled to a thin edge, the *coefficient for submerged discharge is the same as that for free discharge.* Table IV, page 133 shows the coefficients for various thicknesses of plate or wall for square-edged entry; and Fig. 84 shows the effect of rounding the entry for various lengths of such "short-pipe" orifices (also called orifices in thick wall).

Contraction Suppressed on One or More Sides of Orifices.—If a rectangular orifice is so located that it is close to the bottom plate or side wall of the tank, reservoir, canal, etc., the effect is to guide the approaching flow so as to suppress the contraction more or less on those sides of the orifice close to such wall guides. Experiments of Bidone and Weisbach indicate an *increase* of about 0.04 in the sharp-edged coefficient *for each side* on which contraction is entirely suppressed (side flush with wall).

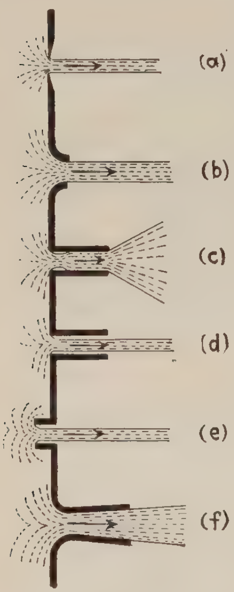


Fig. 82.

70. Various Types of Orifices (Fig. 82). (*a*) *The square-edge orifice* with discharge coefficient = 0.60+ (see Fig. 80). The stream is contracted to about 0.62 of the area of the opening.

(*b*) *Orifice with well-rounded entry* with discharge coefficient = 0.98±. There is little, if any, contraction, the issuing stream being the same size as the opening. But if the rounded approach be too abrupt, there may be some contraction.

(*c*) *The "short-pipe"* or *orifice in thick wall,* with square-edge entry. The coefficient of discharge, *when flowing full,* = 0.82±. There is a contracted stream within the orifice. This expands and fills the downstream end of the tube. The issuing stream is "troubled" or turbulent at low heads, "broomy" at high heads.

(*d*) The same with stream jumping free from tube at upstream corner, so that flow occurs precisely as in case (*a*). Note that this does not occur when the exit is submerged. (See Fig. 84 for coefficients of submerged orifices.)

(*e*) *The reentrant orifice,* also called *Borda's mouthpiece,* when the length is less than the orifice diameter. If tube is several diameters long, it may flow *full* with coefficient $= 0.72\pm$. For flow as in sketch, coefficient of discharge $= 0.53\pm$.

(*f*) *Flaring* or *Venturi* orifice, called by latter name when entry is rounded as in sketch. The velocity in the "throat" of the Venturi orifice may be as much as 55 per cent greater than $\sqrt{2gh}$, because, when the whole tube flows full, the pressure in the throat is less than atmospheric (or less than pressure at exit). Thus the *flow-producing head* for the throat section is greater than the externally apparent head h. (For discussion of the Venturi principle, see p. 142.)

If the downstream end is not submerged, the tendency of the stream to fill the flaring portion of the tube is lessened, and if the divergence angle is greater than above 7.5 deg., or if the tube is much less than five times the throat diameter in length or more than 10 diameters, or if the head is too high (see (*d*) above), the tube may not flow full. The discharge coefficient for the downstream terminal opening is always less than 1.00, being usually less than 0.80.

For case (*c*) the velocity at the contracted stream within the tube is also greater than $\sqrt{2gh}$, surrounded as it is with eddying water at pressure less than atmospheric (or less than the pressure at exit).

71. Loss of Head and Its Relation to the Velocity Coefficient and to the Issuing Velocity-head.—By reversing Eq. (34),

$$h = \frac{1}{0.98^2}\frac{v^2}{2g} = 1.04\frac{v^2}{2g} \tag{37}$$

But for "free fall in vacuo" the height or head $= \dfrac{v^2}{2g}$. Therefore for the stream of water from such an orifice there is a *loss of head* $= 0.04\dfrac{v^2}{2g}$, or the actual issuing velocity-head is $0.96h$.[1]

[1] The *loss of head,* and of available energy in the flowing fluid, goes into heat. See end of par. 100, p. 177.

Example: Experiments show (see p. 132) that the velocity coefficient for a short pipe from 3 to 6 diameters long (with

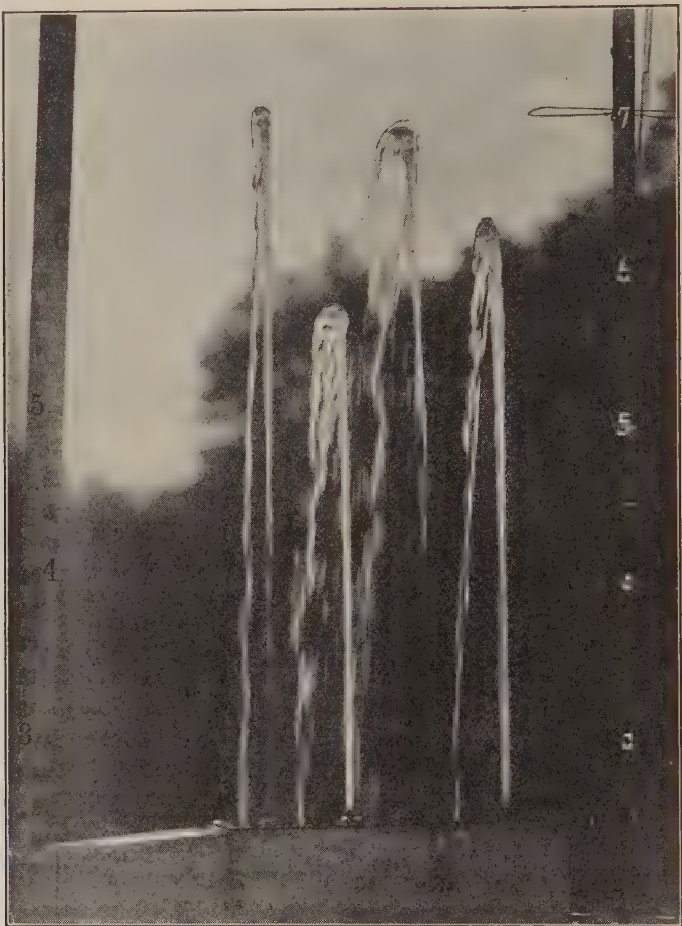


FIG. 82*A*.—The jets in this photograph are from circular orifices each with clear diameter ($\frac{3}{8}$) inch. From left to right: square-edge in thin plate, par. 70(*a*); "short-pipe" with square-edge entry, par. 70(*c*); well-rounded entry, par. 70(*b*); Venturi, ($\frac{3}{8}$) inch throat, ($\frac{1}{2}$ inch) exit, par. 70(*f*).

The head on the orifices was 4.70 ft., and the average summits of the jets failed to attain this height by percentages, respectively: 3.6, 31.9, 4.5, and 20.0. Estimating ($\frac{1}{2}$) of 1 per cent for reduction of height of jet due to air resistance, the indicated exit velocity coefficients are, respectively: 0.985, 0.827, 0.980, 0.896.

square-edge entry and flowing full at the downstream end) is 0.82.

Question: What is the loss of head suffered by the water in passing through the "short-pipe" orifice?

Solution: Given $v = 0.82\sqrt{2gh}$, reversing which, as in deriving Eq. (37),

$$h = \frac{1}{0.82^2}\frac{v^2}{2g} = 1.49\frac{v^2}{2g}, \text{ or } \frac{v^2}{2g} = 0.67h.$$

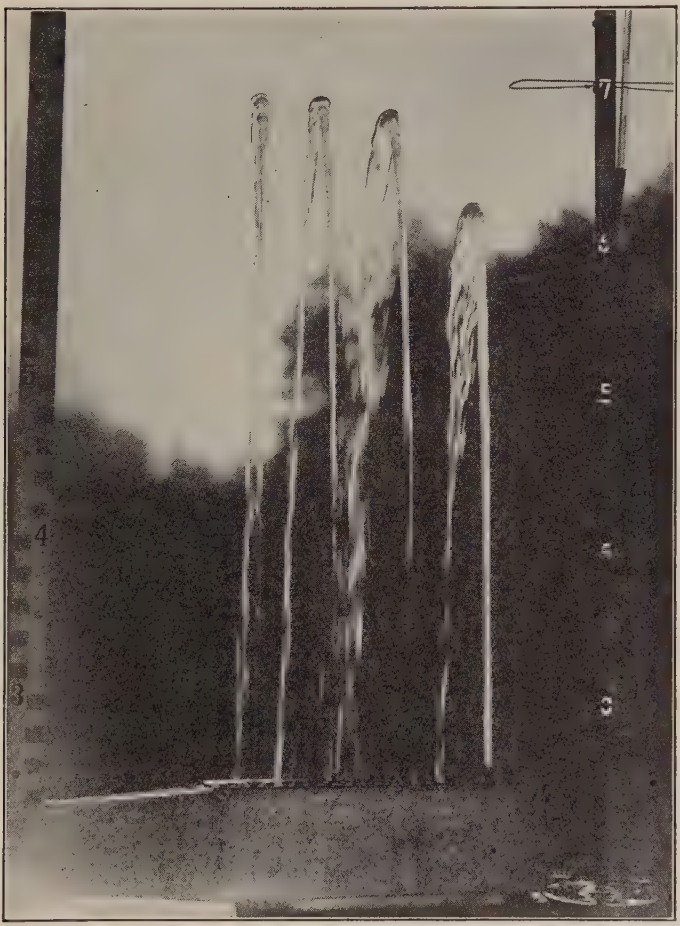


FIG. 82*B*.—In this photograph the jet from the "short-pipe" (second from left) jumps free at the upstream entry corner and acts precisely like the jet from the orifice in thin plate (left orifice). Compare with Fig. 82*A*.

Hence either: (1) loss of head $= 0.49 \times$ *the issuing velocity-head;* or (2) since $(h - 0.67h) = 0.33h$, loss of head $= 0.33 \times$ *the head*

acting on the orifice, the usual phrase being "0.33 of the head is lost." (See Fig. 82*A*, 2nd jet from left.)

Method (1), just mentioned, of expressing loss of head in terms of the velocity head in the stream is quite common in hydraulics. For example, assume that the loss of head, for the case just discussed, is, in round numbers, $0.5 \frac{v^2}{2g}$. Then since, in general, it may always be stated that *total available head* (at the upstream point) equals "*loss of head*" plus *remaining head* (at the downstream point); for this case,

$$h = 0.5 \frac{v^2}{2g} + \frac{v^2}{2g}. \tag{38}$$

(Obviously, this method of subdividing the total head is generally appropriate not only for orifices, but also for all sorts of pipes and open channels or other liquid streams, no matter how great the loss of head is, or how small the remaining head.)

Experimentally, if the stream from the above-discussed "short-pipe" orifice were directed nearly vertically upwards (Fig. 77), it would rise to only 0.67 of the height up to the water surface (air resistance neglected). (See Figs. 82*A* and 82*B*.)

For heights of a few feet, and with crystal-clear jets, the air resistance reduces the rise only about 1 per cent or less below $\frac{v^2}{2g}$, but for fire-extinguishing streams from nozzles as commonly used by fire departments the stream falls some 15 or 20 per cent short of the ideal height, which is perhaps some 100 ft. for the average good fire stream. Even at two-thirds of the ideal height, fire streams "broom out" into a 15- to 30-in. width of heavy spray. Of course, even if the jet remains solid, there is some increase in its diameter as it goes upwards, because the velocity decreases, thus necessitating a greater cross-section. It is just the reverse of the case shown in Fig. 87.

72. Equation of Continuity.—For steady flow in any continuous stream of liquid the discharge passing through any one cross-section is constant. If there is no flow added or taken away as the stream proceeds, all other cross-sections will show the

same rate of flow. Since, in general, $Q = Av$, then for different cross-sections along a stream,

$$Q = A_1V_1 = A_2V_2 = A_3V_3, \text{ etc.,} \tag{39}$$

where the subscripts refer to the several cross-sections.[1]

73. Large Orifices with Low Heads, *e.g.*, Sluice Gates (see also Submergence).—Unless the orifice is in a horizontal plane, *e.g.*, in the bottom of a tank, the top may be so much nearer the surface of the liquid than the bottom that the head at the level of center of orifice does not correspond to the real average velocity of all the water flowing out, but to more than the average, *i.e.*, the actual discharge is less than that shown by the formula. (For theoretical principle, see Weirs, p. 149.) When the head above the center is equal to the vertical dimension of the orifice, the discharge is only about 1 per cent less, and when the head is twice the vertical dimension the diminution is negligible, except for the most precise sort of investigations, for which special calibration should be made to determine the exact coefficient. For lower heads, if h_c represents the head above the center and O the height of opening, Table II gives the percentage reduction in discharge and the corrected discharge coefficient for circular and rectangular orifices. The value of the coefficient, however, is not certain within from 1 to 3 per cent without a special calibration of the particular orifice.

Table II.—Corrections for Low Heads on Rectangular and Circular Orifices

(Using 0.61 as the normal coefficient for high heads)

$\frac{h_c}{O}$	Rectangular orifices		$\frac{h_c}{O}$	Circular orifices	
	Reduction in discharge, per cent	Corrected coefficient of discharge		Reduction in discharge, per cent	Corrected coefficient of discharge
0.50	5.7	0.575	0.50	4.0	0.585
0.62	3.2	0.590	0.55	3.2	0.590
0.87	1.6	0.600	0.73	1.6	0.600
1.00	1.1	0.603	1.00	0.8	0.605

[1] For *gases*, due account must be taken of the changes in volume accompanying changes in pressure and temperature.

74. Orifice meters and pipe orifices, *i.e.*, orifices in diaphragms inserted between pairs of flanges in straight portions of pipe lines, and *orifices in end caps* on pipes, have been found to be reliable measuring devices. The diaphragms are usually of thin non-corrodible metal. The formula is the same as the nozzle formula on page 139, where h in this case is the drop in head between two piezometers located respectively about one pipe diameter upstream from the diaphragm, and close to (but not less than) one-half pipe diameter downstream. When the piezometers are placed several diameters away from the orifice,

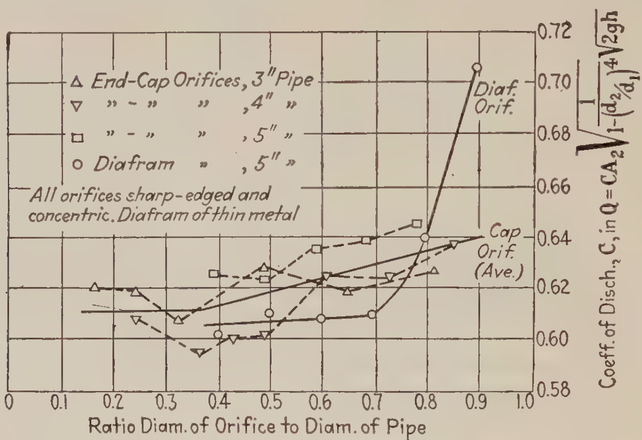


FIG. 82*C*.—Horace Judd's pipe orifices. Orifices in diaphragms and in end-caps. (*Trans. Am. Soc. Mech. Eng.*, 1916.)

the coefficients may be considerably larger. For the end-cap orifices, the data for which are plotted in Fig. 82*C*, the pressure connection was one and one-half pipe diameters upstream from the orifice. Values of the discharge coefficients (averages from Judd's values, *Trans. Am. Soc. Mech. Eng.*, 1916) are given in Fig. 82*C*. For values of the correction factor for velocity of approach, see Table V, page 144.

Judd recommends that no diaphragm orifice be used larger than 80 per cent of the pipe diameter, to assure uniform flow conditions and steady pressure-drop readings. Either a differential gage or open water columns may be used, depending on the pressure in the pipe, head room, etc. There is considerable

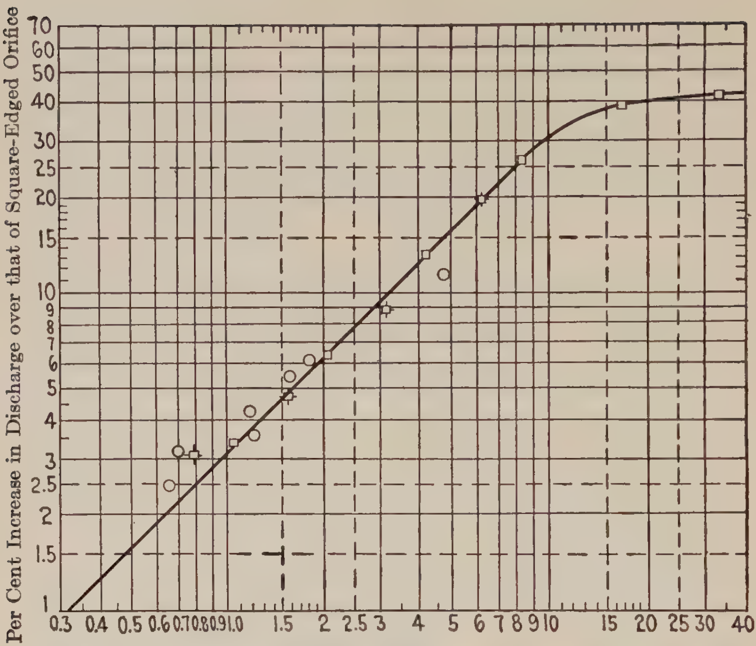
restoration of pressure-head between one-half and four pipe diameters downstream from the diaphragm orifice. The loss of head due to inserting a diaphragm orifice in a 5-in. pipe line was found to average as follows for concentric orifices: In per cent of h, the loss of head is, respectively, 23, 38, 52, 65, 70, 80, 89, and 96, for orifice diameters (in per cent of the pipe diameter) of 90, 80, 70, 60, 50, 40, 30, and 20. (*Trans. Am. Soc. Mech. Eng.*, 1916.)

TABLE III

Effects of Rounding the Entry Edges of Orifices (Experiments at Cornell University).—Free discharge into the atmosphere. (For *submerged discharge*, see Table IV and the plotting in Fig. 84.) All roundings were circular quadrants approximately. Data in table are arranged in the order of the relative *radius* of rounding in percentage of orifice diameter. These data are plotted in Fig. 83.

(Thickness of plate = radius of rounding of edge of orifice)

Shape of orifice	Diameter of circular, or side of square opening, inches	Radius of rounding of upstream edge		Coefficient of discharge C in formula $Q = C(A\sqrt{2gh})$	Per cent increase in discharge over sharp-edged orifice	
		In inches	In per cent of diameter of orifice			
Circle	3/8	Sharp	0	0.609	0	The tests on circular
Circle	15/16	Sharp	0	0.601	0	orifices made by J. O.
Circle	1	Sharp	0	0.601	0	Jones in 1915; the
Circle	1 3/8	Sharp	0	0.600	0	tabulated values being
Square	6	Sharp	0	0.611	0	for 6-ft. head, al-
Square	8	Sharp	0	0.608	0	though the heads in
Circle	1 3/8	0.0090	0.65	0.615	2.5	tests ranged from
Circle	1	0.0070	0.70	0.620	3.2	0.5 to 40 ft.
Square	8	1/16	0.78	0.627	3.13	The tests on 6-in.
Square	6	1/16	1.04	0.632	3.4	square orifices made
Circle	15/16	0.0080	1.16	0.627	4.3	by T. L. Smith and
Circle	3/8	0.0045	1.20	0.631	3.6	C. B. Briggs in 1919;
Circle	1 3/8	0.0215	1.56	0.633	5.5	the tabulated values
Square	8	1/8	1.56	0.6365	4.7	being averages for
Circle	1	0.0180	1.80	0.638	6.2	heads ranging from
Square	6	1/8	2.08	0.650	6.4	0.6 to 2.8 ft.
Square	8	1/4	3.12	0.662	8.9	The 8-in. square ori-
Square	6	1/4	4.16	0.693	13.4	fices were similarly
Circle	3/8	0.0175	4.70	0.680	11.6	tested by E. V. How-
Square	8	1/2	6.25	0.725	19.3	ell in 1918.
Square	6	1/2	8.33	0.772	26.4	
Square	6	1	16.67	0.850	39.1	
Square	6	2	33.33	0.867	41.9	



Ratio $\frac{R}{d}$ in per cent, or radius of rounding of edge of orifice in per cent. of diameter of orifice.

FIG. 83.—Increase of discharge due to rounding the entry edges of orifices. Logarithmic plotting of data in Table III. Free discharge into the atmosphere.

Circles and squares, respectively, indicate those shapes of orifice.

The law of per cent increase in Q, as indicated by the straight portion of the line drawn on Fig. 83, for roundings with radius up to 10 per cent of the diameter of orifice, is

$$\text{Per cent. increase in } Q = 3.1 \times \text{ the per cent. that } R \text{ is of } d,$$

where R is the radius of the quadrant rounding of orifice edge, and d is the orifice diameter.

In drawing the line (Fig. 83), account has been taken of the fact that the roundings of the circular orifice were not always true quadrants of circles and that the equivalent radius could not be determined without considerable probable error. (*See Cornell Civil Engineer*, December, 1917.) On the other hand, the 6- and 8-in. square orifices had their upstream edges rounded to fit steel templates with true circular quadrants. The plotting, including, as it does, size ranging from $\frac{3}{8}$-in. circular to 8-in. square, is probably fairly general in application. (For tubes with *submerged discharge*, see Fig 84.)

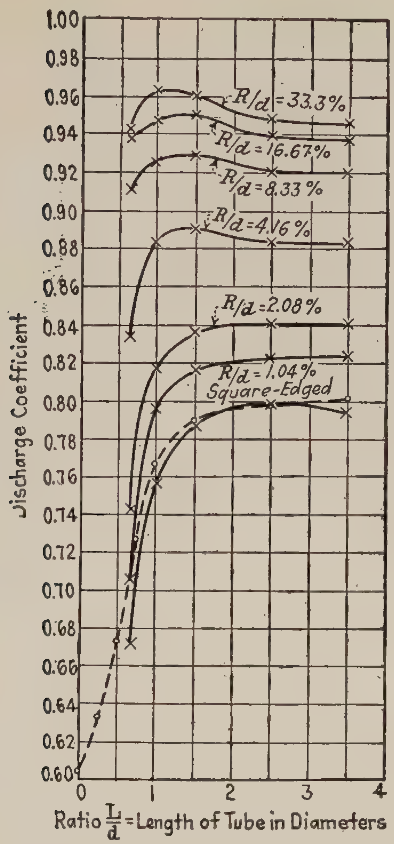


Fig. 84.—Submerged orifices and short tubes (discharge under water) with entry edges rounded to various degrees of rounding.

All roundings to quadrants of circle of radius R.

These tests made on 6-in. square openings and tubes (d = 6 in.).

Circles—tests by T. C. Rogers and T. L. Smith, 1916 (*Eng. News*, Vol. 76, p. 825, 1916). (See Table IV.)

Crosses—tests by T. L. Smith and C. M. Briggs, 1917; both at Cornell University.

The heads (= differences of water levels upstream and downstream) ranged from 0.2 to 2.0 ft. The plotting shows average values of the discharge coefficient.

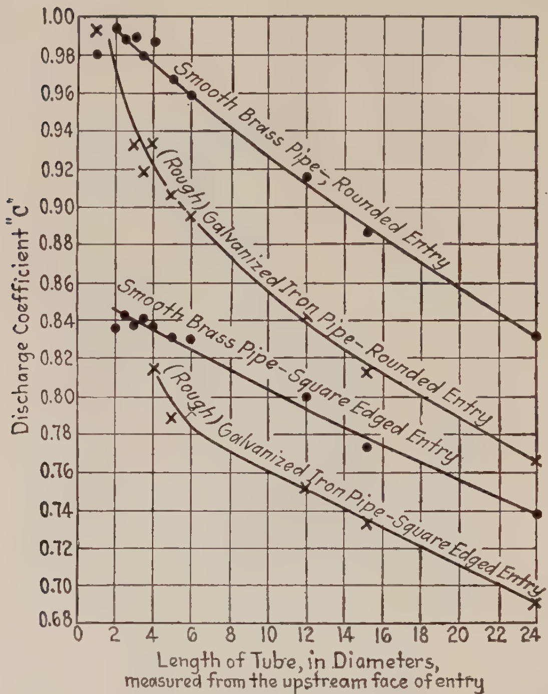


FIG. 85.—Short Tubes, Free Discharge into the Atmosphere, showing reduction of discharge due to addition of various lengths of pipe to the orifices.

All orifices and all pipes were $\frac{3}{4}$ inch nominal pipe size, actual inside diameter = 0.830 in. All experiments made with 3-ft. head on horizontal tubes.

The ordinates in the plotting above are values of the *discharge coefficient* C in formula $Q = C(A\sqrt{2gh})$.

Experiments made at Cornell University, by E. W. Schoder, June, 1919.

Notes on Conditions during Experiments.—Data plotted are for pipes *flowing full* at the downstream end. With square-edged entry, both the smooth and the rough pipes, up to lengths including $L = 6$ diameters, discharged as orifices in "thin plate" *if started that way;* coefficient = 0.64 (see Fig. 82*d*).

(These data are analyzed under The Flow of Water in Pipes on pages 192–196.)

Non-Uniform flow is controlled by the laws of acceleration and conservation of energy,
Uniform flow controlled by the laws of friction

Table IV

Submerged Orifices and Short Tubes (*i.e.*, Discharging under Water).—All openings *square* with *square-edged* entry. Tests on 4-ft. size by C. B. Stewart, University of Wisconsin (see *Eng. News*, Jan. 9, 1908). Tests on 6-, 8-, and 10-in. sizes by T. C. Rogers and T. L. Smith, Cornell University. (See *Eng. News*, Nov. 2, 1916.) The heads (difference between up- and downstream water levels) in Wisconsin tests ranged 0.02 to 0.38 ft., and at Cornell from 0.2 to 2.0 ft. Table gives average coefficients for the 6-, 8-, and 10-in., and 4-ft. square tubes, the mean values for the different sizes differing but slightly. These coefficients are plotted in Fig. 84 by circles and dotted line.

Length of tube in diameters	Coefficient of discharge
$\frac{L}{d} = 0$, "Thin Plate"	0.60
0.25	0.634
0.50	0.674
0.75	0.728
1.00	0.767
1.50	0.790
2.50	0.797
3.50	0.802

Problems

1. For a freely falling body (vertical fall), air resistance negligible, if the velocity at a certain point of the path is 30 ft. per sec., what is the velocity at a point 12 ft. higher? *Ans.* 11.34 ft. per sec.

2. A fountain consists of a vertical jet from a 6-in. pipe. (Area = 0.7854 ÷ 4 = 0.1964 sq. ft.) The discharge is 4.91 c.f.s. How many feet high does the jet rise? (Neglect air resistance.) *Ans.* 9.72 ft.

3. Given the physics law, $s = \frac{1}{2}gt^2$, and the discharge issuing at 45 deg. above horizontal, from a 2-in. circular rounded orifice, area 0.0218 sq. ft., and $Q = 0.436$ c.f.s. How far out horizontally from the orifice is the peak of the jet, if friction of air is neglected? Note that the issuing jet has components of velocity both horizontal and vertical. Sin 45° = 0.707 = cos 45°. *Ans.* 6.21 ft.

4. Define and illustrate with sketches and assumed values: (*a*) the coefficient of discharge; (*b*) coefficient of velocity; (*c*) coefficient of contraction for an orifice in thin plate. (*d*) If an orifice 4 in. in diameter were rounded to a radius of 0.25 in., what change would be made in the discharge coefficient? (Use Fig. 83 in text.) *Ans.* (*d*) 20 per cent.

5. Referring to Table IV for coefficients, compute the discharge (*a*) through a *submerged* square orifice, 3 ft. on each side, cut through a wall 2 ft.

thick, upstream edges square edged. The water is 12 ft. deep above the top of opening on the upstream side, and 3 ft. deep above top of opening on downstream side. (*b*) Find Q for same conditions but with upstream edges rounded to $1\frac{1}{2}$-in. radius. (*c*) Find Q for conditions as in (*b*) except that the water surface of the downstream pool is 5 ft. below the bottom of the opening.

Ans. (*a*) 154 c.f.s.
(*b*) 181 c.f.s.
(*c*) 183 c.f.s.

6. The jets issuing vertically from different orifices, with general conditions as in Fig. 82, when flowing full have discharge coefficients (referred to tips): short pipe, 0.82; Venturi, 0.92. Neglecting air resistance, compute heights of jets from these two orifices, both flowing full, in terms of tank head.

Ans. $0.672h$.
$0.846h$.

7. Compute the discharge in c.f.s. through a rectangular head-gate opening (considered as a sharp-edged orifice) 3 ft. wide and 4 ft. high, water level on the upstream side being 15 ft. and on the downstream side 6 ft. above top of opening. *Ans.* 176 c.f.s.

8. The discharge of a short pipe culvert *flowing full* at exit is known to be exactly *one-half* of the "ideal" value (as computed by assuming no loss due to friction). What is the loss of head expressed: (*a*) in percentage of the issuing velocity-head; (*b*) in percentage of the total head?

Ans. (*a*) 300 per cent.
(*b*) 75 per cent.

9. The "short-pipe orifice," or "orifice in thick wall," when flowing full and with square-edged entry has a discharge coefficient of 0.82. From these data show why the loss of head at a square-edged entry to a pipe is taken at $0.5\frac{v^2}{2g}$. The entrance to a pipe may be taken as a short-pipe orifice.

10. There is a flow of 4.81 c.f.s. from the pipe into the tank (Fig. P.47). (*a*) What should be the size of a square orifice (orifice in thin plate), center of orifice 1 ft. above bottom of tank, so that the water level in the tank will remain at 17 ft.? (*b*) What should be the radius in inches to which the upstream edge of the above orifice should be rounded, so as to keep the water level at 17 ft., if the flow in the pipe is increased to 5.195 c.f.s. (Reference may be made to diagrams in text for part (*b*).)

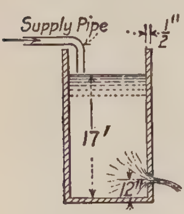


FIG. P.47.

Ans. (*a*) 0.496 ft.
(*b*) 0.153 in.

11. Compute the discharge in c.f.s. of a jet of water of $\frac{1}{2}$ sq. ft. cross-section, if it is known that the energy in the jet is 500 hp. (In foot-second-pound units horsepower $= \frac{Qwh}{550}$.) *Ans.* 41.4 c.f.s.

12. A head gate supplies water from a pond to a canal. The rectangular *submerged* opening is practically a sharp-edge orifice 2.5 ft. wide by 4 ft. high. The pond water level is 7 ft. above the top edge of the opening on one side and the canal water level is 4 ft. above the top edge on the other side. Coefficient is same as for free discharge. Calculate approximate discharge in c.f.s. *Ans.* 83.4 c.f.s.

CHAPTER VIII

CONVERGING AND DIVERGING FLOWS. BERNOULLI'S THEOREM

75. Extension of Physics Law of Free Fall.—By simple extension of the reasoning concerning free fall (p. 115), if a freely falling body has a known velocity v_1 as it passes point 1 (Fig. 86) then at point 2, a distance h ft. below point 1, it will have a velocity

$$v_2 = \sqrt{2gh_2} = \sqrt{2g\left[\left(\frac{v_1^2}{2g}\right) + h\right]} \tag{40}$$

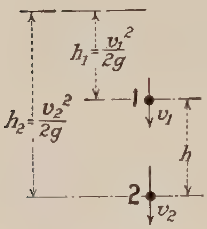


FIG. 86.

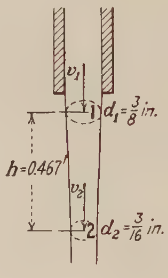


FIG. 87.

because the height h_1 (which corresponds to the velocity v_1) $= \frac{v_1^2}{2g}$, so that the total equivalent height of free fall above point $\underline{2}$ is $h_2 = h_1 + h = \frac{v_1^2}{2g} + h$. From Eq. (40), or directly by inspection of Fig. 86,

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}. \tag{41}$$

76. Clear Jet of Liquid Falling Vertically. (Corresponding Statements May Be Made for a Clear Jet Rising Vertically.)—By opening a water faucet part way, allowing a crystal-clear stream to run out, it is seen that the diameter of the falling stream becomes smaller as it descends. By measurements, in a particu-

lar case, it is found that it takes just 7.4 sec. for ½ lb. of water to be discharged. Other measurements are shown in Fig. 87.

Three separate illustrative calculations will now be made, treating one of the values as unknown, to find how closely the computed results, based on the law of a freely falling body, agree with these measurements.

Note that the jet of water between points 1 and 2 is flowing unconfined (not as it would be if passing through a tapering nozzle) and is surrounded throughout by uniform atmospheric pressure.

1. To calculate h from the other measured (or given) values, namely: Q, the volume of flow per unit of time; d_1 and d_2, the diameters at the upstream and downstream points.

From the equation of continuity, $v_1 = \frac{Q}{A_1}$, where A_1 is the cross-sectional area of the stream at point 1. Also, since it is a case of "steady flow," $v_2 = \frac{Q}{A_2}$. The volume of ½ lb. of water is $\frac{0.5}{62.4} = 0.00801$ cu. ft. and this in 7.4 sec. is at the rate of $0.00801 \div 7.4 = 0.00108$ cu. ft. per sec. (c.f.s.) *at all points along the stream.*

$$v_1 = \frac{Q}{A_1} = \frac{0.00108}{0.000767} = 1.407 \text{ ft. per sec.}$$

Likewise, $v_2 = \frac{Q}{A_2} = \frac{0.00108}{0.000192} = 5.63$ ft. per sec. (More simply, since velocities vary *inversely* as areas of cross-section, or inversely as the square of the diameter, and since in this case $d_1 = 2d_2$, $v_2 = 4v_1 = 4 \times 1.407 = 5.63$ ft. per sec.)

From Eq. (41), $h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} = 0.493 - 0.031 = 0.462$ ft. *Ans.*

(This is a little less than the measured value because no allowance was made for friction of the air.)

(To get the numerical value of $\frac{v^2}{2g}$ on the slide rule, set runner on value of v (lower scale) and divide the opposite upper scale value by 64.4 ($= 2g$ in foot-second units), result on upper scale.)

2. To calculate Q from given values for h, d_1 and d_2. Whether using Eq. (40) or Eq. (41), in either case one velocity must be

expressed in terms of the other and the diameters. As far as the mathematics are concerned, either of the two velocities could be chosen, but from the hydraulics viewpoint it is preferable to deal with the expression for v_2, the downstream velocity, eliminating v_1.

From Eq. (41), $h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$. But $v_1 = \frac{A_2}{A_1}v_2 = \frac{d_2^2}{d_1^2}v_2$

$$\text{therefore, } \frac{v_1^2}{2g} = \left(\frac{d_2}{d_1}\right)^4 \frac{v_2^2}{2g}.$$

Hence

$$h = \left[1 - \left(\frac{d_2}{d_1}\right)^4\right]\frac{v_2^2}{2g}, \tag{42}$$

and

$$v_2 = \sqrt{\frac{1}{1 - \left(\frac{d_2}{d_1}\right)^4}}\sqrt{2gh} \text{ (if there were no friction),[1]} \tag{43}$$

The meaning of the term h in Eq. (44) for nozzles, Venturi meters, etc. is discussed on pages 139, 140, and 142.

For the present case of the falling jet, the value of the first radical in Eq. (43) is 1.033, and $v_2 = 1.033\sqrt{2g \times 0.467}$, (in foot-second units), or $v_2 = 1.033 \times 5.47 = 5.66$ ft. per sec. Then $Q = A_2v_2 = 0.000192 \times 5.66 = 0.001088$ c.f.s., (*Ans.*), or a trifle more than the measured or true discharge.

Above, in Eq. (43), a formula of general utility was developed before solving the particular problem. However, more directly, since $\left(\frac{d_2}{d_1}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$, hence $h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} = \frac{v_2^2}{2g} - \frac{1}{16}\frac{v_2^2}{2g}$. Therefore, $0.467 = \frac{15}{16}\frac{v_2^2}{2g}$, whence $v_2 = 5.66$ ft. per sec. as before.

[1] In this particular case of a clear jet falling a short distance, the friction would have an almost inappreciable effect, but if the flow should be through a smooth tapering nozzle or other convergence, the actual velocity would be some 1 to 3 per cent less than Eq. (43) indicates. For rough nozzles, rough Venturi meter convergences, etc., the velocity might be from 5 to 10 per cent less. For smooth metal or other convergences v_2 may be assumed (in accord with experiments) about 2 per cent less, and therefore for such cases

$$v_2 = 0.98\sqrt{\frac{1}{1 - \left(\frac{d_2}{d_1}\right)^4}}\sqrt{2gh}. \tag{44}$$

3. To calculate the value of d_2 from the other measured or given values of d_1, Q, and h: To find d_2 we must find A_2, which may be found when v_2 is found, because $A_2 = \frac{Q}{v_2}$. Since d_1 is known, A_1 and v_1 may be found as in case 1 (p. 137).

From Eq. (41), $h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$ whence $v_2 = \sqrt{2g\left(h + \frac{v_1^2}{2g}\right)} =$ $8.02\sqrt{0.467 + 0.031} = 5.66$ ft. per sec. Then $A_2 = \frac{0.00108}{5.66} =$ 0.000191 sq. ft. Hence $d_2 = 0.0156$ ft. $= 0.187$ in., (*Ans.*), a trifle smaller than the actual or measured diameter.

77. Flow of Liquid through a Converging Nozzle.—With steady flow proceeding through the nozzle (Fig. 88) it is noted that, instead of an *actual descent* of the liquid from point 1 to point 2, as in Fig. 87, we have a corresponding decrease in the *pressure-head.*

At point 1, if the pressure is p_1, the pressure-head is $\frac{p_1}{w}$, as would be shown by an open water column. At point 2 the pressure is merely atmospheric, the free jet having no tendency to burst out sidewards.

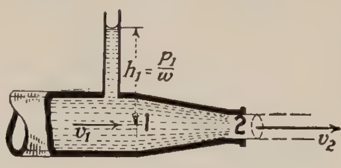


FIG. 88.

In Fig. 87 the water between points 1 and 2 has gained velocity at the expense of *potential head* or head of position in elevation. In Fig. 88 there is likewise a gain of velocity as the water passes through the narrowing passageway of the nozzle, and this gain is at the expense of the pressure-head in the water.

If, then, it is understood that h means the *drop in pressure-head* between the upstream point 1 (sometimes called the *base* of the nozzle), and point 2 at the "tip" of the nozzle, Eq. (43) is applicable.

But since loss by friction between points 1 and 2 cannot be omitted (see footnote, p. 138), the actual value v_2 is less than the ideal calculated by Eq. (43). If C is the coefficient required to reduce this calculated value to the actual velocity, then

$$v_2 = C\sqrt{\frac{1}{1 - \left(\frac{d_2}{d_1}\right)^4}}\sqrt{2gh} \qquad (45)$$

(If the shape of cross-section changes between points 1 and 2 the ratio $\left(\frac{A_2}{A_1}\right)^2$ is to be used instead of $\left(\frac{d_2}{d_1}\right)^4$).

Note.—Equation (45) need not be memorized by the student. It is advisable to calculate the *ideal* velocity v_2 directly from the general basic equation $\frac{v_2^2}{2g} = \frac{v_1^2}{2g} + h$, using the equation of continuity to express v_1 in terms of v_2 and then to modify this ideal v_2 by the proper coefficient.

However, the drop in pressure-head is not always measured by the height called h in Fig. 88, *viz.*, the height of top of water column above the tip of the nozzles. If the nozzle discharges into a partial vacuum (as into the condenser of a steam engine), or into a space having gas pressure above atmospheric, or into another body of liquid, due account must be taken of such facts so as to use the actual *flow-producing difference of pressure-heads* for the h of Eq. (45). The student should never forget that flow is not caused by a pressure but by a difference of pressures.

Equation (45) may also be written

$$\frac{v_2^2}{2g} - \frac{v_1^2}{2g} = C^2h, \tag{46}$$

and

$$h = \frac{1}{C^2}\left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right).^{1} \tag{46a}$$

In words, referring to the application of Eqs. (45), (46), and (46*a*) to such a case as shown in Fig. 88, it may be stated that if the friction causes a coefficient 0.98, *i.e.*, the issuing velocity to be 2 per cent less than that to be inferred from the diameters and full utilization of the *flow-producing difference of head*, or 2 per cent less than by Eq. (43), then the difference of velocity-heads is $C^2h = 0.96h$, or 4 per cent less than the difference of pressure-heads; and the difference of pressure-heads is 4 per cent greater than the difference of velocity-heads (precisely 4.12 per cent).

If the velocity coefficient in Eq. (45) were 0.99 instead of 0.98, then the difference of velocity-heads would fall short only 2 per cent of the difference of pressure-heads (precisely 2.03 per cent). If the velocity coefficients were 0.97, the shortage would be 6 per cent (precisely 6.28 per cent).

[1] Compare these two equations with Eq. (37), p. 123.

Nozzle Problems.—For a nozzle as in Fig. 88:

(*a*) Compute d_2 in inches if $d_1 = 6$ in. and $Q = 1.55$ c.f.s., velocity coefficient 0.97, for a pressure at point 1 of 37 lb. per sq. in. *Ans.* 1.98 in.

(*b*) Compute Q for such a nozzle with $d_1 = 6$ in., $d_2 = 3$ in. and $p_1 = 50$ lb. per sq. in., the velocity coefficient being 0.98. *Ans.* 4.28 c.f.s., or 1920 U. S. g.p.m., or 2,760,000 U. S. g.p.d.

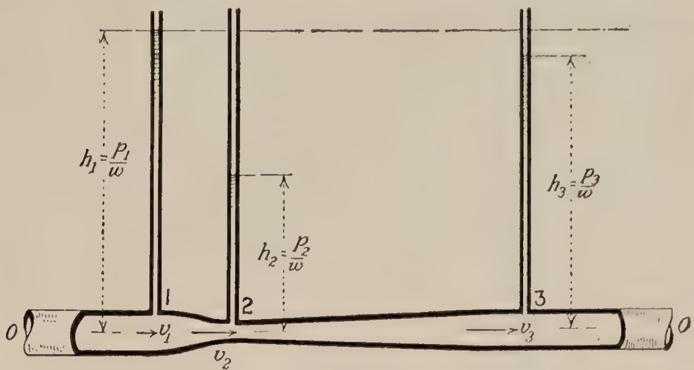


FIG. 89.

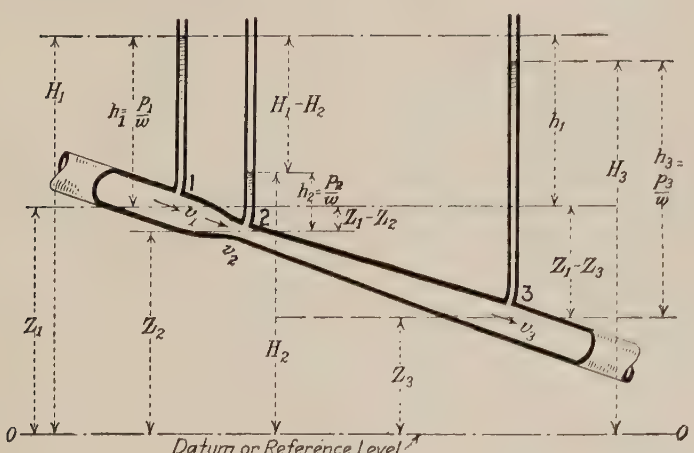


FIG. 89*A*.

78. Flow through Venturi Meter Tubes.—Figure 89 shows a Venturi meter tube laid level, and Fig. 89*A* shows one laid on an incline. Such tubes consist of a rather rapid convergence to a

$H_L = K \frac{V_2^2}{2g}$

short cylindrical "throat" followed by a divergence to the size of the pipe line in which the Venturi tube is inserted.

Equation (45) applies if it is understood that h means the $h_1 - h_2$ of Fig. 89 or the $H_1 - H_2$ of Fig. 89A.

Note that if there were no flow, a valve somewhere downstream being closed, the pipes would be under merely hydrostatic pressure, and both $h_1 - h_2$ and $H_1 - H_2$ would be zero.

In Fig. 89A, z_1, z_2, and z_3 are elevations of points 1, 2, and 3 above any chosen datum; p_1, p_2, and p_3 are the pressures in the pipe at points 1, 2, and 3. To express $H_1 - H_2$ in terms of the actual pressure-heads and potential heads at points 1 and 2, it is noted from Fig. 89A that $H_1 = h_1 + z_1$, and $H_2 = h_2 + z_2$, or $H_1 = \frac{p_1}{w} + z_1$, and $H_2 = \frac{p_2}{w} + z_2$, so that $H_1 - H_2 = \left(\frac{p_1}{w} - \frac{p_2}{w}\right) + (z_1 - z_2)$. Hence, for the ideal case of no friction, Eq. (41) for free fall may be written

$$h = H_1 - H_2 = \frac{v_2^2}{2g} - \frac{v_1^2}{2g},$$

or

$$\left(\frac{p_1}{w} - \frac{p_2}{w}\right) + (z_1 - z_2) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g},$$

or

$$\frac{v_1^2}{2g} + \frac{p_1}{w} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{w} + z_2, \tag{47}$$

which is the usual method of expressing *Bernoulli's theorem.*

79. Bernoulli's Theorem.—In words, Bernoulli's theorem states: Comparing the conditions at two points along a stream line (*i.e.*, the path along which the liquid flows), the *total head* at the *upstream point* is equal to the *total head* at the *downstream point, provided* there is no loss between the two positions due to friction, the giving up of work, etc. and no gain due to the application of outside work. *By total head is meant* the sum of the velocity-, pressure-, and potential-heads.

Bernoulli's theorem is a statement of the *conservation of energy.* It has already been noted that the several kinds of heads are mutually convertible, each into the other's form. Thus, multiplying through Eq. (47), by $W = Qw$, the weight of flowing water passing any section per second, there results $W\frac{v_1^2}{2g} + W\frac{p_1}{w} + Wz_1 = W\frac{v_2^2}{2g} + W\frac{p_2}{w} + Wz_2$. But $\frac{W}{g} = M$, the mass

per second, and it has already been seen that $\frac{p_1}{w}$ is merely a height, say h_1. Hence $\frac{1}{2}Mv_1^2 + W(h_1 + z_1) = \frac{1}{2}Mv_2^2 + W(h_2 + z_2)$, and this is recognizable as a "work and energy" equation.

If both points *1* and *2* are at the same level, the potential heads cancel out in Eq. (47). If one pressure-head is zero, as at *2* in Fig. 88, and the level datum line passes through both points, we have left merely Eq. (41) (p. 136).

Practically, the difference $h_1 - h_2$ of Figs. 89 and 90, and $H_1 - H_2$ of Figs. 89A and 90A, may be replaced by h, so returning to the basic Eq. (41). However, if the fall that causes the increase of velocity is not readily discernible, the formal application, item by item, of Bernoulli's theorem may be of much assistance. Note that each term is a *head* or a *height*.

For *open channels*, Fig. 90 shows conditions where the stream *contracts in width, e.g.*, the flow between bridge piers out in the

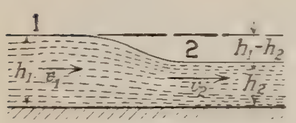


FIG. 90.

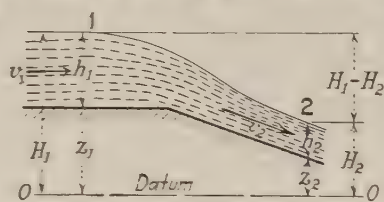


FIG. 90A.

middle of a river. Figure 90A shows the flow over the spillway of a dam or at the head of rapids in a river. Bernoulli's theorem is applicable to each of these cases. (Actually, the presence of a "standing wave" at or near point *2* may somewhat obscure the location of the true hydraulic surface.) In words, the downstream velocity-head = the upstream velocity-head plus the surface fall *minus* the loss of head due to friction (see p. 140).

80. Examples of Application of Bernoulli's Theorem.—For a Venturi meter inclined as in Fig. 89A, point *1* being 2 ft. above point *2*, with d_1 = 12 in. and d_2 = 8 in., velocity coefficient = 0.98 (= discharge coefficient also in this case), what must be the rate of flow so that, with pressure at point *1* = 5 lb. per sq. in., the pressure at point *2* (the throat) shall be 2 lb. per sq. in. *below* atmospheric? The student should draw a sketch properly dimensioned.

Solution.—At first neglecting friction, *i.e.*, calling the coefficient 1.00, using Bernoulli's theorem,

$$\frac{v_1^2}{2g} + \frac{p_1}{w} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{w} + z_2,$$

taking the datum level through point *2*, $\frac{v_1^2}{2g} + (5 \times 2.31) + 2 = \frac{v_2^2}{2g} + (-2 \times 2.31) + 0$, or $\frac{v_2^2}{2g} - \frac{v_1^2}{2g} = 11.55 + 2 + 4.62 =$ 18.17 ft. In this form this is merely Eq. (41) with 18.17 substituted for h. Thus Bernoulli's theorem assures a proper itemization of the values going to make up the *fall in pressure-head* between points *1* and *2* (see p. 141).

But, as on page 138, $\frac{v_1^2}{2g} = \left(\frac{d_2}{d_1}\right)^4 \times \frac{v_2^2}{2g} = \left(\frac{8}{12}\right)^4 \times \frac{v_2^2}{2g} = 0.1975 \times \frac{v_2^2}{2g}$, whence $\frac{v_2^2}{2g}(1 - 0.1975) = 18.17$, or $\frac{v_2^2}{2g} = \frac{18.17}{0.8025} = 22.64$ ft. Hence, by slide rule, $v_2 = 38.2$ *ft. per sec.* (if there were no friction). The actual $v_2 = 0.98 \times 38.2 = 37.4$ ft. per sec., and $Q = A_2 v_2 = 0.349 \times 37.4 = 13.05$ *c.f.s.* (sometimes called "second feet"). *Ans.*

81. Effect of Neglecting the Velocity of Approach.—The value of the expression $\sqrt{\frac{1}{1 - \left(\frac{d_2}{d_1}\right)^4}}$, or its equal, $\sqrt{\frac{1}{1 - \left(\frac{A_2}{A_1}\right)^2}}$, which corrects for the velocity of approach, is as follows for the range of probable values of $\frac{d_2}{d_1}$.

TABLE V.—CORRECTION FACTORS FOR NOZZLES

$\frac{d_2}{d_1}$	$\sqrt{\frac{1}{1 - \left(\frac{d_2}{d_1}\right)^4}}$	$\frac{d_2}{d_1}$	$\sqrt{\frac{1}{1 - \left(\frac{d_2}{d_1}\right)^4}}$
0.1	1.00005	0.5	1.033
0.2	1.0008	0.6	1.072
0.25	1.002	0.67	1.116
0.3	1.004	0.7	1.147
0.33	1.006	0.75	1.210
0.4	1.013	0.8	1.301

The correction factor is seen to be practically unity for any ratio $\frac{d_2}{d_1}$ less than 0.3, and therefore for such nozzles the following simpler formula may be used:

$$Q = CA_2\sqrt{2gh}.$$

Problems

1. A clear jet rises vertically from a well-rounded orifice 1 in. in diameter. Neglecting loss by friction at the orifice and in the air, what is the diameter of the jet halfway up to the summit? *Ans.* 1.19 in.

2. The "Venturi difference" of a 6 in. × 3 in. meter is measured by a U-tube containing mercury. It is found that when the difference in mercury levels is 28.35 in. the discharge is 970 g.p.m. (*a*) What is the discharge coefficient of the meter? (*b*) Is the velocity coefficient of the meter the same?

3. A smooth circular nozzle held horizontally, with diameters upstream 8 in., downstream (tip), 4 in., is used to discharge and to measure a flow of water. A volumetric measurement of the discharge shows 939 cu. ft. in 150 sec. A pressure-gage located 8 ft. higher than the center of nozzle communicates by a small pressure pipe with a piezometer hole in the wall of the 8-in. pipe just upstream from the nozzle convergence. The gage indicates 29.8 lb. per sq. in. (*a*) Calculate the discharge coefficient of the nozzle. (*b*) How much is the velocity-head in the jet just outside the nozzle? State units with answer. (*c*) What per cent of the total head is lost in the passage of the water through nozzle? (*d*) Express this loss also in per cent of the velocity-head in the issuing jet.

N. B.—Make a sketch to show nozzle and gage. Calculate as accurately as the slide rule permits, say to within one-third to one-fifth of 1 per cent. Roughly approximate values will not do in this case.

Ans. (*a*) 0.988.
(*b*) 79.8 ft.
(*c*) 2.44 per cent.
(*d*) 2.51 per cent.

4. If the discharge from the nozzle shown in sketch Fig. P.48 is 0.80 c.f.s., and the discharge coefficient = 0.98: (*a*) Compute the reading of the pressure gage. (*b*) What would the same gage read if d_1 were 10 in.?

Ans. (*a*) 54.5 lb. per sq. in.
(*b*) 58.5 lb. per sq. in.

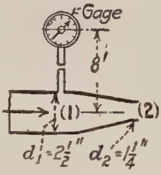


Fig. P. 48.

5. When the nozzle of a common garden hose, is held horizontally 3 ft. above the ground, it is noted that the issuing stream strikes the ground a distance 30 ft. from the nozzle. The tip diameter of the nozzle is ¼ in. and the upstream diameter is ⅝ in. (*a*) Compute the discharge through the nozzle in gallons per minute. (*b*) Compute the available horsepower in the

issuing stream. (*c*) Assuming that the nozzle has a discharge coefficient 0.92, compute the pressure in pounds per square inch at the base of the nozzle. (*d*) Compute the lost head (in feet) through the nozzle and express it as a decimal part of the issuing velocity head.

6. (*a*) A fireman's nozzle (easy taper, no contraction of issuing jet) is $1\frac{3}{8}$ in. in diameter at the tip. The fire engine pumps 300 g.p.m. through the hose and nozzle from a pool whose surface is 50 ft. below the level of the nozzle. How many cubic feet and how many pounds of water are flowing per second? (*b*) What is the velocity of the issuing jet from nozzle in feet per second? (*c*) What is the velocity-head of the jet at the tip? (*d*) To what height in feet would the stream rise above nozzle if there were no air resistance to overcome? (*e*) How many foot-pounds per second does it require merely to lift the water from the pool to the nozzle, changing potential head only, at 100 per cent efficiency? (*f*) How much energy is represented in the issuing nozzle jet in foot-pounds per second? (*g*) If the back pressure due to friction in forcing the water through 200 ft. of hose amounts to an extra lift of 70 ft., how many horsepower must the fire engine produce to force the water from the pool through the nozzle?

Memo.—1 hp. = 550 ft.-lb. per sec. 1 cu. ft. = 7.48 gal.

7. (*a*) From the nozzle shown in Fig. P.48 but changing d_1 to 3 in. and d_2 to $1\frac{1}{2}$ in., the actual discharge is 0.80 c.f.s. The discharge coefficient is 0.98. Write in algebraic form Bernoulli's theorem in its complete general form for the ideal (frictionless) case, applying it between points 1 and 2. Indicate by a cancellation mark those terms that equal zero. (*b*) From data and equation of (*a*) write the algebraic expression for actual v_2, eliminating any other velocity and introducing the dimensions of the nozzle. (*c*) Now, from (*a*) and (*b*) write the expression for actual v_2 in terms of actual pressure-head at point 1, and compute this actual pressure-head (in feet). (*d*) Compute the pressure-head at gage and gage reading in pounds per square inch. Gage is 8 ft. above nozzle.

8. (*a*) With data as in problem 7 (*a*), compute the velocity head at 1. What is it if d_1 is 12 in.? (*b*) Would this larger d_1 change the gage reading? If so, which way and how much? State this last item exactly, if possible, otherwise approximately.

9. Clear jet rising nearly vertically, air resistance negligible. At issue point A the cross-section = 4 sq. in. Compute the cross-section at B, a height $\frac{h}{3}$ above A. *Ans.* 4.90 sq. in.

10. Venturi meter similar to Fig. 89. With a flow of 15 c.f.s. in a main of 24-in. diameter (= also upstream diameter of meter) find the throat diameter so that the pressure at throat point 2 will just support a water column 10 ft. high, the pressure at 1 being 9.5 lb. per sq. in. The coefficient of the meter is 0.985.

N.B.— First find the theoretical expression for the throat velocity. This simplifies the work. *Ans.* 9.96 in.

11. Fire-stream nozzle diameters: upstream, $2\frac{1}{2}$ in.; tip, $1\frac{1}{4}$ in. Gage at base of nozzle 80 lb. per sq. in. Velocity coefficient, 0.99. How high above nozzle should stream rise if there were no air resistance and the stream remained a solid jet?

12. In a Venturi meter, 6 c.f.s. are flowing. The ratio of upstream to throat diameters is 2 to 1, the upstream or pipe diameter is 12 in. If the loss of head through the meter is one-eighth of the throat velocity-head, what is its amount in feet of water? *Ans.* 1.81 ft.

13. A special Venturi meter (with large throat) has upstream diameter = 15 in. and throat diameter = 10 in. The discharge coefficient is 0.99. In a test, the gage difference is 0.95 ft. on a U-tube gage with liquid of sp. gr. 1.25 in the U-bend. Gage is connected to upstream and throat piezometers, and hose and tops of U columns are full of water. (*a*) Apply Bernouilli's theorem and, at first neglecting friction, derive an expression for the throat velocity in terms of the dimensions and the difference of pressure-heads (water). (*b*) By hydrostatics, derive (using plainly labeled sketch) an expression for the difference of pressure-heads (water) in terms of the observed gage difference. (*c*) Compute the discharge in cubic feet per second.

14. Jets of water issue vertically from orifices as in Fig. 82. Measured discharges in 100 sec. are: Venturi, 2.00 cu. ft.; short pipe, 1.00 cu. ft. Head is 4 ft. Diameters are: Venturi, $\frac{3}{8}$ in. and $\frac{1}{2}$ in., short pipe, $\frac{3}{8}$ in. (*a*) Compute the discharge coefficient for the short pipe. (*b*) For the Venturi orifice (referred to tip). (*c*) Neglecting air resistance, how high does short pipe jet rise? (*d*) The Venturi jet? (*e*) Apply Bernouilli's theorem to flow between throat and tip of Venturi orifice and, neglecting friction, compute the pressure at the throat referred to atmosphere as zero. (*f*) If friction be considered, is the throat pressure greater or less than calculated? Why?

15. A 24- by 18-in. Venturi meter has a coefficient 0.98. What is the flow in cubic feet per second proceeding through the meter when the mercury difference in a U-tube mercury gage is 4.21 in.? Connecting lines from pipe to U-tube are filled with water.

16. Water-wheel nozzle (see Fig. 139). If 500 hp. is the energy in the issuing jet 6 in. in diameter, what is the Q in cubic feet per second, assuming a discharge coefficient = 0.98, the upstream diameter of the nozzle being 18 in.?

Ans. 22.2 c.f.s.

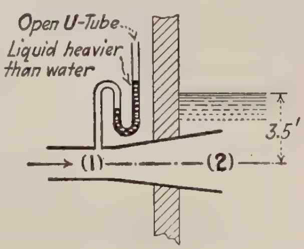


Fig. P. 49.

17. Horizontal flaring tube, discharge under water, as in Fig. P.49. Given: diameters, $d_1 = 2$ ft.; $d_2 = 4$ ft., $v_1 = 20$ ft. per sec. Compute the water pressure at point 1, (*a*) first assuming no friction, then (*b*) allowing 0.8-ft. loss between points 1 and 2. As an important detail in problems of this sort, first express the pressure conditions at 1 in terms of

pressure-head (usually feet of water). (*c*) Would the pressure-gage (U-tube) indicate as shown in sketch? *Ans.* (*a*) -1.0 lb. per sq. in.

(*b*) -0.66 lb. per sq. in.

(*c*) Should indicate opposite to sketch.

18. The vertical draft tube of a turbine flares as shown in (Fig. P.50). This reduces the exit velocity-head (which represents lost power). It is full of water flowing downwards and the bottom end of the tube is 5 ft. under water. Assuming that the loss due to friction is one-twentieth of velocity-head at 1, compute the water pressure in the top of the tube at point 1, stating the result first in feet of water column, then in pounds per square inch. Given the velocity at point 1 = 20 ft. per sec.

Ans. 30.5 ft. water suction.

13.2 lb. per sq. in. suction.

19. Suction pipe of a drainage pump (see Fig. P.51). Pipe is full of water with flow proceeding as shown by the arrow. Diameters: at point 1, 4 ft.;

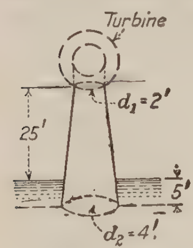


Fig. P.50.

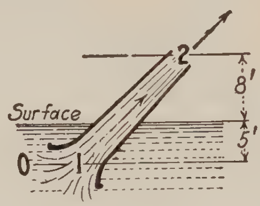


Fig. P.51.

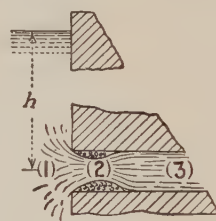


Fig. P.52.

at point 2, 2 ft. If the pressure at point 2 is 9.5 in. mercury vacuum, and it is known that the loss of head by friction between points 1 and 2 is just one-tenth of the velocity-head at point 2, compute the discharge in cubic feet per second. Negligible loss between 0 and 1. *Ans.* 39.9 c.f.s.

20. (*a*) Through Beebe Lake dam there had been cut horizontally a hole 5 ft. in diameter (circular) closed on the upstream face of dam by a sluice gate. When the gate is wide open the hole acts as a short-pipe orifice, *i.e.*, with loss of head = 0.5 × the issuing velocity-head. For water surface in Beebe Lake 20 ft. above the center of the hole, compute the discharge in cubic feet per second. (*b*) Referring to Fig. P.52 with loss of head from 1 to 3 as given in (*a*) and assuming that the contracted stream area $A_2 = 0.60A_3$, so that $v_2 = \frac{5}{3} \times v_3$, first, find v_3 in terms of $\sqrt{2gh}$, then find $\frac{v_2^2}{2g}$ in terms of h. Second, apply Bernoulli's theorem between 1 and 2 (neglecting the small friction in this part) and find value of h in feet so that there shall be a perfect vacuum at 2 (sea-level conditions).

Ans. (*a*) 575 c.f.s.

$(b)_1$; $v_3 = \sqrt{\frac{2}{3}}\sqrt{2gh}$, $\frac{v_2^2}{2g} = 1.85h$.

$(b)_2$ 40 ft.

CHAPTER IX

THE FLOW OF WATER OVER WEIRS

82. Definitions.—By a *weir* is meant a bulkhead, barrage, or dam, over which water flows, or a notch in the top of such a structure through which water flows.

The *crest* of a weir is the topmost part of the structure, or the bottom of the notch cut in the topmost part, to which level the water must rise before any can flow over.

The *head* on a weir is the height above the crest level of the surface of the water in the pool immediately upstream from the weir.

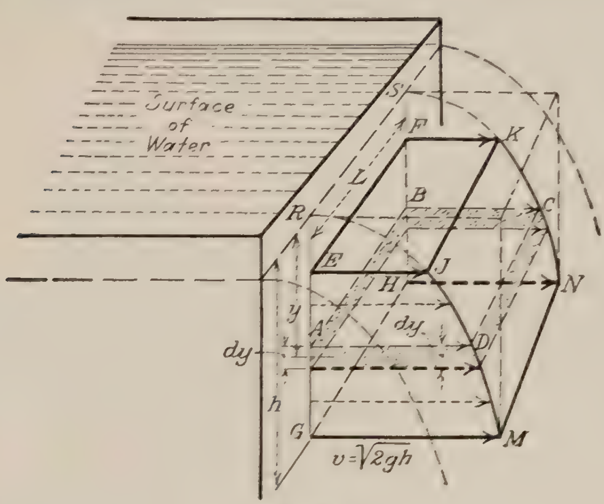


Fig. 91.

83. The Weir Flow Theory.—In Fig. 91, *EFGH* represents a large rectangular opening in the vertical side of a tank. Obviously, the water flowing out near the bottom of such an opening does so under the pressure of a greater head than does the water near the top of the opening.

If the sides of the opening extend up above the water surface, a rectangular weir notch exists in the side of the tank.

The *common theory* regards a case of flow over a weir as the summation of the elemental flows through a *series of horizontal slits* one above the other, each discharging in accordance with the orifice law.

In ordinary notation $Q = Av$, and in differential notation $dQ = dA \times v = (Ldy)v = Ldy\sqrt{2gy}$, where L is the horizontal length of the elemental slit (AB in Fig. 91), and dy is its height, y being the average head on the slit considered as an individual orifice.

This elemental discharge per second, dQ, is represented in Fig. 91 by the dotted *layer ABCD*. The *total ideal discharge* is the summation of all such elemental discharges from surface to crest (top to bottom).

For a rectangular weir notch with head $= h$, *i.e.* with the water surface back in the tank or pool at a height h above the level of the bottom of the notch:

$$Q = \int_0^h L\sqrt{2gy}dy,$$

or

$$Q = \int_0^h y^{1/2}\, dyL\sqrt{2g} = \frac{2}{3}h^{3/2}\ L\sqrt{2g} = \frac{2}{3}Lh\sqrt{2gh}. \qquad (48)$$

This is seen to be two-thirds of the ideal discharge through an opening of height h, length L, and under the head h. Graphically, the ideal volume discharged per second through a rectangular weir notch such as $RGHS$ (Fig. 91) is the paraboloidal prism $RGMSHN$. Its volume is known from calculus to be *two-thirds* that of the circumscribing parallelopiped with volume $Lh\sqrt{2gh}$, *i.e.*, $Q = Av = (Lh)(2/3\sqrt{2gh})$, as in Eq. (48).

84. Actual Discharge over Weirs.—By experimental measurements it is found that this ideal or "theoretical" law of variation of the discharge with the head, *i.e.*, Q varies as $h^{3/2}$, holds closely in actual flow over weirs. However, the value of the constant of Eq. (48), $2/3\sqrt{2g}$ (for unity L), which is 5.35 in the foot-second system of units, must be reduced considerably to designate correctly the actual rate of discharge.

Under the most favorable conditions for maximum discharge, such as that of an A-shaped dam with rounded crest, this value is seldom more than 4.00, still representing a reduction from the ideal of 25 per cent. In most cases of weirs and dams, the reduction is still more. This is in decided contrast to the flow through a well-rounded orifice with a coefficient 0.98.

Contraction Effects.—In the "channel of approach" upstream from the weir all the water moves forward, not only that above the level of the crest of the weir but also that below the crest level. The forward velocity of the water below the crest level is changed to an upward flow owing to the barrier interposed by the upstream face of the weir. The kinetic energy of this water flowing upwards towards the crest along the upstream face of the weir causes the lower profile of the overfalling sheet of water actually to rise as it passes over the crest, if free to do so.

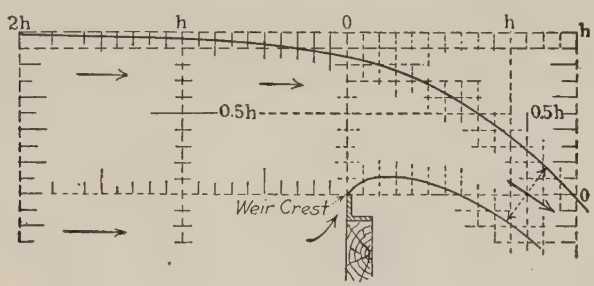


Fig. 92.

Figure 92, drawn to scale, shows the shape of the profiles of the upper and lower water surfaces for flow over a *vertical sharp-crest weir.*[1] At a point vertically above the weir crest, the upper surface has fallen $0.15h$ from the upstream water level and is $0.85h$ above the crest level. The lower surface rises to $0.11h$ at about $0.25h$ downstream from the crest, and is back down to the crest level at $0.67h$ downstream from the crest. Where the falling sheet of water crosses the crest level, its thickness is about $0.43h$.

[1] This is taken from Bazin's experiments reported in *Ann. Ponts et Chaussées*, October, 1888, and translated in *Proc. Eng. Club of Phil.*, July, 1892. These curves have been verified for heads up to 7.5 ft. on weirs up to 10 ft. high by unpublished experiments made at Cornell University.

Experimental measurements show that the discharge over such a vertical *weir crest,* edge square and sharp, is about 0.62 times the ideal theoretical value of Eq. (48), or

$$Q = \left[0.62\frac{2}{3}Lh\sqrt{2gh}\right], \text{ or } Q = 3.33Lh^{3/2} \text{ (foot-second units)}, \quad (49)$$

which last form is commonly called the *Francis formula.*

A diagram for calculating discharges by this formula is given in Fig. 93.

85. Applicability of the Francis Formula.—Unless modified as hereinafter indicated, the formula assumes conditions similar to those prevailing during the experiments on which the formula is based, *viz.:*

1. Weir with vertical upstream face of the bulkhead, built squarely across the channel, level top (or crest) across the stream, the crest piece either a "thin plate" or square edged and sharp at the top upstream corner (yet not thick enough to be called a "broad-crested" weir), so that the water jumps free at the upstream top corner.

2. Free admission of air beneath the overfalling sheet of water on the downstream side close to the crest, so as to prevent formation of a partial vacuum under the sheet (with associated clinging effects and increase of discharge).

3. Weir built in a channel whose sides are vertical and parallel, and which extend some distance both upstream and downstream (the latter necessary only above the crest level), from the weir crest, thus having water flowing over it for the full width of the channel.

4. Deep channel of approach, so that the head on the weir is much less than the height of the weir crest above the bottom of the channel, or so that to the eye there is not a swift current in the upstream pool or "channel of approach."

5. The head to be measured far enough upstream from the weir to avoid effects of the surface curve (Fig. 92), say a distance upstream from the crest at least $3h$, to be safe.

6. The head should be greater than 0.30 ft. For lower heads the discharge is greater than by the formula, necessitating an additive term, the percentage effect of which is given in Fig. 96.

WEIR HEAD & DISCHARGE SCALES, FRANCIS FORMULA, $Q=3.33h^{\frac{3}{2}}$

FIG. 93.

If there is a considerable departure from any one of these conditions, Francis' formula must be modified or else the measurements adjusted to suit the formula. The quantitative effects of some of these modifying influences will now be discussed.

86. End Contractions.—For a *rectangular notch*, cut in "thin plate," or with its ends square edged as well as the bottom, experiments show that the discharge is reduced as though the length (= width of notch) were diminished by $0.1h$ *for each end contraction*, *i.e.*, the issuing sheet of water is that much less than the width of the notch due to the throttling effect of water flowing in from the sides. Hence,

$$Q = 3.33(L - 0.2h)h^{3/2}, \tag{50}$$

as the modified Francis formula *for a rectangular weir notch with two end contractions* in addition to the bottom contraction of the overfalling sheet of water.[1]

87. Velocity of Approach.—Strictly speaking, the *head* on each of the imaginary elemental horizontal slits (p. 150) is the sum of the pressure-head and the velocity-head. Whenever the latter is large enough to modify the static pressure-head sensibly, it should be taken into account. The Francis formula for a weir without end contractions may be modified for these cases involving more than a negligible kinetic energy (velocity-head) in the approaching water so as to read:

$$Q = 3.33L\left(h + a\frac{v^2}{2g}\right)^{3/2}, \tag{51}$$

where v is the mean velocity in the *whole stream* approaching the weir.

The value of a is often taken as 1.5, although it ranges from less than 1.00 to more than 2.00 for different cases, depending on the *distribution of velocities* in the channel of approach, *i.e.*, whether high-bottom velocities, uniform distribution from surface to bottom, or high-surface velocities prevail at a cross-section some $3h$ ft. upstream from the weir where the overfall curve has not yet commenced (Fig. 92). Figure 94 shows percentage effects on Q due to addition of $1.5\frac{v^2}{2g}$ to h.

In using Eq. (51), since $v = \frac{Q}{A}$, then $Q = 3.33L\left[h + a\frac{\left(\frac{Q}{A}\right)^2}{2g}\right]^{3/2}$.

[1] Eq. (50) is not to be used unless L is greater than $2h$. For narrower and deeper rectangular notches, see Par. 91, p. 161.

To avoid having to deal with such an equation and to solve it by trial, it is more convenient to compute by Eq. (51) itself, *by successive approximations*, at first assuming $\frac{v^2}{2g} = 0$, *i.e.*, using the simple Francis formula (Eq. (49)). Then from the approxi-

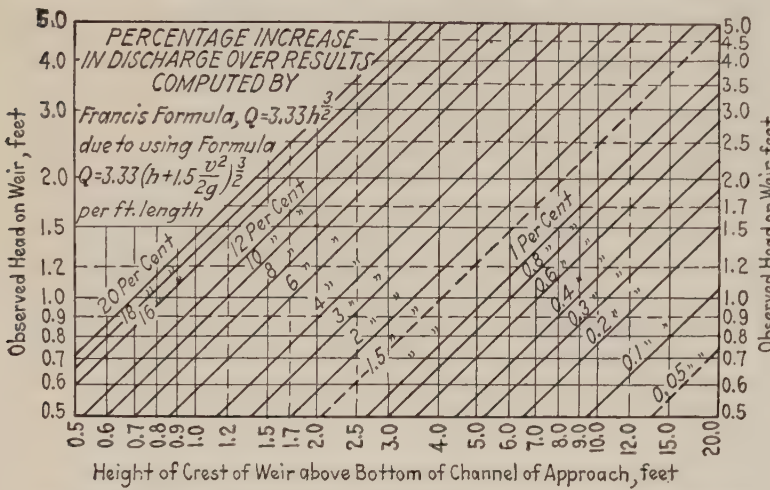


FIG. 94.—Diagram for $a = 1.5$, (see Eq. (51)), showing the per cent increase in Q over that by the simple Francis formula $Q = 3.33\ Lh^{3/2}$, the increase being due to velocity of approach.

mate value of Q thus obtained a value of v may be found by the relation $v = \frac{Q}{A}$, where A is the cross-sectional area of the *whole channel of approach*, from surface to bottom.

This permits the use of Eq. (51) with all its terms, and the making of a second approximation to the correct Q, from which another value for v may be obtained to use in a third approximation, if necessary to compute Q to a greater refinement.

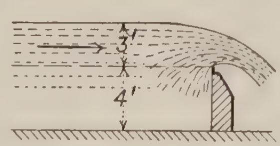


FIG. 95.

Example: What is the discharge over a sharp-crest weir occupying the full width of a rectangular canal, 16 ft. wide, the head being 3 ft., the weir 4 ft. high (Fig. 95), the value of a being assumed $= 1.4$?

Solution: For the first approximation, let $\frac{v^2}{2g} = 0$ in Eq. (51), so that $Q = 3.33L3^{3/2} = L \times 17.30$ c.f.s. (From Fig. 93.)

Second Approximation.—For 1-ft. length of weir, $v = \frac{17.30}{(4 + 3)}$ $= 2.47$ ft. per sec., and $1.4\frac{v^2}{2g} = 0.132$ ft. Then $Q = 3.33 \times 3.132^{3/2} = 3.33 \times 5.54 = 18.44$ c.f.s. (per ft.). This is 6.6 per cent more than in the first approximation.

Third Approximation.—$v = \frac{18.44}{7} = 2.63$ ft. per sec. and $1.4\frac{v^2}{2g} = 0.151$ ft. Then $Q = 3.33, \overline{3.151}^{3/2} = 3.33 \times 5.595 =$ 18.63 c.f.s. This is 1 per cent greater than in the second approximation.

A fourth approximation gives $Q = 18.66$ c.f.s., or only two-tenths of 1 per cent more than by the third trial, and this is as close as it is justifiable to compute. The value 18.66 is nearly 8 per cent greater than the 17.30 by the simple Francis formula.

For the 16-ft. length, $Q = 16 \times 18.66 = 299$ c.f.s. *Ans.*

88. Precautions Necessary for Precision. *Effect of Height of Weir.*—In general, for the same *head* on a weir, the velocity of approach increases (and Q also increases) as the depth of water becomes shallower in the channel of approach, or as the height of the weir becomes less. Figure 96 shows the percentage magnitude of this effect on the discharge according to the experiments of Fteley and Stearns, Bazin, and Rehbock.[1] The effect is clearly evident in the results of each of these experiments, although they do not agree with one another as to the absolute values of the discharge.

The *differences* between the findings of these men, as shown on the plotting (Fig. 96), are due to other causes hitherto commonly regarded as of minor importance, but which cannot be neglected for precise work.

Effect of Distribution of Velocities in Upstream Channel. 1. *Velocities Near Surface.*—The simple weir theory (p. 150), takes account only of the upstream water *above the crest level.* Hence, logically, only the additive effect of the velocity-head in

[1] Fteley and Stearns, *Trans. Am. Soc. Civil Eng.*, vol. 12, 1883; Bazin, translation in *Proc. Eng. Club*, Phil., vol. 7, January, 1890; Rehbock, "Handbuch der Ingenieur Wissenschaften," III Teil, "Wasserbau," band II, p. 59, Stauwerke, 1912. (Formulae stated below Fig. 96.)

DIAGRAM FOR CALCULATING DISCHARGE OF WATER OVER SHARP CREST WEIRS of same width as channel of approach (without end contractions) **BY THE FORMULAE OF FTELEY AND STEARNS, BAZIN, AND REHBOCK,** showing Percentage variation of Discharge calculated by these Formulae from Discharge calculated by Francis Formula ($Q = 3.33Lh^{3/2}$) without considering velocity of approach.

A change of 0.01 in "μ" = 1.6 per cent variation from Francis Formula.

A change of 0.10 in "m" in formula, $Q = mh^{3/2}$ = 3.0 per cent variation from Francis Formula.

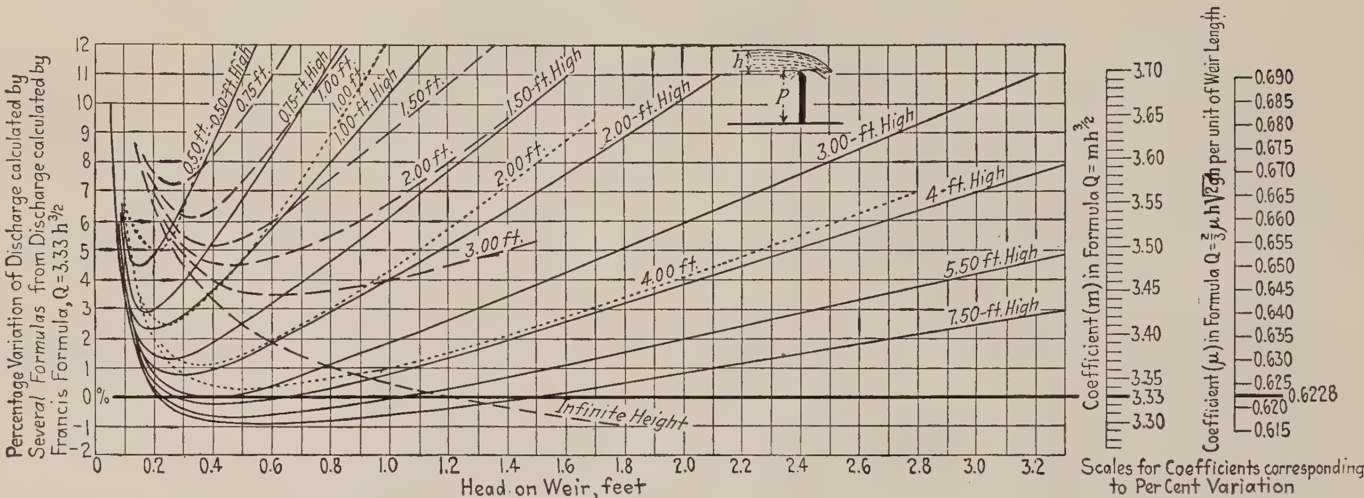


FIG. 96.—Percentage departures from Francis (simple) formula are shown for convenience in calculating because of the wide use of this formula and the many Head-discharge tables and diagrams available. (See Fig. 93.)

When μ = 0.6228, m = 3.33 = 0.0 per cent Variation.

When μ = 0.660, m = 3.53 = 6.0 per cent Variation.

Fteley & Stearns—Dotted lines. $Q = 3.31L\left(h + 1.5\frac{v^2}{2g}\right)^{3/2} + .007L.$

Bazin—Dashed lines. $Q = \left(0.405 + \frac{.0098}{h}\right)\left[1 + 0.55\left(\frac{h}{p + h}\right)^2\right]Lh\sqrt{2gh}.$

Rehbock—Full lines. $Q = \frac{2}{3}\left(0.605 + \frac{1}{320h - 3} + \frac{0.08h}{p}\right) Lh\sqrt{2gh}.$

the approaching water *above* the crest level should be considered when modifying the simple weir formula. Strictly the head y on each elementary horizontal slit (Fig. 91) should be increased by the prevailing velocity-head back of that slit, before integration. A simpler practical formula results, however, if v_a is called the mean velocity of approach *above* the crest level in the channel upstream from the weir (through the section where h is measured), and if Eq. (48) is altered by adding $\frac{v_a^2}{2g}$ to h, to give the total effective head. Thus Eq. (49) becomes

$$Q = 3.33L\left(h + \frac{v_a^2}{2g}\right)^{3/2}. \qquad (52)$$

Many measurements of velocities, in channels of approach to weirs and dams, show that (v_a), the mean velocity above the crest level, is usually greater than the average velocity in the whole cross-section of the stream, as, indeed, is the case for water flowing in rivers and canals generally. For normal flow the surface velocities are higher than the bottom velocities whether or not there is a weir or dam in the stream.

If, for example, it is found in a particular case that $v_a = 1.22 \times v$, then $\frac{v_a^2}{2g} = 1.5\frac{v^2}{2g}$. This illustrates the chief cause for values of a greater than unity in Eq. (51).

Practically, then, for precision in estimates of flow over weirs, it is necessary to know the *distribution* of velocities. Recent tests[1] covering a large variety of distributions of velocity have substantiated the use of Eq. (52), when auxiliary velocity measurements are available, as yielding results superior in precision to those by Eq. (51) with such a value of a as is suggested by judgment based on dimensions and appearances.

2. *Velocities below Level of Crest.*—The kinetic energy of the water approaching the weir *below* the level of the crest does not expend itself entirely in elevating the lower profile of the overfalling water (Fig. 92). Some part of it merges with the kinetic energy of the water above the crest level. The formula[1]

$$Q = 3.33L\left[\left(h + \frac{v_a^2}{2g}\right)^{3/2} + \frac{h}{3.33}\frac{v_b^2}{2g}\right] \qquad (52a)$$

[1] "Precise Weir Measurements," *Trans. Am. Soc. C. E.*, Vol. 93 (1929), p. 1038.

has been found to satisfy a wide range of conditions. In this, v_b is the mean velocity *below* the crest level.

Effect of Rounding or Dulling Edge of Crest.—It has been found[1] that slight roundings of the upstream corner of the crest increase the discharge approximately according to the formula:

$$\text{Percentage increase in } Q = 274 \times \frac{r}{h^{0.75}} \text{ (both } r \text{ and } h \text{ in feet).}$$

Thus for heads of 0.3 and 1 ft. on a weir with rounding to a radius of 1 mm. (= 0.04 in.) the increases in discharge are 2.2 and 0.9 per cent.

Effect of Roughness of Upstream Vertical Face of Weir.—Tests show[2] that the effect of roughening 4 in. of the weir face just below the crest by cutting adjoining 60-deg. V-grooves (24 per inch) horizontally in a 12-in. brass plate was to cause an increase in Q of 1 per cent for heads up to 1.3 ft. Additional grooves (15 per inch) on the lower 8 in. of the plate caused an increase = 1.7 per cent.

89. Triangular Weir Notch.—For small flows the triangular weir notch has greater heads for a given discharge than does a rectangular notch of the same width at water surface, and hence is a more sensitive measuring device.

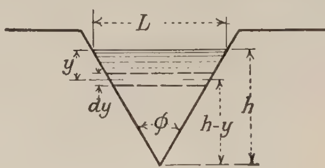


FIG. 97.—Triangular weir.

For a triangular notch (Fig. 97), the theory given on p. 150 is modified by expressing the horizontal length of the imaginary elemental slit in terms of h.

By similar triangles, the variable length of slit $= \frac{L}{h}(h - y) = L - \frac{Ly}{h}$. The differential equation then becomes $dQ = dA \times v$, or $dQ = \left[Ldy - L\frac{y}{h}dy\right]\sqrt{2gy} = L\sqrt{2g}\left[y^{1/2}\,dy - \frac{1}{h}y^{3/2}\,dy\right]$, which upon integration gives

$$Q = \frac{4}{15}Lh\sqrt{2gh}. \tag{53}$$

[1] J. O. JONES, *Cornell Civil Eng.*, November, 1917; also reference in footnote, p. 158.

[2] *Idem* also for C. M. Weber. For roughness effect on V-notch weirs, see James Barr, *London Engineering*, Apr. 8 and 15 and Oct. 28, 1910.

Practically, on account of friction and effects of contraction,

$$Q = C\frac{4}{15}Lh\sqrt{2gh}, \tag{54}$$

in which C is found by experiments to be on 60- and 90-deg. square-edged notches close to 0.58. The coefficient varies slightly with the head and with the angle of the notch.[1]

For a notch with angle $= \phi$, $L = 2h \tan\frac{\phi}{2}\cdot$ Hence,

for $\phi = 90$ deg., $Q = 2.48h^{5/2}$, or approximately $Q = 2.5h^{2.5}$; (55)
for $\phi = 60$ deg., $Q = 1.43h^{5/2}$, (56)
in the foot-second system of units.

These formulas assume sufficient width and depth of the channel of approach to insure complete contraction effects.

90. The Cipolletti weir is the result of an effort so to *flare the ends* of a previously rectangular notch that the discharge will be increased just as much as the end contractions with vertical ends would decrease the discharge (see p. 154 and Eq. (50).

From Eq. (54), for a triangular notch, with $L = \frac{h}{2}$,

$$Q = C\frac{2}{15}h^2\sqrt{2gh}.$$

But this is the same as the subtractive term in the Francis formula for a rectangular weir notch with two end contractions, *viz.*: $3.33(-0.2h)h^{3/2}$, the value 3.33 being $= 0.62 \times 2/3 \times \sqrt{2g}$, whence the Francis subtractive term $= -0.62(2/15)h^2\sqrt{2gh}$.

If the value of C for this narrow triangular notch is close to 0.62, which is approximately true, then it may be stated that the addition of a triangle with $L = \frac{h}{2}$ to a rectangular notch compensates for the end contractions; and for such a modified rectangle (now a trapezoid) the simple Francis formula $Q = 3.33Lh^{3/2}$ may be used.

The triangular addition is made by placing half of the triangle on each end, so that the ends of a Cipolletti weir have a slope of 4 vertical to 1 horizontal, as shown in Fig. 98.

N. B.—The value L is the crest length, not the width of the notch at the surface level.

[1] Marks, "Mechanical Engineers' Handbook," 2nd ed., p. 268.

It is assumed that the head h is not greater than $\frac{L}{2}$.

Experimental measurements have verified the theory above stated. The Cipolletti weir is now widely used in the western United States as well as in other countries for measuring irrigation water.

91. Narrow Rectangular and Trapezoidal Sharp-edged Notches with Crest Length Less than Two Times the Head h.—Experiments on narrow rectangular notches beyond the applicability of the Francis formula, covering heads up to 3 ft. and widths down to 2.7 in., have shown that the formula $Q = 0.56(\frac{2}{3}Lh\sqrt{2gh})$ *i.e.*, $Q = 3.00Lh^{3/2}$, is reliable when h is as great as, or greater than, L. Hence, for such cases, 90 per cent of the discharge given by the Francis formula may be taken and the

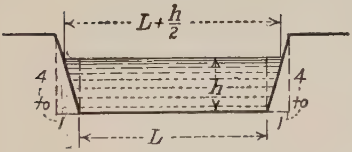


FIG. 98.—Cipolletti weir notch.

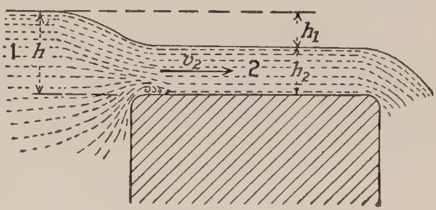


FIG. 99.—Broad crest weir.

full width of the notch may be used for L (nothing subtracted as in the Francis formula for end contractions). For notches with L between h and $2h$ the coefficient increases and merges into values given by the Francis formula for two end contractions. For narrow trapezoidal notches, the discharge may be expressed as the sum of that through a central rectangle and two triangles at the ends (see formula above and Eq. 54). A discharge coefficient of from 0.58 to 0.60 has been found to apply.

92. Broad Crest Weirs. *Theory of Maximum Discharge.*—For a weir with broad level crest, smoothly finished, and rounded upstream corner (Fig. 99), the velocity across the flat top beyond the main drop in the water surface where stream lines first become approximately parallel may be considered as due to free fall through a height h_1. With no friction, the ideal velocity $v = \sqrt{2gh_1}$, and the discharge per unit length of dam (across the stream) is given by $Q = Av = h_2\sqrt{2gh_1} = (h - h_1)\sqrt{2gh_1}$. Assuming

that the flow adjusts itself to such velocity and depth as to give the maximum discharge for a fixed head h on the dam, $\frac{dQ}{dh_1} = 0$, or

$$0 = \frac{d[(h - h_1)\sqrt{2gh_1}]}{dh_1} = \frac{\sqrt{2g}}{2}\frac{h}{\sqrt{h_1}} - \frac{3\sqrt{2g}}{2}\sqrt{h_1},$$

whence

$$\frac{h}{\sqrt{h_1}} = 3\sqrt{h_1}$$

or

$$h = 3h_1, \text{ and } h_1 = \frac{h}{3}; \text{ also } h_2 = \frac{2}{3}h.$$

Substituting these values in the expression for Q,

$$Q = \frac{2}{3}h\sqrt{2g\frac{h}{3}} = 0.577\frac{2}{3}h\sqrt{2gh}.$$

For foot-second units, ideal $Q = 3.09h^{3/2}$ (per foot). Tests show a coefficient of about 0.91 for rounded corner, and 0.83 for square corner (see *U. S. Geol. Survey Water Supply and Irrigation Paper* 200). Hence, actual Q (round corner) $= 2.81h^{3/2}$ and actual Q (square corner) $= 2.56h^{3/2}$.

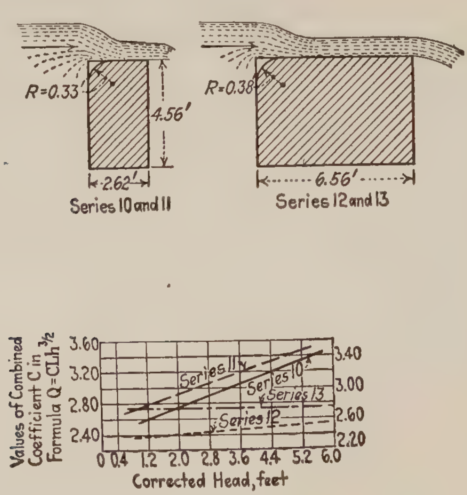


FIG. 100.—Broad-crested weirs. U. S. deep waterways board experiments at Cornell University, 1899. (*From U. S. G. S. W. S. Paper* 200).

Series 10 & 12 were run with the upstream top corner *sharp*.
Series 11 & 13 had the corner *rounded* to radii 0.33′ & 0.38′ respectively.
The plotting shows coeffs. as if the head due to vel. of approach were zero, *i.e.* for a height of dam at least four or five times the head on the dam.

93. Flow over Weirs and Dams Other than Sharp-crested.—A valuable compilation of experimental measurements on a great variety of forms is given in *Water Supply and Irrigation Paper* 200 (Reprint of No. 150), U. S. Geological Survey, "Weir Experiments, Coefficients, and Formulas," by Robert E. Horton.

Only a very few results will be given here, and these chiefly to illustrate the range in value of coefficient, and the effect of head and shape of dam on the rate of discharge (Figs. 100 and 101).

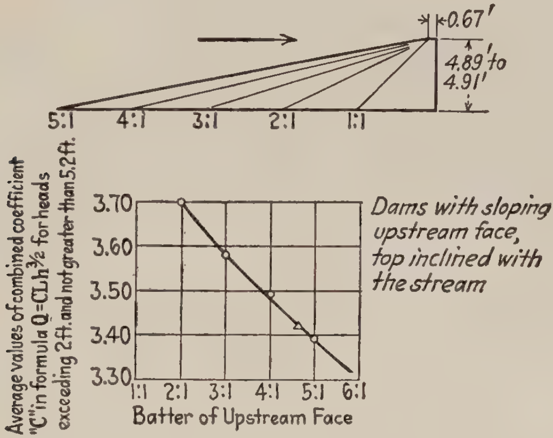


Fig. 101.

Inclined Weirs.—If a weir bulkhead be imagined hinged at the bottom, then if it is inclined so that the top is moved upstream from the vertical position the discharge for a given head is less or the head for a fixed discharge is greater. If the top is moved downstream from the vertical, the discharge for a given head is greater or the head for a fixed discharge is less.

For square, sharp-crest weirs, Bazin found the combined coefficient to be 3.10 for a 1 to 1 slope against the stream and 3.73 for a 2 horizontal to 1 vertical slope with the stream.

94. Range of Coefficients of Discharge.—A resumé of the range of values covered may be condensed into the statement that the discharge varies from about 25 per cent less to 20 per cent more than by the simple Francis formula. The lowest discharges are for broad, flat crests with square corner; and the

highest for A-shaped dams with rounded apex, also a few other favorable shapes.

95. Submerged weirs and dams, where the water surface downstream from the dam is at higher level than the crest, do not show much reduction in discharge as compared with the unsubmerged condition, until the downstream head or backwater is nearly one-half the upstream head above the crest. This is especially true for dams with broad crests, whether flat or rounded, and for dams with easy upstream sloping faces, since in these cases there is a considerable surface drop to the water and a decided increase in the velocity, even before the water leaves the crest. This condition remains practically unchanged with submergence until the downstream backwater head is over half the upstream head, *i.e.*, high enough to smother the flow.

Theory.—When a weir or dam is submerged or drowned so that the water level downstream is higher than the crest of the

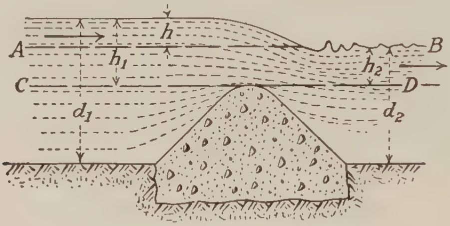


FIG. 102.—Submerged weir.

weir and also when water flows between bridge piers set midstream in a river as shown in Figs. 90 and 102 the flow may be regarded in two ways.

From one viewpoint, Bernoulli's theorem may be applied to the whole stream between points 1 and 2.

From another viewpoint, the water *above* the downstream river level may be regarded as discharging like a *weir*, and the water *below* the downstream river level as discharging through a *submerged orifice* of height h_2 and under a head equal to the difference of water levels, *viz.*, $h_1 - h_2$, in Fig. 90 and h in Fig. 102.

(Note that in the case of the submerged weir the height h_2 is measured from the level of the top of the weir, and not, as in Fig. 90, from the bottom of the river.)

Adopting the latter view, (Dubuat's theory), the discharge per unit length of dam is given by Q (per foot) = weir discharge + orifice discharge.

$$Q = C_w \frac{2}{3} h\sqrt{2gh} + C_o h_2 \sqrt{2gh}.$$

The values of C_w and of C_o are not readily found. As a crude approximation, let $C_w = C_o$. Then

$$Q = \left(C_w \frac{2}{3}\sqrt{2g}\right)\sqrt{h}\left(h + \frac{3}{2}h_2\right),$$

$$Q = C\sqrt{h_1 - h_2}\left(h_1 - h_2 + \frac{3}{2}h_2\right),$$

$$Q = C\sqrt{h_1 - h_2}\left(h_1 + \frac{h_2}{2}\right).$$

Note that this formula is in terms of the upstream and downstream heads as measured above the crest level. However, h_1 should include the effect due to velocity of approach.

For sharp-crest weirs, assuming $C_o = C_w = 0.62$, the coefficient $C = 3.33$ for this special formula. The experiments of Francis and Fteley & Stearns indicate this value for low and high submergence ratios, (h_2/h_1), but the lower value 3.10 for medium submergence ratios 0.5 to 0.8. For rounded crests and high heads the coefficient is greater. (See par. 94.)

It is very convenient to express the discharge over a submerged weir or dam as a multiple of the discharge over the same dam when unsubmerged or free.

For example, the "submerged weir model"[1] of the U. S. Deep Waterways Commission, with $h_1 = 6.6$ ft., showed discharge ratios for various submergence ratios, as follows:

$\frac{h_2}{h_1}$	$\frac{Q_{subm.}}{Q_{free}}$	$\frac{h_2}{h_1}$	$\frac{Q_{subm.}}{Q_{free}}$
0.0	1.000	0.5	0.937
0.1	0.991	0.6	0.907
0.2	0.983	0.7	0.856
0.3	0.972	0.8	0.778
0.4	0.956	0.9	0.62

In this case the unsubmerged discharge was 11 per cent. greater than by the simple Francis formula.

[1] *U. S. Geol. Survey Water Supply and Irrigation Paper* 200, p. 146.

96. Backwater.—Evidently the construction of a weir, dam, bridge piers, or any form of obstruction in a river will cause the water to back up and stand higher than formerly at points upstream from the structure.

If the rate of discharge to be passed in floods is known, the engineer can compute the head on the dam, or the difference of heads at the bridge, and hence the height of the new water level just upstream from the obstruction. The regimen of the river and its height for a certain discharge remain the same at *downstream points* as before the obstruction is built. This refers to points a considerable distance downstream beyond the hydraulic jump, local turmoil, standing waves, etc.

The extent of backwater upstream from the obstruction and the heights of the new (elevated) water surface at various points upstream are special problems in the flow of water in open channels, a subject treated in Appendix C of this text.

Problems

1. Compute the proper length of a sharp-edged, vertical, rectangular weir, 3 ft. high, with two end contractions, to install at the end of a rectangular canal 12 ft. wide and 4 ft. high to pass a discharge of 10 c.f.s., keeping the depth of water in the canal at 3 ft. 6 in., *i.e.*, head on weir = 0.50 ft.

Ans. 8.59 ft.

2. (*a*) If the rectangular notch of problem 1 is to be replaced by a Cipolletti weir, other conditions the same, compute the proper length of weir crest. (*b*) Draw an end view of the canal showing all dimensions for its construction.

Ans. (*a*) = 8.49 ft.

3. Bazin's experiments on sharp-edged weirs without end contractions show an increase of 15.5 per cent in discharge over the simple Francis formula for the case of a head of 1.40 ft. on a weir 1.25 ft. high. Compute the exact value of the multiplier a, for the mean velocity head in the channel of approach corresponding to the given values, having reference to the modified Francis formula involving velocity of approach. *Ans.* (*a*) 1.585.

4. Selection of a proper measuring weir. It is sometimes desired to measure flows of water ranging from a very small minimum to a fairly large maximum, and at the same time, in estimating the discharge, to avoid errors of more than a small percentage. (Say, as an illustration, 2 per cent maximum permissible error in discharge due solely to expected error in field measurement of head.) At the same time it is not permissible to have such a weir that the head, or the depth of pool, or the backwater shall be too great. Assume that the choice is finally between a 90-deg. triangular weir notch

and a Cipolletti weir with crest length of 3.00 ft. The error in head measurement may be $\frac{1}{32}$ in. for the low heads, and $\frac{1}{16}$ in. for the high heads. Assume for this case that the formula for the 90-deg. notch is $Q = 0.60(\frac{4}{15} Lh\sqrt{2gh})$. In both cases velocity of approach effects may be neglected.

(*a*) Compare the errors both in cubic feet per second and in per cent. of estimated discharge, due to head error, for the expected minimum discharge of 0.05 c.f.s. (*b*) Compare the errors for the maximum discharge of 15 c.f.s.

Ans. (*a*) Weir, 0.0067 c.f.s. 13.4 per cent.
Notch, 0.0168 c.f.s. 3.15 per cent.
(*b*) Weir, 0.089 c.f.s. 0.595 per cent.
Notch, 0.096 c.f.s. 0.64 per cent.

5. (*a*) Show how the value 3.33 in the Francis formula is derived from the ideal equation for flow over a sharp-crested weir. (*b*) Is the factor a in the modified Francis formula for the flow of water over weirs greater than, or less than, unity in most cases, and why?

6. Given a weir without end contractions 6 ft. high; head on weir 2 ft.; velocity-head coefficient $a = 1.4$, mean velocity in the channel of approach 1.21 ft. per sec. What is the percentage effect on the quantity flowing if the velocity-head increment be neglected? *Ans.* 2.4 per cent.

7. Sharp-crested weir. Bazin's researches show that the thickness of the falling sheet of water *A-B* (see Fig. P.53) at the level of the crest is $0.43h$. His volumetric measurements showed that the discharge for a head

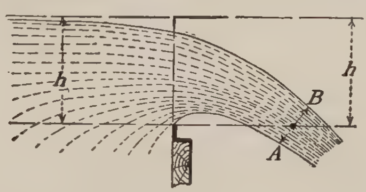


FIG. P.53.

of 1.00 ft. was 3 per cent greater than by the simple Francis formula. What is the velocity coefficient for the *A-B* cross-section, if it is assumed that the flow at that section is due to all the water falling through a height $1.013h$ (the excess over h being due to velocity of approach)?

Ans. $C_v = 0.988$.

8. Compute the length of a rectangular weir notch "in thin plate" cut out of the top of a side of a tank 10 ft. wide and 10 ft. high, so that under a head of 9 in. the discharge may be just 8 cu. ft. per sec. (*Note:* For *each* end contraction the *effective length* is reduced by one-tenth of the head.)

Ans. 3.85 ft.

9. The *ideal* discharge through a triangular weir notch is $\frac{4}{15}$ of that through an *ideal rectangle* (with the same width as the triangle at level of surface of water), all portions of which rectangle are imagined under the same head as the water at the crest level and the same as the head on the

vertex of the triangle. The coefficient is the same as for a simple sharp-crested weir or a sharp-edged orifice. What head is necessary to discharge 10 c.f.s. through a triangular notch with angle between the sides equal to 120 deg.? *Ans.* 1.363 ft.

10. The theory of flow over a broad-crested weir (Fig. 99), is that the discharge is the maximum possible. If *all* the head h were converted into velocity there would be a zero depth h_2 and hence no area in $Q = A_2v_2$. If there were no fall h_1 there would be zero velocity v_2. Apply Bernoulli's theorem between points 1 and 2 (velocity-head = zero at 1) and find the ratio $\frac{h_1}{h}$ or $\frac{h_1}{h_2}$ for maximum Q. Then write the equation for Q in terms of h and compare it with Francis' formula. *Ans.* $h_2 = \frac{2}{3}h$.

11. Using the formula $Q = 3.33L\left(h + a\frac{v^2}{2g}\right)^{3/2}$ for a weir occupying the entire width of channel, compute the value of a if the discharge is known to be 10 per cent greater than by the Francis formula, $Q = 3.33Lh^{3/2}$, when the upstream water surface is 4.00 ft. above the crest level, the bottom of the stream being 6.0 ft. below the crest level. Make a sketch and show clearly how you obtain the result. *Ans.* $a = 1.97$.

12. The actual discharge through a rectangular weir notch with end contractions is given by the formula $Q = 3.33(L - 0.2h)h^{3/2}$, in which L is the length of crest and h is the head. The actual discharge through a certain triangular notch is $Q = 1.332L'h^{3/2}$, where L' is the width of water surface and h is the depth of water on the vertex of the triangle. From these data show why the discharge over a Cipolletti weir is given by the formula $Q = 3.33\ Lh^{3/2}$, where L is the length of crest and h is the head.

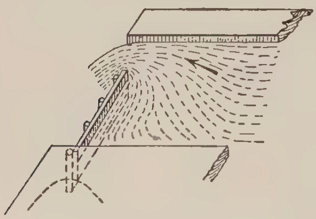


Fig. P.54.

13. A high dam with rounded crest (Fig. P.54) has a discharge factor $m = 3.95$ in the formula $Q = mLh^{3/2}$ (corresponding to the 3.33 of the simple Francis formula). Flashboards stop the flow over all except 9 ft. of the length of the dam, leaving a gap for the flow with full contraction on one end and no contraction on the other end. The measured head is 3.70 ft. Compute the discharge in cubic feet per second. *Ans.* 243 c.f.s.

14. If the experimental measurements show $Q = 4.44h^{5/2}$ for a 120-deg. triangular weir notch, compute the coefficient of discharge necessary to make the ideal or fundamental formula represent the facts. *Memo:* The value $\frac{2}{3}$ for the ordinary weir becomes $\frac{4}{15}$ for the triangular notch.

Ans. 0.60.

CHAPTER X

EXPONENTIAL LAWS OF VARIATION, AND PERCENTAGE EFFECTS DUE TO SMALL CHANGES IN THE VARIABLES

97. Laws of Variation.—In hydraulics (as in mechanics generally) there are many so-called "exponential" formulas.[1] The tendency of the student is to regard these formulas merely as expressions in which known values may be substituted in order to find the unknowns, overlooking the important fact that these formulas are *laws of variation.*

The engineer should have clearly in mind the nature of the variation of one variable due to changes in another, even when the less important details of the formula are forgotten. This is merely equivalent to stating that it is of prime importance to have in mind the fundamental nature of the physical law involved.

For example, applying $Q = 3.33Lh^{3/2}$ to a *particular weir*, note first that L is constant. For each particular value of h, *e.g.*, h_1, h_2, h_3, etc., there is a corresponding Q_1, Q_2, Q_3, *i.e.*, $Q_1 = 3.33Lh_1^{3/2}$, $Q_2 = 3.33Lh_2^{3/2}$, etc. Hence the *ratio* between any two discharges, such as $\dfrac{Q_1}{Q_2} = \left(\dfrac{h_1}{h_2}\right)^{3/2}$. Notice that the constants of the formula cancel out.

In words, it may be stated either "the discharge varies as the three-halves power of the head;" or "the *ratio* of any two discharges equals the *ratio* of the corresponding heads (raised to the three-halves power)."

[1] A few examples follow:

$$P_H = \frac{1}{2}wh^2;\ v = \sqrt{2gh}, \text{ or } \sqrt{2g} \times h^{1/2};\ Q = 3.33Lh^{3/2};\ Q = 2.5h^{5/2};$$

$$t = K\frac{\text{Vol.}}{CA\sqrt{2g}}h^{-1/2};\ v = C\sqrt{Rs}, \text{ or } CR^{1/2}s^{1/2};\ v = \frac{1.486}{n}R^{2/3}s^{1/2};$$

$$h_F = f\frac{L}{d}\frac{v^2}{2g};\ h_F = 0.36\frac{v^{1.86}}{d^{1.25}}; \text{ etc.}$$

With this sort of information in mind, it is possible, as a simple process of mental arithmetic, or very quickly on the slide rule, to work out such cases as the following:

Examples.—If the head on a weir is doubled, the discharge is increased $2^{3/2} = 2 \times 1.414 = 2.83$ times. If the head is increased 4 times, the discharge is 8 times as great. If the *discharge* is halved, the *head* is $(\frac{1}{2})^{2/3} = 0.63$ times as great.

So, in mechanics, a 2-in. by 10-in. joist is nearly three times as strong as a 2-in. by 6-in. joist (being the ratio of $\overline{10}^2$ to $\overline{6}^2$); a single 10-in. timber (either round or square) acting as a beam is as strong as eight 5-in. timbers of the same shape and with the same span (since strength varies as width times $(\text{depth})^2$, or in this case $2 \times 2^2 = 8$).

98. Percentage Effects Due to Small Changes.—In hydraulics particularly, there are many cases where there is likely to be a variation in one of the variables of from a small fraction of 1 per cent to several per cent. Such variation is due to some such causes as:

1. Uncertainties (practically unavoidable) as to the absolute correctness and completeness of assumed data for design.
2. Natural variability, as illustrated by change in dimensions due to deterioration of structures.
3. Errors in measurements.
4. Known changes in conditions, or assumptions necessitating definite percentage changes in one or more of the variables.

In such cases the question often arises as to how great a percentage change in one of the main quantities is caused by, or corresponds to, a stated per cent change in one of the variables controlling the dependent quantity.

Mathematically, let $y = mx^n$, and $y' = m(x \pm \Delta x)^n$, where Δx is a small percentage of x, either an increment or a decrement, as shown by the appropriate sign. It is desired to know the value of the ratio $\frac{y'}{y}$, which $= \frac{(x \pm \Delta x)^n}{x^n}$.

By the binomial theorem, $(x \pm \Delta x)^n = x^n \pm nx^{n-1}\Delta x + \frac{n(n-1)}{2}x^{n-2}\overline{\Delta x}^2 \ldots$, the further terms involving higher powers of Δx. If $\frac{\Delta x}{x}$ is a small fraction, not over 5 or 10 per cent,

then $\left(\frac{\Delta x}{x}\right)^2$ will be so small as to be practically negligible, and of course higher powers will not be worth considering. If Δx is a large fraction of x, this is not true, mathematically speaking, but it may still be substantially true from a practical viewpoint, as will be explained presently.

The ratio

$$\frac{y'}{y} = \frac{(x \pm \Delta x)^n}{x^n} = 1 \pm n\frac{\Delta x}{x} + \frac{n(n-1)}{2}\left(\frac{\Delta x}{x}\right)^2 \pm \ . \ . \ .$$

only the first two terms of which are of present consequence.

Hence, practically, $$\frac{(x \pm \Delta x)^n}{x^n} = 1 \pm n\frac{\Delta x}{x}. \qquad (57)$$

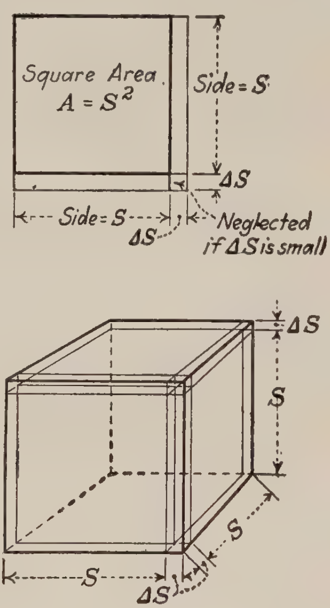


FIG. 103

Note that n is the exponent of the original formula, and that the ratio $\frac{\Delta x}{x}$ is *the fractional change in the variable* x.

Hence

the percentage change in $y = n \times$ *the percentage change in* x, (58) or, in brief form, y per cent $= n \times x$ per cent, where n is the exponent or power of x in the original equation between y and x.

Geometrical Examples Illustrating the Theory.—If a small amount Δs is added to, or subtracted from, the side (each side) of a square, there is added to the area two times $s\Delta s$ (Fig. 103) neglecting the small corner square $(\Delta s)^2$.

$$\text{Ratio of } \frac{\text{added area}}{\text{original area}} = \frac{2s\Delta s}{s^2} = 2\left(\frac{\Delta s}{s}\right)$$

$$= 2 \times \text{ratio of } \frac{\text{added length}}{\text{length of one side}}.$$

Likewise for a *cube*,

added volume $= 3 \times s^2\Delta s$, and ratio $\frac{3s^2\Delta s}{s^3} =$

$$3\left(\frac{\Delta s}{s}\right) = 3 \times \text{ratio of } \frac{\text{added length}}{\text{length}}.$$

Here three small prisms of volume $3 \times s(\Delta s)^2$ and a corner cube $(\Delta s)^3$ are neglected.

Examples.—From Eq. (58), applied to the flow through orifices, since $v = C\sqrt{2gh}$ (C being the velocity coefficient due to friction), and $Q = CA\sqrt{2gh}$, *i.e.*, since both v and Q vary as $h^{1/2}$, then: 1 per cent change, error, or uncertainty in head means one-half of 1 per cent change in velocity and discharge; 3 per cent change in discharge means 6 per cent change in head (since h varies as v^2 or Q^2).

For *weirs*, where Q varies as $h^{3/2}$, and h varies as $Q^{2/3}$, 1 per cent change in head means 1.5 per cent change in discharge; 2 per cent change in discharge means $2/3 \times 2 = 1\frac{1}{3}$ per cent change in head.

Note that for such calculations, as far as the formula is concerned, all it is necessary to have in mind is the exponent of the variable.

Large percentage variations often fall under the scope of Eqs. (57) and (58) because, in engineering practice, if an uncertainty must be admitted as great, say, as 30 per cent in a value, it may mean anywhere near this, or say from 25 to 35 per cent.

In fact, it is quite logical to state that where considerable uncertainty exists there is also considerable doubt as to the exact extent of the uncertainty in any particular case, *i.e.*, in estimating it beforehand.

For example, if it is admitted that there exists an uncertainty of some 30 per cent as to the probable flood discharge over a dam, then the uncertainty in head by Eq. (58) is $2/3 \times 30 =$ 20 per cent, based on the weir law. Now, by the exact method, if Q is *increased* 30 per cent, the ratio of the new to the old head is $1.30^{2/3} = 1.19$; and for 30 per cent *decrease* in Q the ratio of heads is $0.70^{2/3} = 0.79$, or 19 and 21 per cent variation in head, respectively. But the value 20 per cent, so easily obtainable by mental calculation, probably comes much closer to the mathematically exact values 19 and 21 per cent than the estimated 30 per cent departure in Q agrees with the truth.

Note, however, in the example just given, if it is known by measurement that a certain discharge was definitely just 30

per cent greater than by Francis' formula, $Q = 3.33Lh^{3/2}$, due, say, to effects of velocity of approach, then the exact method should be used, giving 19 per cent of h as the effective velocity-head term, stating $h + a\frac{v^2}{2g} = 1.19h$, or $\left(a\frac{v^2}{2g}\right) = 0.19h$ (see par. 87).

Problems.—What percentage effect is caused:

1. In the total horizontal pressure against a dam by a 4 per cent increase in the height of water? *Ans.* 8 per cent.

2. In the discharge through a triangular weir notch by an error of 2 per cent in measuring the head? *Ans.* 5 per cent.

3. How many times is the power in a nozzle jet of water increased by doubling the issuing velocity? Note that power involves Q and h, the h being in the $\frac{v^2}{2g}$ form. *Ans.* Eight times.

4. If one dam is twice as high as another, how do the total horizontal water pressures compare? *Ans.* 4 to 1.

Problems

In all cases, first state proper complete formula, *next* state law of variation between variables in the order mentioned in question, *finally* show computation leading to answer.

1. To change the area of $\left\{\begin{matrix}\text{an equilateral triangle}\\ \text{a square}\\ \text{circle}\end{matrix}\right\}$ $\left\{\begin{matrix}1\\ 1\tfrac{1}{2}\\ 2\end{matrix}\right\}$ per cent how many per cent must $\left\{\begin{matrix}\text{each side}\\ \text{each side}\\ \text{the diameter}\end{matrix}\right\}$ be changed?

2. If the $\left\{\begin{matrix}\text{radius}\\ \text{volume}\end{matrix}\right\}$ of a sphere is measured in error $\left\{\begin{matrix}2\\ \tfrac{1}{2}\\ 3\end{matrix}\right\}$ per cent, how many per cent in error is the computed $\left\{\begin{matrix}\text{volume}\\ \text{radius}\end{matrix}\right\}$?

3. If the total depth of water in the pond back of a dam in time of highest flood is $\left\{\begin{matrix}\text{underestimated 3 per cent}\\ \text{overestimated 2 per cent}\end{matrix}\right\}$ how many per cent in error is the estimated moment of the water pressure on the upstream face, about the toe, and is it on the safe or unsafe side?

4. If the $\left\{\begin{array}{l}\text{discharge}\\ \text{discharge coefficient}\\ \text{head for fixed discharge}\end{array}\right\}$ of a sluice gate (which is a variety of orifice) is uncertain to the extent of $\left\{\begin{array}{c}4\\ 5\\ 8\end{array}\right\}$ per cent, what is the corresponding uncertainty in the $\left\{\begin{array}{l}\text{side of square orifice}\\ \text{head for fixed } Q\\ \text{discharge coefficient}\end{array}\right\}$? Is it in the same or opposite direction?

5. If the head back of a nozzle is measured by a pressure gage which is later found to read $\left\{\begin{array}{l}\text{2 per cent}\\ \text{3 per cent}\\ \text{4 per cent}\end{array}\right\}$ too $\left\{\begin{array}{l}\text{low}\\ \text{high}\\ \text{low}\end{array}\right\}$, and if the assumed diameter of the nozzle tip is later found to be $\left\{\begin{array}{l}\text{1 per cent}\\ \text{2 per cent}\\ 1\tfrac{1}{2}\text{ per cent}\end{array}\right\}$ too $\left\{\begin{array}{l}\text{high}\\ \text{low}\\ \text{high}\end{array}\right\}$, how many per cent too high or too low is the discharge computed from the wrong data?

6. When the $\left\{\begin{array}{l}\text{discharge passing through}\\ \text{head of water on}\end{array}\right\}$ a triangular weir notch $\left\{\begin{array}{l}\text{increases 10 per cent}\\ \text{decreases 3 per cent}\end{array}\right\}$, how many per cent does the $\left\{\begin{array}{l}\text{head}\\ \text{discharge}\end{array}\right\}$ $\left\{\begin{array}{l}\text{increase}\\ \text{decrease}\end{array}\right\}$?

7. For 19 per cent error in estimated discharge, compute the corresponding percentage error in the measured head on a dam (considered as a weir).

Ans. 12.65 per cent.

8. Apply the approximate laws of percentage effects. (*a*) The discharge through a certain Venturi meter is found to be 10 c.f.s. when the "Venturi difference" was measured as 4.00 ft. of water. It was afterward found that the correct Venturi difference was 4.16 ft. of water. What was the true discharge? (*b*) An error of 3 per cent is made in measuring the diameter of the stream of water issuing from a sharp-edge orifice. What percentage error will this make in computing the contraction coefficient?

9. If the head on a dam (flood water) is assumed 10 per cent too low (as shown by a subsequent actual flood) and if the coefficient used, 3.33, is 10 per cent too high, how many per cent excess or deficiency does the actual discharge have as compared to computed value?

10. The discharge over a weir without end contractions was computed to be 10 c.f.s. when the head was observed as 0.980 ft. It was afterwards found that the correct head should have been 0.970 ft. By applying the approximate law for percentage effects due to small changes in one variable compute the correct discharge.

CHAPTER XI

STEADY UNIFORM FLOW OF WATER IN PIPES AND OPEN CHANNELS

GENERAL THEORY

99. Preliminary Discussion of Physical Aspects, with Definitions.—By a *pipe* is meant any kind of a closed conduit which usually flows *full* "under pressure."

By an *open channel* is meant any kind of water course with top surface of the flowing water exposed to the atmosphere, such as a canal, aqueduct, flume, sewer, or river.

In some cases a stream belongs to one of these categories at one stage of the water and to the other category at a different stage. A sewer pipe ordinarily is an open channel, but when forced to carry run-off waters from excessive rainfall it may act as a pipe under pressure, with water rising part way up in the manholes. Likewise, depending on the water level or on the volume of flow, a culvert under a railroad or a sluiceway through a dam may act either as an open channel or as a pipe.

Uniform flow (par. 63), which is now being dealt with, is commonly described as being free from changes of velocity along the course of the stream. But in most cases that come up in practice the flowing water, in fact, undergoes many small local accelerations, retardations, and deviations at numerous places. These minor disturbances to perfectly "uniform" flow are due to lack of perfect alignment of the banks and bottom of an open channel, and to irregularities in the walls of a pipe caused by inherent imperfections in manufacture in factory or construction in field, and to joints, deflections, incrustations, etc. So-called "uniform flow," however, proceeds without any such major changes of velocity as is the case at rapids in a river; or flow between bridge piers; or flow through a Venturi meter, a reducer, or a nozzle in a pipe line.

These latter cases are proper subjects for Bernoulli's theorem or for the orifice flow law. The law controlling *uniform flow,* on the other hand, requires separate consideration, as follows:

100. The Phenomena of Loss of Head in Pipes and of Surface Slope in Open Channels, for Uniform Flow.—In Fig. 104 are illustrated portions of long pipe lines showing the pressure effects accompanying the flow of a liquid. The liquid is shown flowing in various directions, point *1* always being the *upstream* point and point *2* being a place further *downstream.* Note that the

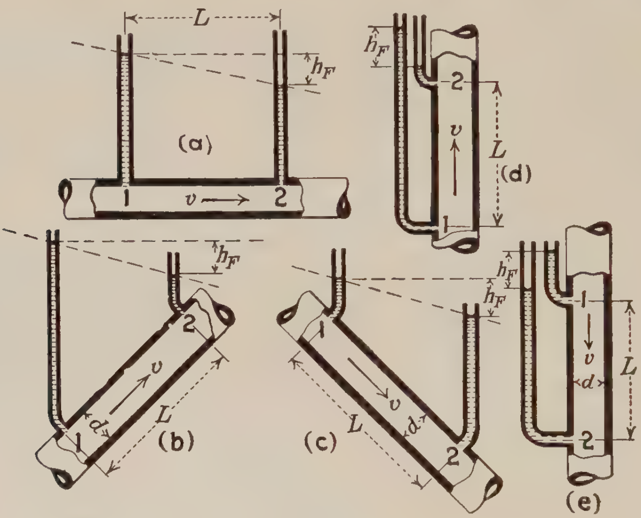


Fig. 104.

upstream point may be at the downhill end, as in cases (*b*) and (*d*).

In cases (*a*), (*b*), and (*c*) there is shown a line joining the tops of real or imaginary open water columns along the pipe. This line slopes *downward downstream* whenever there is flow in the pipe. It is called the "hydraulic grade line," "the hydraulic gradient," or the "hydraulic slope." If the pipe is not horizontal, and the length is different from the horizontal projection, it is to be remembered that, for uniform flow, the hydraulic slope, $s = \frac{h_F}{L}$, where h_F is the loss of head by friction and L is the actual length of pipe.

When there is no flow, as when, for instance, a valve downstream from point *2* is closed in any of the cases shown in Fig. 104, the hydraulic grade line is level.

Note that this hydraulic slope (often called simply the "slope") is different and independent from the *grade* on which the pipe is laid.

It should be noted that there is no increase in velocity between points *1* and *2*, neither does the water run slower at *2* than at *1* because of the loss of head, since this would imply that the water is compressed into smaller bulk. Nevertheless, the water has less energy at *2* than at *1* due to the loss of head (or loss of compressive stress in the water).

If the height, here called h_F, were a "Venturi difference" in a Venturi meter, such as $h_1 - h_2$ in Fig. 89 or $H_1 - H_2$ in Fig. 89*A*, it would not represent "loss of head," because the velocity energy would have been increased in going from *1*

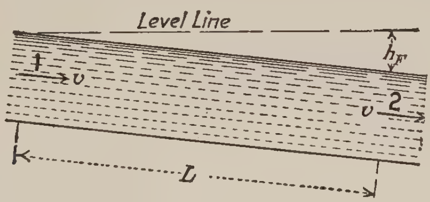


Fig. 105.

to *2*, thus offsetting the apparent loss of energy due to drop in pressure-head.

In Fig. 105 is shown a portion of a long open channel with uniform flow of water proceeding. The slope of the surface of the water is parallel to the general slope of the bed of the stream. In a distance L, the water has fallen through a height h_F, but there is no increase in velocity nor gain in energy. The energy due to the fall has been consumed in overcoming "friction," *i.e.*, in *maintaining the flow* (previously started at some place upstream). The loss of energy goes mostly into heat, but the rise of temperature is very small, even if none of the heat were radiated. If 1 lb. of flowing water (each pound of a large mass) suffered a "loss of head" of 1 ft. in flowing a certain distance, this would imply a rise of temperature of only $\frac{1}{778}$°F. In

tumbling over the Horseshoe Falls at Niagara, approximately 154 ft. high, the heat equivalent is only $^{154}/_{778}$ or $\frac{1}{5}$°F.

101. Nature of Resistance to Flow (Turbulent Flow Assumed, See Chap. XVI).—The chief resistance encountered by a liquid flowing in a long pipe or in an open channel, such as a canal, flume, or river, is the so-called "fluid friction" due to the motion of the fluid along and between the enveloping walls of the conduit, or over the bed and between the banks of the open stream. It is a matter of common observation that water flows with less velocity near the shore than out in the middle of a canal or river. Measurements show a corresponding retardation of velocity near the bottom of a river and near the walls of a pipe. These facts indicate that the outermost portions of the liquid are retarded by "rubbing" against the pipe or stream bed, and that the next inner portions of the liquid, in turn, meet a resistance in rubbing against the slower-moving outer portions.

These changes in velocity are evidences of the influence of *viscosity* (see p. 276), but for the turbulent type of flow there is an actual bodily sliding through the pipe of a *turbulent* mass of liquid. This is true even for smooth pipes, and with increase in roughness the flow becomes more turbulent.

Such being the intricate nature of the flow, it is useless to attempt to analyze the path pursued by an individual particle of the flowing fluid, particularly since direct experimental measurements may be made to find the net effect on loss of head, which is the head consumed in maintaining various rates of flow in various sizes and kinds of pipes and open channels.

The theory of flow now to be presented has to do only with the obvious dimensions of the channels and pipes, and with the mean forward velocities corresponding to lump rates of flow or total volumes per unit of time, so that $Q = Av$. In fact, *it is substantially assumed that a prism of liquid is sliding along and rubbing against the waterway formed by the channel or pipe.*

102. Approximate Theory and Common Approximate Formulas for Flow of Fluids in Pipes and Open Channels (Turbulent Flow is Assumed[1]).

[1] For the flow of water through *very small tubes*, and for the flow through sand, as well as for the flow of *very viscous oils* or other liquids through com-

This theory applies to all fluids whether liquid or gaseous, but for gases the changes of volume and density due to changes of pressure and temperature may become important enough to require special allowance. Ordinarily, for moderate ranges of pressure and temperature along a pipe line conveying air or other gas, average values of these variables may be taken for the whole pipe, and this common theory may be used for gases also (see p. 281).

Let f_1 be the resistance (in pounds or similar unit) offered to the flow by a unit area of rubbing surface at unit velocity for a fluid of unit density. The simplest way of measuring this is to apply a spring balance to find the pull required to hold a thin board or a thin sheet of metal lengthwise against the frictional "drag" of a current of water or air. Froude's classic experiments[1] were made by measuring the pull required to drag long, thin boards through still water at various speeds. The boards were coated with substances of various roughnesses, such as varnish, fine sand, coarse sand, etc.

Experiments show that this resistance varies:

a. With the degree of roughness of the surface which moves through the fluid or along which the fluid flows.

b. Directly as the area of rubbing surface.

c. As the square, nearly, of the velocity.

d. Directly as the density of the fluid.

---

mercial sizes of pipes, there is true *stream-line* flow, without eddies, cross-currents, or turbulence. In these cases the resistance to flow depends on the *adhesion* of the fluid to the walls of the conduit and on the *cohesion* (or rather on the resistance to shearing) of the liquid itself. The more viscous or "sticky" the liquid the more resistance does it offer to motion through the conduit, which motion involves the sliding of the faster-moving central portions ahead of the slower-moving outer portions of the liquid.

Since liquids become more fluid with rise of temperature, even water being so affected, the loss of head for this stream-line variety of flow depends very much on the temperature of the liquid. The loss of head, moreover, varies with the first power of, or directly as, the average velocity, while for the ordinary "eddy" or "turbulent" flow the internal commotions coexistent with the forward motion cause the loss of head to vary with a power of the velocity decidedly greater than 1, in fact, nearly 2 (see Fig. 119*B* also Chap. XVI).

[1] British Association Reports, 1874.

Varies as the viscosity

e. The resistance is independent of the pressure of the fluid.

For pipes and open channels the "rubbing area" is the product of the length times the *wetted perimeter*, which latter is that por-

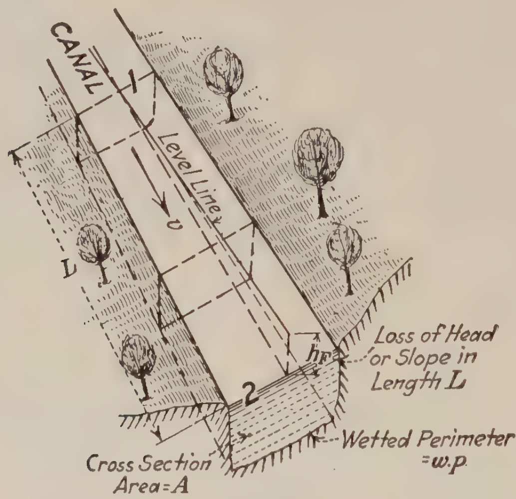


FIG. 106.

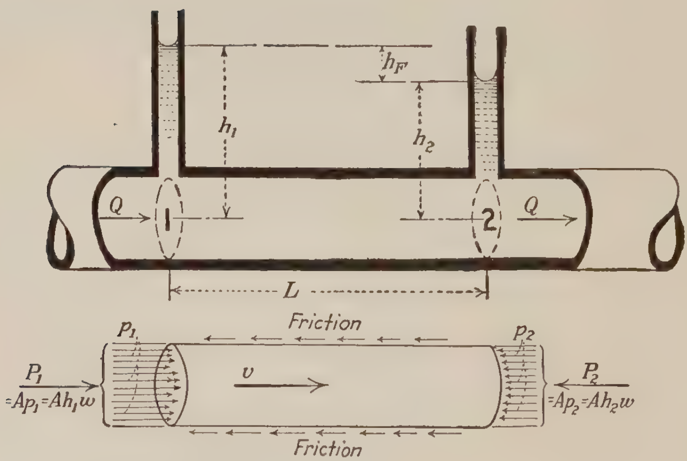


FIG. 106*A*.—Total friction (pounds) $= A(h_1w - h_2w) = f_1(w.p. \times L)v^2w.$

Friction-head $= h_F = h_1 - h_2 = f_1\left(\frac{w.p.}{A} \times L\right) v^2.$

tion of the perimeter of the cross-section of the waterway in contact with the liquid (see Figs. 104, 106, and 106*A*).

Calling the wetted perimeter *w.p.*, the cross-sectional area of the stream A, the length L, the mean velocity v, the weight per unit volume w, then the frictional resistance to motion, which must be continuously overcome while motion proceeds, is $f_1(w.p.)Lwv^2$. (See Fig. 106*A*.)

To maintain the flow of the fluid against this friction there must be this much excess total pressure (from a hydrostatic viewpoint) on the upstream cross-section of the length L of stream in question over and above the total pressure on the downstream cross-section. Calling this *loss of pressure* due to friction p_F *per unit of cross-section*,

$$A \times p_F = (w.p.)Lf_1wv^2, \text{ or } \frac{p_F}{w} = f_1L\left(\frac{w.p.}{A}\right)v^2.$$

It is known that a term of the form $\frac{p_F}{w}$ is a *head* or a height of liquid column. In an open channel it represents the surface fall (see Figs. 105 and 106); and in a pipe it is the loss of head, or fall in the hydraulic grade line, or the friction-head (see Fig. 104). It will be designated by h_F. Then

$$h_F = f_1L\frac{(w.p.)}{A}v^2. \tag{59}$$

This basic formula is variously modified. The most widely used modification is the Chezy formula, derived as follows:

The ratio $\frac{A}{w.p.}$, or the cross-sectional area of the stream divided by the wetted perimeter, is called the *hydraulic radius* (also called the *hydraulic mean depth* because for a wide river this ratio practically equals the depth of the water). Calling this ratio R (not to be confused with the geometric radius r of a circle), the formula becomes $h_F = f_1\frac{L}{R}v^2$, or $v^2 = \frac{1}{f_1}\frac{Rh_F}{L}$. The ratio $\frac{h_F}{L}$ is the rate of fall of the water surface of an open channel or, generally, the slope of the hydraulic grade line, and is commonly called the hydraulic slope or simply the slope, and is designated by s. Hence $v = \sqrt{\frac{1}{f_1}}\sqrt{Rs}$.

Calling $\sqrt{\frac{1}{f_1}} = C$, $\quad v = C\sqrt{Rs}$ (*the Chezy formula*). (60)

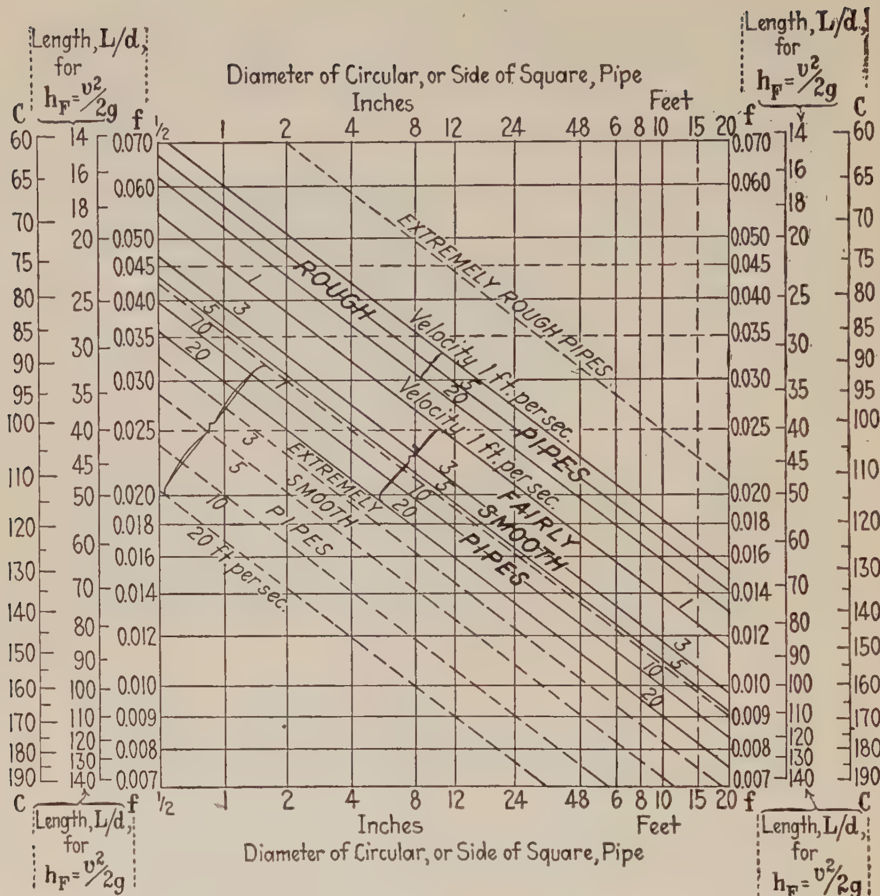


Fig. 107.—Flow of water in pipes.

NOTES ON THE SEVERAL CATEGORIES OF ROUGHNESS

"*Extremely smooth pipes:*" New seamless-drawn brass, block-tin and lead, glass, porcelain-like glazed pipes; all with interior surfaces both appearing very even to the eye and feeling very firm and smooth to the touch.

Intermediate between the above category and the one below are all sorts of *newly-laid* so-called "smooth" common commercial pipes such as coated cast-iron, wrought-iron, and wood-stave. High grade rubber-lined fire hose causes about one-third less loss of head than the following category.

"**Fairly smooth pipes:**" All ordinary pipes after a few (say about five, more or less) years in ordinary service, such as asphalt-coated cast-iron and spiral-riveted steel pipes (latter of thin metal and with very flat rivet heads), wrought-iron, both "black" and galvanized (but the latter in the small sizes may be "rough pipes" even when new), wood-stave, reinforced-concrete, galvanized, spiral-riveted steel. This category is rough enough to be called fairly conservative for general water supply designing purposes.

Intermediate between the above category and the one below are the above-mentioned pipes after being fairly long (say about ten years or so) in service

Values of C are best found by measuring $v\left(=\frac{Q}{A}\right)$, R, and s, rather than f_1. Values of f_1, however (found by measuring the "drag" exerted by water flowing along a submerged surface, or else the pull necessary to drag a surface lengthwise through still water), are found to be in fairly close agreement with values of C found from the measured v, R, and s of actual pipes and open channels. This agreement establishes in a general way the validity of the approximate assumptions made in the foregoing theory.

Values of C for pipes are given on Fig. 107, and for open channels in Table XI.

The numerical value of C, for most cases of engineering practice, ranges from about 50 to 150 (foot-pound units), the value 100 being customarily kept in mind by engineers for rough offhand estimates and checks.

103. Formula for Circular (also Square) Pipes Flowing Full.—If Eq. (59) be applied to either a circular or a square pipe with diameter or side $= d$, and with $\frac{(w.p.)}{A} = \frac{4}{d}$ in each case, then $h_F = 4f_1\frac{L}{d}v^2$. Instead of using this formula as it stands, it is useful for several reasons to introduce the velocity-head $\frac{v^2}{2g}$ into the

---

and subjected to average deterioration. Unlined linen "mill fire hose" causes about one-third more loss of head than the previous category of "fairly smooth pipes."

"**Rough pipes**:" Originally "fairly smooth pipes" that have deteriorated fairly rapidly for some ten or fifteen years after being laid; also ordinary lap-riveted steel pipes some years in service; also large well-laid brick storm-water sewers flowing full. This category represents a roughness such that its use in design is quite conservative in cases where full capacity will probably not be demanded for some dozen years after laying.

Intermediate between the above category and the one below are pipes having more local roughness or more frequent joints than ordinary water pipes, *e.g.*, ordinary glazed clay sewer pipes in average good service condition, also small brick lined sewers, also small riveted steel pipes made of sections only some two or three diameters long.

"*Extremely rough pipes:*" This category represents a degree of roughness or deterioration beyond anything that would ordinarily be allowed for in design of water pipes, say the condition of small street mains after some thirty or forty years of service. In this category come small sewer pipes considerably fouled by slime and deposits or laid with poor alignment.

formula. Therefore, using a new friction factor f, such that

$$f_1 = \frac{f}{4 \times 2g},$$

$$h_F = f \frac{L}{d} \frac{v^2}{2g}. \tag{61}$$

For values of f see Fig. 107.[1]

Note that $\frac{L}{d}$ is the *length of pipe in diameters.* Also note that when $L = d$, or for a length of 1 *diameter,* the loss of head is $f \times \frac{v^2}{2g}$. Experiments show that the value of f, Fig. 107, is a fraction ordinarily between .01 and .05 *i.e.* between $1/100$ and $1/20$. Hence f is the loss of head in each one-diameter length of pipe, expressed as a fractional part of the velocity-head, $\frac{v^2}{2g}$, of the flowing stream.

The total available head that causes flow through the pipe must supply, (see Eq. 38, par. 71), *in addition to the* $\frac{v^2}{2g}$ *of the stream and loss due to entry,* as many times $f \frac{v^2}{2g}$ as there are diameters in the total length of the pipe line. Hence, (as in par. 112, Example 3),

$$\begin{aligned} h_{\text{Total}} &= h_{\text{Entry}} + h_{\text{Friction}} + \frac{v^2}{2g} \\ &= 0.5 \frac{v^2}{2g} + f \frac{L}{d} \frac{v^2}{2g} + \frac{v^2}{2g} \\ &= \left(0.5 + f \frac{L}{d} + 1\right) \frac{v^2}{2g} \end{aligned}$$

Obviously *the greater the length of the pipe,* (h, d, and f remaining the same), *the less is the velocity and discharge.*

By thus expressing the loss due to friction as a multiple of $\frac{v^2}{2g}$, and introducing the additional term into Eq. 38, par. 71, all simple pipe problems such as given at the end of this chapter may be handled. See examples worked in par. 112.

[1] See footnote, p. 182 and 183, for discussion on variation in f values.

[Some authors let $f_1 = \frac{f'}{2g}$, and then the final formula becomes $h_F = 4f'\frac{L}{d}\frac{v^2}{2g}$. The textbooks use the same symbol f in both cases, but there need be no confusion if the formula is stated with accompanying tables of values of the friction factor f.]

Relations between C and f.—At the bottom of p. 181 the symbol C was used in place of $\sqrt{\frac{1}{f_1}}$, and at the top of p. 184 f_1 was placed equal to $\frac{f}{4 \times 2g}$. Evidently, then,

$$C = 2\sqrt{\frac{2g}{f}}; \text{ and } f = \frac{8g}{C^2}. \tag{62}$$

Hence the rough average value 100 for C corresponds to the value 0.026 for f, in foot-second units.

For a pipe this means that a length of $\frac{1}{0.026} = 38+$, say about 40, diameters will cause a loss of head $= \frac{v^2}{2g}$. Placing h_F, which $= f\frac{L}{d}\frac{v^2}{2g}$, equal to $\frac{v^2}{2g}$ then $\frac{L}{d} = \frac{1}{f}$. In words, to cause a loss of head equal to the velocity-head of the flowing water, the length of pipe equals the reciprocal of f, length being expressed in diameters.

104. Analogies between Loss of Head in Hydraulics and Increments and Decrements of Tensile or Compressive Stress in Mechanics.—If a rope be drawn through a pipe in which it fits tightly, the tension in the rope is a maximum at the end where the pull is applied and decreases uniformly to the other end where it may be either zero, if the rope be loose there, or may have an initial value due to a pull on that end.

If the rope be pushed (piston-like) through a pipe, the compressive stress is greatest at the upstream end and becomes less and less in going downstream along the "flowing" rope. The tendency of the rope to crumple or bulge out *sidewise* resembles somewhat the tendency of a fluid to burst out through the restraining walls of a pipe.

A rope wound around a snubbing post and being "paid out" shows an analogous decrease and increase of stress due to friction in what may be termed uniform flow.

A liquid, of course, transmits freely any compressive stress in all directions, so that an open water column or a pressure-gage attached to a hole in the wall of a pipe measures the degree of compression or the pressure-

head. In the rope, a spring balance would show the *total* tension, or a coil spring would show the *total* compression.

As compared to the flow of water in an open channel, one may find certain analogies in a sled sliding down a long uniform slope. Soon after the start, the sled ceases to accelerate further, having then acquired a velocity at which the friction (chiefly of the runners against the snow) balances the tangential component of the weight. Thereafter the velocity is uniform, the energy due to the descending weight being entirely absorbed by friction, but until the velocity of uniform motion is attained, the work of the falling weight is partly used in accelerating the sled and partly in overcoming friction. It may be noted also that, after uniform motion is attained, *if* the *friction* at some point along the course *increases*, due, for instance, to ashes on the roadway, the *speed decreases* until the (negative) work of friction again balances the work of falling at the new reduced velocity. During the transition from high to low velocity, some of the momentum of the sled has been used to overcome the greater resistance due to the increased roughness. The occupant of the sled is well aware of the sudden demand on him to "hang on" to prevent being "pitched" forward.

With water flowing in an open channel, a locally increased resistance likewise decreases the velocity, and the surplus velocity-head tends to change into pressure-head. What actually occurs, as a result, is a combination of a "standing wave" on the surface of the stream together with considerable eddying or "boiling" water. The *decreased velocity*, moreover, calls for an *increased* cross-section for the stream of water, because in steady flow there is a constant rate of discharge at all points along the stream, and, from the equation of continuity, $Q = A_1v_1 = A_2v_2 = A_3v_3$, and $A_1 = \frac{Q}{v_1}$; $A_2 = \frac{Q}{v_2}$, etc.

105. Power, or Energy, Gradient.—Generally speaking, in a portion of a stream *where the velocity is changing*, the surface slope, or the hydraulic gradient, does not directly show the rate of loss of head, since conversions of pressure and potential heads to or from velocity-head are involved. In fact, in addition to the hydraulic gradient, there may be drawn what may be termed the "power gradient" at a height $\frac{v^2}{2g}$ *above* the hydraulic gradient. As long as the velocity is constant these two gradients show the same rate of fall, but when the velocity changes, as at a Venturi meter or at a nozzle, the true loss of head is shown by the *power gradient*, not by the hydraulic gradient.

106. Recapitulation with Illustrative Experimental Assembly. The pipe line shown in Fig. 108 is of uniform diameter except

for a contracted section at N and a nozzle at the downstream end R. A portion of the pipe at J extends above the level of the water surface in the supply tank. This particular combination of features is here assembled into one pipe line merely for convenience in study.

If such a pipe line is plugged at the downstream end R, and, having first been filled with water, is put into communication with the water in the tank at the left by opening the valve D, the tops of the open water columns will all stand at O-O', the level of the water in the tank.

If the stopper at R is removed, flow starts and soon becomes steady, provided the inflow into the tank is adjusted to equal the outflow through the pipe line.

With flow proceeding, the open water columns respond to the now reduced pressures in the pipe, and when settled, the columns appear as shown in Fig. 108.

At G the Pitot tube inserted in the pipe shows the pressure-head *plus* the velocity-head of the flowing water (see p. 334). The velocity is not uniform throughout the cross-section (see p. 336).

First examine the open columns attached at places where the pipe is of the *same size, i.e.*, at F, I, K, M, and Q. A gradual decrease in pressure is shown in proceeding downstream.

With steady flow proceeding, the flowing liquid has the same kinetic energy at each of these points, because the velocity is the same, being $= \frac{Q}{A}$, where A is the area of the cross-section of the pipe and Q is the volume per second flowing.

Since the gradual decrease in heights of the open water columns is not to be accounted for by hydrostatics, nor by change in kinetic energy, it must be due to friction (*i.e.*, resistance to steady motion), encountered by the flowing liquid and overcome by the consumption of some of the pressure energy in the liquid. (See p. 185 for analogies between such loss of head and other phenomena in mechanics.)

Aside from friction, as the water enters the pipe at E, some pressure-head is converted into velocity-head, so column F would be expected to show a somewhat greater falling off in

pressure-head than by friction alone. Also (see p. 205), the elbows B and C make the loss greater than if there were simply straight pipe between E and F, and the valve D, even when wide open, makes some disturbance to the flow, causing additional loss of head.

Such losses of head are to be noted as phenomena always accompanying the flow of fluids, no matter how smooth the conduit.

The two places in the pipe line (Fig. 108) where there are quite pronounced changes in the water columns, one at M-N, where the stream contracts due to the convergence of the pipe towards

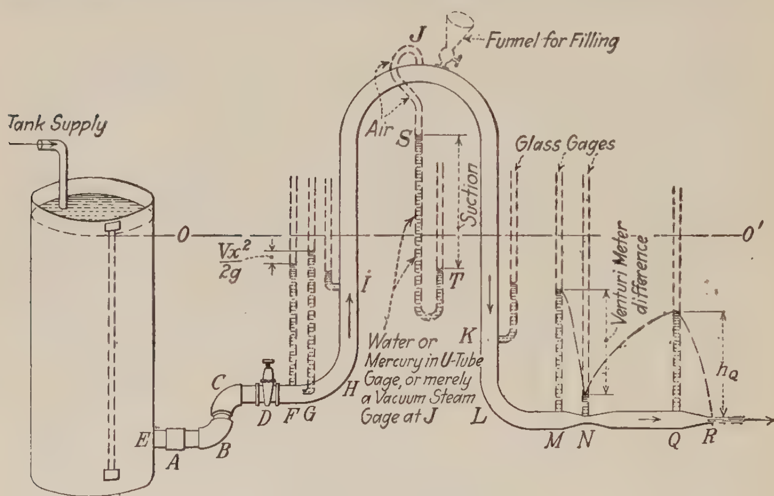


Fig. 108.—Experimental apparatus for illustrating some of the conditions accompanying the flow of liquids.

the throat of the Venturi meter, and the other at Q-R, where the pipe terminates in a converging nozzle, have already been treated in Chap. VIII.

The *rise* of the level in the columns between N and Q shows clearly that the previous fall from M to N is not a loss of head and not therefore a loss of power or of energy, except for the small real loss. It will be found later that, under favorable arrangements, nine-tenths of the throat velocity-head may be *reconverted* into pressure-head. The line connecting the tops of such open columns is called by engineers the "*hydraulic grade line,*"

Weisbachs Formula. Chezy

$h_f = \frac{FLV^2}{2gD}$ $V = C\sqrt{RS}$.

and its slope, the "*hydraulic gradient*," whether the columns actually exist or not.

More precisely, the hydraulic gradient is the quotient of the change in pressure-head between any two points along the line of flow and the actual length of stream between the points. This ratio, $\dfrac{\text{change in } \frac{p}{w}}{L}$, or slope of the hydraulic grade line is commonly called the *hydraulic slope*, or often simply the *slope*.

Such changes in pressure-head may be caused, as Fig. 108 shows, by friction alone or by changes in velocity-head accompanied by friction.

Wherever the pipe line lies *above* the hydraulic gradient, the pressure is less than atmospheric. The summit J (Fig. 108) represents such a condition.

Problems

1. A 2-in. wrought-iron pipe 100 ft. long discharges 7.48 gal. per sec. What is the actual velocity-head at points distant 1 and 99 ft. from the discharge end? Internal diameter of a 2-in. wrought-iron pipe, 2.067 in.; area, 3.356 sq. in.

2. A portion of a fairly smooth level pipe line, 8 in. diam., with gage readings of 20.9 lb. per sq. in. and 12.24 lb. per sq. in. at points 2000 ft. apart. (*a*) What is the flow in the pipe? (*b*) If the smaller gage reading is found to have a correction of 0.24 lb. per sq. in., what percentage error would this make in the discharge? *Ans.* (*a*) 1.54 c.f.s.
(*b*) 1.4 per cent.

3. If the approximate theory shows that $h_F = f_1 L \frac{w.p.}{A} v^2$ where f_1 is resistance for unit area, velocity, and density, and $w.p.$ is the wetted perimeter, derive the Chezy formula from this relation.

4. If the efficiency of the pumps is 75 per cent, how many horsepower will the engine driving the pumps exert when forcing 3 c.f.s. from a well to a reservoir through 3000 ft. of rough pipe, 10 in. diam.? The reservoir is 150 ft. above the level of the water in the well. *Ans.* 92 hp.

5. An 8-in. pipe line 2000 ft. long conveys water from a pump to a reservoir whose water surface is 300 ft. higher than the pump, which is pumping at the rate of 2.0 c.f.s. (*a*) Using constants for rough pipe (Fig. 107), find gage pressure at the pump. Neglect losses of head except "friction-head." (*b*) If this pipe is replaced by a new 10-in. pipe (to which the fairly smooth pipe coefficients apply), find the gage pressure at the pumps when operating. (*c*) How many horsepower are saved by changing from the old 8-in. to the new 10-in. pipe? *Ans.* (*a*) 152 lb. per sq. in.
(*b*) 135 lb. per sq. in.
(*c*) 9.3 hp.

6. It is necessary to pump 3 c.f.s. through a pipe line 4000 ft. long to a reservoir 200 ft. above the pumps. What minimum size of new cast-iron pipe should be installed (assuming it will be in service only 2 years) if the lost head is not to exceed 10 per cent of the static lift?

7. A 900-ft. lateral is to supply a factory with 125 g.p.m. The pressure in the main is constant at 30 lb. per sq. in. The discharge in the factory takes place under a pressure of 5 lb. per sq. in. What size of fairly smooth pipe should be installed?

8. A pipe line is 2500 ft. long with a total drop in pressure head of 20 ft. What commercial-sized pipe is required (making allowance for the future when the pipe will have become foul and tuberculated) to furnish a flow of 2,000,000 g.p.d?

9. (*a*) If the elevation of the surface of a reservoir is 410, and a pipe line 4060 ft. long of 10-in. rough pipe leads to a lower reservoir at elevation 205, compute the discharge Q in gallons per day. (*b*) If the elevation of the reservoir surface is 410, what size of fairly smooth pipe should be installed in a pipe line 4060 ft. long if it discharges $Q = 1.6$ c.f.s. into lower reservoir at elevation 205? *Ans.* (*a*) 3,400,000 g.p.d.

10. A lateral 6-in. line 1500 ft. long is taken from a 24-in. main. The pressure in the main may be taken as constant at 50 lb. per sq. in. The 6-in. line terminates in a fire hydrant. The highest "tap" in the house near the hydrant is 36 ft. above the 6-in. lateral. In case of fire, how many "good fire streams" of 250 g.p.m. each could be taken from the hydrant, and still have water at the above-mentioned house tap?

11. A fire pumper takes water from a hydrant and supplies a line of 2½-in., high-grade, rubber-lined hose 300 ft. long, terminating in a good nozzle with tip diameter of 1¼ in. The pressure back of the nozzle is to be kept at 28 lb. per sq. in. The discharge is to be 250 g.p.m. High-grade, rubber-lined hose causes one-third less loss of head than fairly smooth pipe. (*a*) At what pressure must the pumps be operated? Draw hydraulic grade line. (*b*) Given that the horsepower in a stream of water is $\frac{Qwh}{550}$, where Q is expressed in cubic feet per second, w equals 62.4 lb. per cu. ft., and h is the head, what would be the number of horsepower supplied to the pumps with the pump efficiency at 80 per cent? Neglect minor losses.

12. Find the head necessary to discharge 4 c.f.s. through a 12-in. (fairly smooth) pipe, 75 ft. long, allowing $0.5\frac{v^2}{2g}$ for loss of head due to the square-edge entry. (*b*) Same for same pipe 2000 ft. long. *Ans.* (*a*) 1.2 ft.
(*b*) 16.7 ft.

13. Two reservoirs, 10 miles apart, are connected by a pipe line. What size of rough pipe would discharge 10 c.f.s. with a drop of 3 ft. in the hydraulic grade line in the middle 2 miles? *Ans.* 3.56 ft.

14. Of two channels in the same material, one the shape of a half hexagon and the other the shape of a segment of a circle, both having the same

stream cross-section and slope, which is the most "efficient" (has maximum discharge)? Why?

15. In an open channel of triangular section with each side equal to 1.414 times the depth, Chezy C equal to 122, discharge of 16 c.f.s. on a slope of 0.001, what is the depth? *Ans.* 2.18 ft.

16. In a circular sewer flowing first full, then at 0.9 depth, indicate the corresponding change in the hydraulic elements—area, hydraulic radius, velocity, and discharge; whether they are the same, greater, or less at the 0.9 depth.

17. A semicircular canal of radius 5 ft. discharges 300 c.f.s. with Chezy C equal to 127. What is the necessary slope? *Ans.* 0.00144.

CHAPTER XII

THE FLOW OF WATER IN PIPES (*Continued*)

COEFFICIENTS AND SPECIAL LOSSES

Before proceeding with applications, the flow in pipes will be reconsidered from a different viewpoint, viz., from the indications of lump velocity cœfficients similar to those of orifices and short tubes.

(Turbulent flow is assumed. See p. 276.)

107. General Experimental Findings. 1. *Effects of Added Friction in Decreasing the Discharge.*—The plotting of experimentally found data in Fig. 85 shows how the discharge of a tube decreases as the length increases. Also it appears that the rougher pipe (galvanized iron) has less discharge for the same diameter and length than the smoother pipe (drawn brass), if the comparison is between pipes having the same kind of entry edges.

The data show that friction is increased in a tube of a fixed diameter either by increasing the roughness of a tube of a fixed length, or by increasing the length of a tube of a fixed degree of roughness, or by both changes combined.

Beyond the range of the plotting in Fig. 85 the coefficients continue to decrease. For instance, for a length of 11.08 ft. (= 160 diameters for this size pipe) coefficients are 0.453 and 0.385, respectively, for the brass and galvanized pipes with square-edged entry.

2. *Uniform Flow Conditions and Consequent* **Uniform Rate of Loss of Head** *along Portions of a Pipe* **away from Entry Disturbances.**—(The student should review the discussion on the hydraulic grade line on pp. 187–189 referring to Fig. 108, also he should refer to plotting on Fig. 115 for actually observed hydraulic gradients, beyond square-edged contractions.) The hydraulic grade line for steady flow through any pipe of uniform size shows

that the loss of head occurs uniformly along the pipe except for a short portion just downstream from the entry. In fact, the flow conditions beyond a square-edged entry to a long pipe closely resemble those already considered for a short-pipe orifice (see example, p. 125 and notes on Fig. 82*c*). From the curves on Fig. 85, it appears that flow conditions, as judged by the coefficients, become fairly uniform at a distance of about six diameters downstream from the entry. The actual gradients on Fig. 115 substantiate this. (To attain *perfect* normalcy of flow, a considerably longer distance is required.)

108. Comparison of Heads Required on Two Pipes of Different Lengths, but with Same Rate of Flow. (Pipes of Same Diameter and Same Degree of Roughness.)—If adding to the length of a

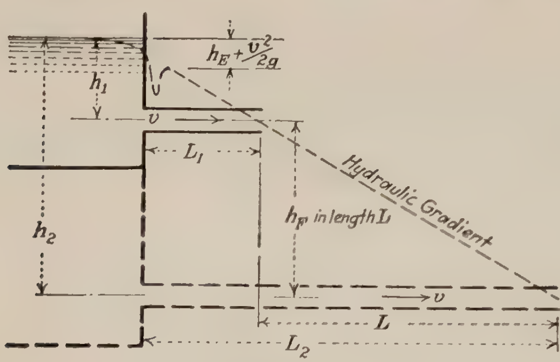


FIG. 109.

pipe decreases the discharge under the same head, it follows that more head is necessary for the longer pipe to yield the same velocity and discharge as for the shorter pipe.

Assuming that experiments have given coefficients for a variety of lengths, one may reason as follows:

If $v = C\sqrt{2gh}$, then $\frac{v^2}{2g} = C^2h$, and

$$h = \frac{1}{C^2}\frac{v^2}{2g}. \quad \text{(See Eqs. (37) and (46).)} \qquad (62a)$$

Experiments show that C remains practically constant for different heads for any one length of pipe, and that a different C is

obtained only when the length is changed (assuming the same degree of roughness for both lengths). Consequently, the facts may be represented as in Fig. 109.

If the head h_1 causes a velocity v through the pipe of length L_1, and it is found necessary to increase the head to h_2 when the length is increased to L_2 in order to maintain the same velocity v, and if the coefficients in the two cases are C_1 and C_2, then it is obvious that the *added head* $(h_2 - h_1)$ is necessary to overcome the added friction in the added length $L_2 - L_1$, or, as designated in Fig. 109, h_F is the loss of head due to friction in length L. But if from the reversed orifice flow law (Eq. 62*a*)

$$h_1 = \frac{1}{C_1^2}\frac{v^2}{2g}, \text{ and } h_2 = \frac{1}{C_2^2}\frac{v^2}{2g},$$

then

$$(h_2 - h_1) = h_F = \left(\frac{1}{C_2^2} - \frac{1}{C_1^2}\right)\frac{v^2}{2g}. \tag{63}$$

109. Experimental Illustration.—Picking off several of the coefficients from Fig. 85, and extending the computations to find the friction-head in the added length, the results are given in Table VI.

From the last column of the table it appears that for the smooth brass pipe a loss of head $\frac{v^2}{2g}$ is caused by a length of about $45 \times d$, taking the value for the long length $136d$. The shorter lengths show a small departure from this value, the portions nearer the entry showing the highest values, possibly because of a disturbed condition of flow.

For the rougher pipe the values show greater variation. No two portions of such a pipe (galvanized iron) are exactly alike in degree of roughness. The shorter the specimen the less likely is it that another short piece will be of closely the same roughness (in a hydraulic resistance sense). The average value, weighting the separate values in proportion to lengths, is $31d$.

Hence the galvanized-iron pipe causes about 50 per cent more loss of head for the same rate of flow than does the smooth brass pipe, because in a length of $45d$ the rougher pipe would cause a loss of about $1.5\frac{v^2}{2g}$.

TABLE VI.—TABULATION OF DATA FROM CURVES OF FIG. 85 WITH COMPUTATIONS BASED ON RELATION EXPRESSED IN EQ. (62a)

(All pipes are standard ¾ in., internal diameter = 0.83 in.)

(1) Kind of pipe and nature of entry	(2) Total length of pipe expressed in diameters	(3) Coefficient of velocity and discharge found by experiment, C from plotted curves (Fig. 85)	(4) Values of $\frac{1}{C^2}$ (see Eq. (62a)) = total head as a multiple of the issuing velocity-head, *i.e.*, $h = \frac{1}{C^2}\frac{v^2}{2g}$	(5) Differences between successive values in column (4) = loss of head in added length of pipe, usually = $8d$, the loss as a multiple of $\frac{v^2}{2g}$ (see Eq. (63))	(6) Length of pipe, expressed in diameters, causing loss of head = $\frac{v^2}{2g}$
Smooth, seamless-drawn brass pipe, rounded entry.	$8d$	0.942	1.127		
				$0.158\frac{v^2}{2g}$	$50.6d$
	$16d$	0.882	1.285		
				$0.163\frac{v^2}{2g}$	$49.0d$
	$24d$	0.831	1.448		
Smooth, seamless-drawn brass pipe, square-edged entry.	$8d$	0.814	1.509		
				$0.156\frac{v^2}{2g}$	$51.3d$
	$16d$	0.775	1.665		
				$0.181\frac{v^2}{2g}$	$44.2d$
	$24d$	0.736	1.846		
				$3.027\frac{v^2}{2g}$ (in $136d$)	$44.9d$
	$160d$	0.453	4.873		
Rough, galvanized-iron pipe, rounded entry.	$8d$	0.874	1.309		
				$0.211\frac{v^2}{2g}$	$37.9d$
	$16d$	0.811	1.520		
				$0.189\frac{v^2}{2g}$	$42.3d$
	$24d$	0.765	1.709		
Rough, galvanized-iron pipe, square edged entry.	$8d$	0.772	1.678		
				$0.193\frac{v^2}{2g}$	$41.4d$
	$16d$	0.731	1.871		
				$0.223\frac{v^2}{2g}$	$35.9d$
	$24d$	0.691	2.094		
				$4.652\frac{v^2}{2g}$ (in $136d$)	$29.2d$
	$160d$	0.385	6.746		

110. Development of Friction-head Formula.—If the loss of head per one diameter length of pipe is called $f \times \frac{v^2}{2g}$, where f is a friction factor or friction fraction of the velocity-head, the loss of head in a pipe of any length L may be stated as:

$$h_F = f \frac{L}{d} \frac{v^2}{2g}, \qquad \text{(same as Eq. (61))}$$

a formula of extensive use in hydraulics, and which has been derived (p. 184) on another basis.

Example: Let the pipe of Fig. 109 be increased in length to 300 diameters (= 20.75 ft.). Then, in view of the additional friction, the equation for a short tube orifice with square-edged entry (Eq. (38), p. 126) must be changed to read (for $f = \frac{1}{30}$),

$$h = 0.5 \frac{v^2}{2g} + \frac{300}{30} \frac{v^2}{2g} + \frac{v^2}{2g}, \tag{64}$$

the value 0.5 being the loss of head due to a square-edged entry.[1]

Then, for the problem in hand, with $h = 3$ ft. (see Fig. 85),

$$3 = 11.5 \frac{v^2}{2g}, \text{ or } \frac{v^2}{2g} = \frac{3}{11.5} = 0.261 \text{ ft.}$$

The remaining 2.739 ft. of the original 3 ft. is lost in entry resistance and pipe friction, mostly the latter. If $\frac{v^2}{2g} = 0.261$ ft., $v = 4.10$ ft. per sec. and $Q = Av = 0.0154$ c.f.s. or 4140 U. S. g.p.h. (The coefficient $C = \frac{4.10}{\sqrt{2g \times 3}} = \frac{4.10}{13.9} = 0.288$.)

If the entry is *rounded*, thus almost eliminating the loss, the term $0.5 \frac{v^2}{2g}$ may be omitted, and the calculation then gives $v =$ 4.19 ft. per sec.; the corresponding coefficient $C = 0.301$. Note how much smaller the velocity is for this pipe 300 diameters long than for the shorter pipes as given in column (3) of Table VI.

111. Long Pipes.—If comparison is made between the discharges through considerable lengths, say 100 ft. $= \frac{100 \times 12}{0.83} =$ 1445 diameters of the two kinds of pipe, taking entry loss as in Eq. (64),

[1] As on p. 184, $h_{\text{Total}} = h_{\text{Entry}} + h_{\text{Friction}} + \frac{v^2}{2g}$ (At exit).

$$\textit{Brass pipe, } h = \left(0.5 + \frac{1445}{45} + 1\right)\frac{v^2}{2g} = \left(32.1 + 1.5\right)\frac{v^2}{2g}.$$

$$\textit{Galvanized pipe, } h = \left(0.5 + \frac{1445}{30} + 1\right)\frac{v^2}{2g} = \left(48.2 + 1.5\right)\frac{v^2}{2g}.$$

It will be seen that for such a case (a long pipe) the entry loss and the issuing velocity-head may be omitted since this sum is insignificant in comparison with the pipe friction, *i.e.*, practically $h = 32.1\frac{v^2}{2g}$ for the brass pipe, and $h = 48.2\frac{v^2}{2g}$ for the galvanized pipe, without involving an error as large as the uncertainty in the pipe friction.

Velocity Coefficient.—Since for the brass pipe,

$$h = 33.6\frac{v^2}{2g}, \quad v = \sqrt{\frac{1}{33.6}}\sqrt{2gh} = 0.172\sqrt{2gh}, \textit{ i.e.}, 0.172$$

is the velocity (also discharge) coefficient. Likewise, the coefficient is 0.142 for the galvanized pipe. This means that only about one-sixth to one-seventh of the head goes into creating velocity, the rest being used up in maintaining the flow against the friction of the pipe walls.

Problem for the Student: If the pipe were *smoother* than in the example on p. 196, so that a length of 46 diameters (instead of 30) causes a loss of head equal to the velocity-head, *i.e.* if the $\frac{L}{d}$ value $\left(\text{for } h_F = \frac{v^2}{2g}\right)$ is 46, or if $f = 1/46$, the head is 12 ft., entry rounded (loss negligible), other data as in the example on p. 196, what is the velocity and what the discharge? *Ans.* $v = 10.1$ ft. per sec., $Q = 0.0379$ c.f.s. or 17.0 U. S. g.p.m. (1 cu. ft. = 7.48 U. S. gal.).

The diagram (Fig. 107) shows values of the friction factor f for various sizes, roughness categories, and velocities. Note that, since f represents the fraction of $\frac{v^2}{2g}$ lost in one diameter length ($L = d$), the reciprocal of f is the length in diameters $\left(\frac{L}{d}\right)$ causing a loss of head $h_F = \frac{v^2}{2g}$.

112. Simple Pipe Problems.—(By formula.)

Example 1: Find the head (the h in either Fig. 110 or Fig. 111) necessary to discharge 1 million U. S. g.p.d. (24 hr.) through a fairly new 6-in., cast-iron, asphalt-coated pipe 2000 ft. long. Assume that the loss of head at entry E is negligible. (*N. B.*: This item is not always negligible. See example 3 on page 201.)

Solution with Discussion: One cubic foot = 7.48 U. S. gal. Hence $Q = \dfrac{1,000,000}{7.48 \times (24 \times 60 \times 60)} = 1.547$ c.f.s. (Note this value; also that 1 c.f.s. = 0.646 million g.p.d. Roughly, 1 million U. S. g.p.d. = $1\frac{1}{2}$ c.f.s., and 1 c.f.s. = $\frac{2}{3}$ million g.p.d.)

General Analysis of the Problem: The head h (really a difference of heads, as in all cases of flowing fluids) is used up in overcoming friction h_F, and in supplying the *velocity head* $\dfrac{v^2}{2g}$ of the stream, *i.e.*,

$$h = f\frac{L}{d}\frac{v^2}{2g} + \frac{v^2}{2g}.$$

$$\text{But } v = \frac{Q}{A} = \frac{1.547}{0.196} = 7.9 \text{ ft. per sec.,}$$

and $\dfrac{v^2}{2g} = 0.97$ ft. To find the value of f, interpolate on Fig. 107, for fairly smooth pipes with $d = 6$ in. and v about 8 ft. per sec., finding $f = 0.022$.

$$\text{Hence } h = \left(0.022 \times \frac{2000}{0.5}\right)\frac{v^2}{2g} + \frac{v^2}{2g}$$

$$h = (88 + 1)\frac{v^2}{2g} = 89 \times 0.97 = 86 \text{ ft.}$$

(Although as a mere matter of mathematics it is not essential to keep the expression $\dfrac{v^2}{2g}$ as an entity until near the end of the problem as is done above, there are certain advantages from the viewpoint of hydraulics, and these advantages will appear to the student with the solution of further problems.)

Comments.—If the pipe is *rough* inside, so as to fall in the category of "rough pipes," it is found from Fig. 107 that $f = 0.036$ and $h = (144 + 1)\dfrac{v^2}{2g} = 145 \times 0.97 = 140$ ft. as compared with only 86 ft. required for the smoother pipe to discharge water at the same rate.

Minor and Negligible Head Items.—It should be noted that in this case of a long pipe, the head necessary to produce the velocity could have been neglected, and practically the same result could have been obtained. In fact, as regards the proper

value for f, an uncertainty of one unit in the last place (*e.g.*, whether to take 0.022 or 0.023; 0.035 or 0.036, etc.) causes more difference than the omission of the head corresponding to the velocity. In example 1, for smooth pipe, this degree of uncertainty in f makes about four times as great a differenceas the inclusion or omission of the solitary $\frac{v^2}{2g}$ item.

Because of this fact, it is customary among engineers, in computation for long pipes (say where $\frac{L}{d}$ is as great as or greater than 1000, or perhaps 500), to neglect such items as (*a*) loss of head at

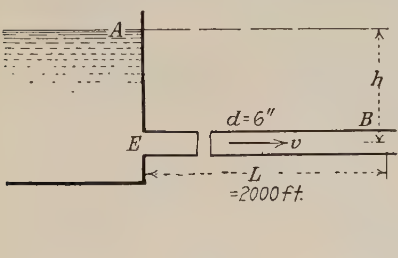


FIG. 110.

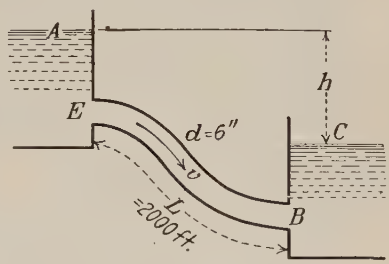


FIG. 111.

entry, (*b*) the velocity head item, (*c*) loss due to an occasional elbow or bend, and, in general, all such minor losses, if their total is not more than a few per cent (say more than 5 to 10 per cent) of the friction-head item. (But see example 3 on page 201 for a typical case where these items are not "minor" and cannot be neglected.)

Example 2: Find the *discharge* (dependable after a dozen or more years of service) of an 8-in. pipe 1500 ft. long with a head of 20 ft., the general arrangement in other respects as shown in Figs. 110 or 111. Consider only the friction-head in this case.

Solution: If the minor head items are neglected, the general analysis is simply that the total available head equals the friction-head, or from Eq. (61) $h = f\frac{L}{d}\frac{v^2}{2g}$. Since the discharge is unknown, also the velocity (upon which the value of f depends to some extent), the exactly correct value of f cannot be obtained

at once. But Fig. 107 shows that there is but little change in f with change in velocity, *especially for rough pipes.*

If, as a guess, v is between 3 and 5 ft. per sec., $f = 0.035$, and $20 = 0.035 \frac{1500}{2/3} \frac{v^2}{2g}$, or $\frac{v^2}{2g} = \frac{20}{78.8} = 0.254$ ft., whence $v = 4.04$ ft. per sec. (showing that the assumed value of f need not be changed). Then $Q = Av = 0.349 \times 4.04 = 1.41$ c.f.s. *Ans.*

Comments.—If the value of f had been estimated greatly in error due to a very wrongly assumed v, the first computed v could be taken as an approximation, and used merely to select a more nearly correct f for the second (usually final) computation.

A good exercise for the student is to use the suggestion in the example on page 197, for making a rapid "scientific guess" close to the correct v at the start, *i.e.*, close enough so that the first value used for f need not be changed.

N. B.—There is no occasion for making a wild guess for v. The value is *bound to be less* than $\sqrt{2gh}$, and for a long pipe very much less, since the total head must supply not only the one $\frac{v^2}{2g}$ represented in the velocity of the stream, but also the friction-head and any other special losses equal to many times $\frac{v^2}{2g}$. The multiple of $\frac{v^2}{2g}$ depends on how many times $\frac{L}{d}$ (the length in diameters) is greater than the value appropriate for the pipe in question, namely, the reciprocal of f (see Fig. 107), a sort of average value for medium-sized pipes being some 30 to 50 diameters length for $h_F = \frac{v^2}{2g}$. Thus in the example on page 196 it was found that not h, but $\frac{h}{11.5} = \frac{v^2}{2g}$.

Note how a very much smaller head (20 ft. in example 2 as compared with 140 ft. in example 1, both for rough pipes) supplies nearly as great a discharge through 1500 ft. of 8-in. pipe as does the larger head through 2000 ft. of 6-in. pipe. The great importance of selecting the *proper size* of pipe for a definite or fixed service is thus emphasized. (See p. 170 for the general rule, and page 227 for particular application on the effects on head and discharge of small errors or variations in the diameter.)

Problem for the Student.—Referring to example 1, compute the discharge of the rough pipe for the same 86-ft. head that gives

1,000,000 g.p.d. for the smoother pipe. *Ans.* 800,000 g.p.d. or 20 per cent less than for the fairly smooth pipe.

Example 3: Find the proper size of a *square* culvert of concrete, 150 ft. long, to discharge 400 c.f.s. with 5 ft. difference of water levels (see Fig. 112).

Solution: In this case, as will appear presently, $\frac{L}{d}$, the length in diameters, seems likely to be less than 100, so the item of the issuing velocity head cannot be neglected. Also, since the

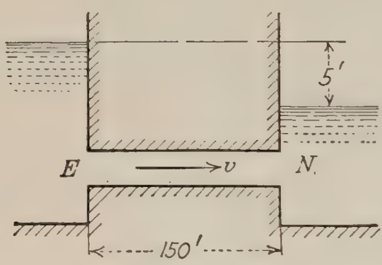


FIG. 112.

entry at E is square-edged and (see p. 206) a loss of head = about $0.5 \times \frac{v^2}{2g}$ occurs there, this item cannot be neglected. The student should try to give the reasons for these several statements.

At the submerged exit N, the velocity head is lost in eddies and turbulence because the enlargement is *sudden* rather than *gradual* as in the Venturi meters pictured in Figs. 89 and 89*a*.

Therefore, $h = 0.5\frac{v^2}{2g} + f\frac{L}{d}\,\frac{v^2}{2g} + \frac{v^2}{2g}$, or, in words, the total head, really *the flow producing difference of heads,* is used up in overcoming the entry resistance, in supplying the friction along the pipe, and in supplying the kinetic energy or velocity-head of the stream. (For convenience, rather than from necessity a system is adopted wherein the first two of these items are expressed as multiples of $\frac{v^2}{2g}$.)

Then $5 = 0.5\frac{v^2}{2g} + f\frac{L v^2}{d\, 2g} + \frac{v^2}{2g}$, or $5 = \frac{v^2}{2g}\left(1.5 + f\frac{150}{d}\right)$.

The velocity $v = \frac{Q}{A} = \frac{400}{d^2}$ for the square cross-section specified $\left(\text{or } \frac{400}{0.7854d^2} \text{ if the section were circular}\right)$.

(At this point of the solution the natural impulse is to get rid of v by substituting $\frac{400}{d^2}$ for it. One would have, then, $5 = \frac{\frac{400^2}{d^4}}{2g}\left(1.5 + f\frac{150}{d}\right) = \frac{400^2 \times 1.5}{2g}\left(\frac{1}{d^4} + \frac{100f}{d^5}\right)$, or $\frac{1}{d^4} + \frac{100f}{d^5} = 0.00134$, or $0.00134d^5 - d = 100f$, or $d^5 - 746d = 74{,}600f$. After having derived this expression, it would have to be solved by trial. For each assumed value of d there is a certain corresponding value of v, which $= \frac{Q}{A} - \frac{400}{d^2}$, and for this v the diagram or tables show a corresponding value of f.

To avoid the necessity of dealing with such an equation, which also obscures the hydraulic terms, it is simpler to solve the original equation by trial as follows:)

Dealing directly with the expression

$$5 = \frac{v^2}{2g}\left(1.5 + f\frac{150}{d}\right)$$

try as a guess, $d = 4$ ft., then the corresponding $v = \frac{Q}{A} = \frac{400}{16} = 25$ ft. per sec. But this velocity is impossible, since it is greater than $\sqrt{2g \times 5}$, hence it would require more than the available 5-ft. head. Moreover, the several losses of head must be supplied in addition to the velocity-head $\frac{v^2}{2g}$. In fact, the value $\sqrt{2g \times 5} = 17.9$ ft. per sec. represents the upper ideal limit, with no friction at all; and at the start an assumed velocity much lower than this should be taken.

Try $d = 6$ ft., then $v = 400/36 = 11.1$ ft. per sec.; and, for this v, Fig. 107 shows, for the rough-pipes category, $f =$ about 0.018. Then $h = \frac{11.1^2}{2g}(1.5 + 0.45) = 3.74$ (which is less than the available 5-ft. head). This shows that the assumed d is too large for the 400 c.f.s. But note that a small change in d has a greater relative effect on the necessary head.

Try $d = 5.6$ ft.; then $v = \dfrac{400}{31.4} = 12.75$ ft. per sec. and $h = 2.53\ (1.5 + 0.496) = 5.05$ (using 0.0185 for f). This is a close enough check. *Ans.* Hence $d = 5.6$ ft., the correct size.

Comments: Note that the sum of the entry loss and the velocity-head is here three times the pipe friction-head; hence none of the three items may be neglected, as done in examples 1 and 2.

Problems: 1. Compute the effect on the required diameter of rounding the entry so as practically to eliminate the loss of head at that point. *Ans.* The diameter is reduced from 5.6 to 5.22 ft., the area from 31.4 to 27.3 sq. ft.

2. Compute the effect on the size of the culvert of both rounding the entry and making the interior surface fairly smooth instead of rough.

113. Compound pipes (composed of two or more sizes in series with the same discharge through each, or pipes with terminal nozzles) may be handled as in examples 1 to 3, except that care must be taken to distinguish between the *different velocities* in the several sizes of pipe (see also p. 231 and Figs. 120 and 121).

Example 4: What is the discharge through the pipe system, Fig. 113, if pumps to the left of A maintain a steady flow with a pressure of 100 lb. per sq. in. at A? Pipes are fairly smooth.

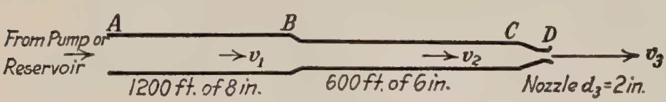


Fig. 113.

Solution: At first, subject to later correction or justification, as the case may be, neglect all losses except those due to friction in the pipes, and include only the terminal velocity-head at the nozzle tip D.

Since 100 lb. per sq. in. is equivalent to 231 ft. of water head,

$$231 = {}_1f\frac{1200}{\frac{2}{3}}\frac{v_1^2}{2g} + f_2\frac{600}{\frac{1}{2}}\frac{v_2^2}{2g} + \frac{v_3^2}{2g}.$$

But $v_1 = \left(\frac{2}{8}\right)^2 v_3 = \frac{v_3}{16}$, and $\frac{v_1^2}{2g} = \frac{1}{256}\frac{v_3^2}{2g}$; also $v_2 = \left(\frac{2}{6}\right)^2 \times v_3 = \frac{v_3}{9}$ and $\frac{v_2^2}{2g} = \frac{1}{81}\frac{v_3^2}{2g}$.

As in example 2, f_1 and f_2 cannot be found exactly until v_1 and v_2 are approximately known. But it is obvious that $\frac{v_3^2}{2g}$, the terminal velocity-head, must be less than the total head, 231 ft.; or v_3 must be less than 122 ft. per sec., or v_2 less than $^{122}\!/_9 =$ 13.5, and v_1 less than $^{122}\!/_{16} = 7.6$ ft. per sec.

Therefore, assume temporarily, in order to find approximate values for f, that v_2 is *roughly* about (or of the order of) 10 ft. per sec. and v_1 roughly about 5 ft. per sec. Then from Fig. 107 for fairly smooth pipes, f_1 = about 0.022 for 8-in. pipe with 5 ft. per sec., and f_2 happens in this case to be nearly the same for 6-in. pipe with 10 ft. per sec. (Why is this so? Does f change (increase or decrease) the same way as the diameter and velocity? What does Fig. 107 show?)

$$\text{Hence } 231 = \left(0.022 \times 1800 \times \frac{1}{256} + 0.022 \times 1200 \times \frac{1}{81} + 1\right)\frac{v_3^2}{2g}$$
$$= (0.155 + 0.326 + 1)\frac{v_3^2}{2g} = 1.481\frac{v_3^2}{2g},$$

whence $\frac{v_3^2}{2g} = 156$ ft.

Then $v_3 = 100.3$ ft. per sec. and $Q = A_3v_3 = 0.0218 \times 100.3 =$ 2.18 c.f.s., equivalent to 980 U. S. g.p.m. The corresponding value of v_1 is $\frac{100.3}{16} = 6.27$ ft. per sec., and of $v_2 = \frac{100.3}{9} = 11.14$ ft. per sec.; and Fig. 107 shows that there is no perceptible change in f_1 or f_2 for these changes in v_1 and v_2 from the assumed 5 and 10 ft. per sec.

If a velocity coefficient = 0.975 for the nozzle had been assumed, corresponding to a loss of head $= 0.05\frac{v_3^3}{2g}$, the computed discharge would be only 1.5 per cent smaller, a variation much less than that due to uncertainties in the pipe friction (see also p. 237).

Problems: 1. With the nozzle CD of Fig. 113 removed, what head at A is necessary to discharge this same Q = 2.18 c.f.s.? (Note that the values of v_1 and v_2 already found remain the same.)

Ans. Head = 75 ft. (From the data available, what is the very simplest way of finding the desired head?)

2. If the whole pipe line were 6-in. size, how much would the discharge be *reduced*, keeping the same 75-ft. head? *Ans.* Q is *reduced* from 2.18 to 1.53 c.f.s., a reduction of 30 per cent. On the other hand, if 1200 ft. of 8-in. pipe were substituted for an equal length of a pipe line having originally 1800 ft. of 6-in. pipe, the discharge would be *increased* 42 per cent.

SPECIAL LOSSES OF HEAD

114. Curves, elbows, tees, enlargements, contractions, hydrants, meters, and all such fittings or "specials" in a pipe line cause losses of head which, if of consequence, must be allowed for in addition to the plain pipe friction.

The special losses may be expressed either as: (a) fractions or multiples of $\frac{v^2}{2g}$, as already done for plain pipe in using the formula $h_F = f\frac{L}{d}\frac{v^2}{2g}$; ($b$) or in terms of the length of straight pipe that will cause the same loss (see p. 184 and the middle scales on Fig. 107). For a pipe that is obviously of length less than a "long pipe" (see pp. 196 and 199), such that the velocity-head of the flowing stream or of the issuing stream may not be neglected, it is convenient, as in example 3 (p. 201), to have each item of loss of head expressed as a multiple of $\frac{v^2}{2g}$.

But if, by diagram or formula, it is preferred to deal with the equivalent straight pipe, of length somewhat longer than the actual pipe containing the specials, then the transformation from the $K\frac{v^2}{2g}$ form to the $\frac{L}{d}$ form (or extra length or straight pipe) is readily made by means of the middle scales of Fig. 107. (See pp. 247–248 for example.)

For any particular special fittings, such as an elbow, tee, etc., experiments show that the loss of head is not in all cases a constant fraction of $\frac{v^2}{2g}$ (v being the mean velocity in the pipe), but that the fraction varies somewhat with the velocity. For most practical purposes, however, the values given in Table VII may be relied on as **safe average values** and yet not needlessly high.

TABLE VII.—SPECIAL LOSSES OF HEAD IN TERMS OF $\frac{v^2}{2g}$

(To find the equivalent length of straight pipe causing the same loss, use middle vertical scales of Fig. 107, p. 182)

Nature of the special resistance	Loss of head as decimal or multiple of $\frac{v^2}{2g}$
Square-edged entry; or upstream end of pipe perpendicular to, and flush with, inside face of reservoir wall. (Under good conditions value is about 0.40 ±.)	0.50
Projecting pipe at entry; or reentrant pipe end	1.00
Well-rounded entry. Loss is from 0.02 to 0.05 × $\frac{v^2}{2g}$, practically zero	
Easy curves or bends of very large radius, and smooth inside, of the same inside diameter throughout as the pipe line. As far as *excess loss* over the same length of straight pipe is concerned, due to curvature, h_F = about 0.05 to 0.15 × $\frac{v^2}{2g}$, and in many cases may be disregarded in view of other uncertainties	
90-deg. curves, fairly smooth, same inside diameter as pipe; with radius of center line = diameter of pipe (*i.e.*, a "short turn")	0.50
Easier 90-deg. curves than above, with center-line radius equal to two to eight times the pipe diameter	0.25
90-deg. common screw-end pipe elbow, short turn, inside larger than pipe, made both in malleable and cast iron	0.75
Tees, common screw-end, full-size branch	1.50
Square-elbow (intersection of two cylinders, corner not rounded at outside nor at inside of curve)	1.25
Obtuse-angled elbows (deflection of pipe less than 90 deg.; multiply values for square-elbow and 90-deg. curve by $\left(\frac{\text{Deflection, degrees}}{90\text{ deg.}}\right)^2$	
Hydrants, from street main to hose, with as many fire streams flowing as there are outlets on hydrant, valve wide open, ample-sized connection to main; loss 1 to 3 lb. per sq. in.	maximum 5 lb. per sq. in.
Water meters (subject to much variation):	
Disk, or "wobble-disk" type	3.4–10
Rotary type (star or cog-wheel-shaped disk as piston)	10
Reciprocating piston type (like a piston pump)	15
Turbine-wheel type (double flow, balanced)	5–7.5
Diaphragm and orifice meters (see Fig. 82*a*)	
Diaphragm of thin material across pipe, with concentric hole, or an inside projecting washer or gasket, or a "burr" due to cutting pipe with a wheel cutter (data from washers in joints of fire-hose):	
Diameter of hole = 0.91 × diameter of pipe	0.34
Diameter of hole = 0.80 × diameter of pipe	1.88
(Note that $\frac{v^2}{2g}$ in this particular case refers to pipe, not to the area of opening, nor to the contracted stream through the opening.)	
Nozzles: When the coefficient of velocity and discharge ranges from 0.99 to 0.975, h_F ranges from 0.02 to 0.05 × $\frac{v_2^2}{2g}$, where v_2 is the velocity of issuing jet, therefore	0.02–0.05
A particular rough nozzle, a 6- by 3-in. reducer used as a nozzle, having inside projecting flat rivet heads along a longitudinal joint, and also circumferentially at the tip, gave velocity coefficient = 0.91, or h_F = about $\frac{1}{6} \times \frac{v_2^2}{2g}$, where v_2 is the tip velocity	0.17
Venturi Meter Tubes: The loss of head occurs mostly in and downstreamwards from the *divergence*, the loss between upstream end and throat being only 0.03 to 0.06 × $\frac{v_2^2}{2g}$, where v_2 is the *throat velocity*. The total loss through meter is: For total angle of divergence = ±5 deg., $h_F = \frac{1}{7}$ to $\frac{1}{10} \times \frac{v_2^2}{2g}$; For total angle of divergence = ±15 deg., $h_F = \frac{1}{3}$ to $\frac{1}{6} \times \frac{v_2^2}{2g}$	

TABLE. VII.—(*Continued*)

Nature of the special resistance	Loss of head as decimal or multiple of $\frac{v^2}{2g}$
(The lower values go with large throats, and the larger values with small throats whose $d_2 = \frac{d}{3}$ to $\frac{d}{2}$, d being the diameter of the pipe.) *Sudden enlargement,* from a small pipe ending abruptly where a larger pipe begins, is found to involve loss of head fairly closely in accordance with *Borda's formula, viz.:* $h_F = \left(\frac{v_1 - v_2}{2g}\right)^2$. (*N. B*: This is not the difference of the velocity-heads, but is the head corresponding to the difference of the velocities.) The above-stated formula may be written $h_F = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{v_1^2}{2g}$, or, for a circular or square pipe, $h_F = \left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2 \times \frac{v_1^2}{2g}$ where v_1, A_1, and d_1 refer to the smaller upstream pipe (see Fig. 116*a*), while the subscript 2 refers to the larger downstream pipe (see Table VIII, p. 210).	

115. Derivation of Borda's formula for loss of head due to sudden enlargement of cross-section of stream, with consequent *impact* of fast-moving liquid against slower-moving liquid.

In Fig. 114 the water flowing from the smaller pipe into the larger pipe has its mean velocity reduced from v_1 at 1 to v_2 at 2.

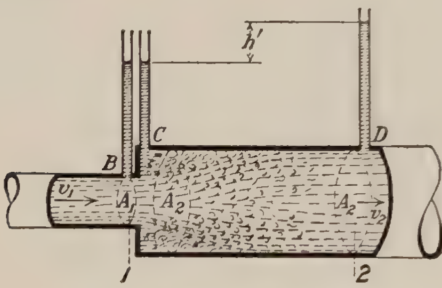


FIG. 114.—Sudden enlargement.

The indications of the open water columns at B, C, and D are as shown in the sketch, except that column C actually stands a little lower than B, not only because of the hydraulic gradient in the smaller pipe but also in part due to the abnormal conditions in the corner where C is connected, involving curvilinear motion of the water in the expanding stream and a viscous drag by the flowing water on the eddying mass in the corner, thus making for some degree of *suction*. Figure 114 shows columns

B and C separated, but they are to be thought of as each showing the pressure-head of the stream just as it leaves the small pipe. Although there is a gain in pressure-head between C and D due to the conversion of velocity-head, there is considerable loss in the process. The pressure at D is not as high as it would be under idealized conditions with no loss by impact (see actual hydraulic gradients, Fig. 115).

The Theory.—A mass of water has had its velocity reduced from v_1 to v_2 in a time t required to flow from 1 to 2. If the volume rate of flow is Q, the mass of water flowing between points 1 and 2 in time t is $\frac{Qwt}{g}$. In general, force = mass times acceleration, or $P = Ma$. In this case, the retarding force is the difference of the total pressures on the right and left circular cross-sections of the cylinder of water between 1 and 2. At present neglecting ordinary pipe friction, the excess head at 2 over the head at 1 is h' (Fig. 114), or the accelerating force is $(-h'w)A_2$. Then $P = Ma$ becomes

$$(-h'w)A_2 = \frac{Qwt}{g}\left(\frac{v_2 - v_1}{t}\right), \text{ or } h'A_2 = \frac{A_2 v_2}{g}(v_1 - v_2),$$

$$\text{or } h' = \frac{1}{2g}(2v_1 v_2 - 2v_2^2).$$

By Bernoulli's theorem:

(Total head at 1) $- h_{\text{Enl.}} =$ (total head at 2), or

$$h_1 + \frac{v_1^2}{2g} - h_{\text{Enl.}} = h_2 + \frac{v_2^2}{2g},$$

where h_1 and h_2 are the pressure-heads at 1 and 2.

But $(h_2 - h_1) = h'$, or $\frac{v_1^2}{2g} - h_{\text{Enl.}} = \frac{v_2^2}{2g} + h'$.

$$h_{\text{Enl.}} = \frac{v_1^2}{2g} - \frac{v_2^2}{2g} - \frac{(2v_1v_2 - 2v_2^2)}{2g} = \frac{v_1^2}{2g} - \frac{2v_1v_2}{2g} + \frac{v_2^2}{2g},$$

or

$$h_{\text{Enl.}} = \frac{(v_1 - v_2)^2}{2g} \quad \textit{(Borda's formula)}. \tag{65}$$

This may also be written $h_{\text{Enl.}} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{v_1^2}{2g}$, or $h_{\text{Enl.}} = \left(\frac{A_2}{A_1} - 1\right)^2 \frac{v_2^2}{2g}$, the first of which may be expressed,

$$h_{\text{Enl.}} = \left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2 \frac{v_1^2}{2g}. \tag{66}$$

HYDRAULIC GRADIENTS FOR PIPES WITH SUDDEN CONTRACTION AND SUDDEN ENLARGEMENT OF CROSS-SECTION.

These hydraulic gradients were observed on apparatus used in demonstration lectures at Cornell University.

The rates of flow for the three gradients shown were 0.1144, 0.0967 and 0.0762 cu. ft. per sec.

Smooth brass pipes with square abrupt contractions.

Diameters	Nominal	Actual
(1)	2½ inch	0.206 feet
(2)	1¼ inch	0.112 feet
(3)	2. inch	0.171 feet

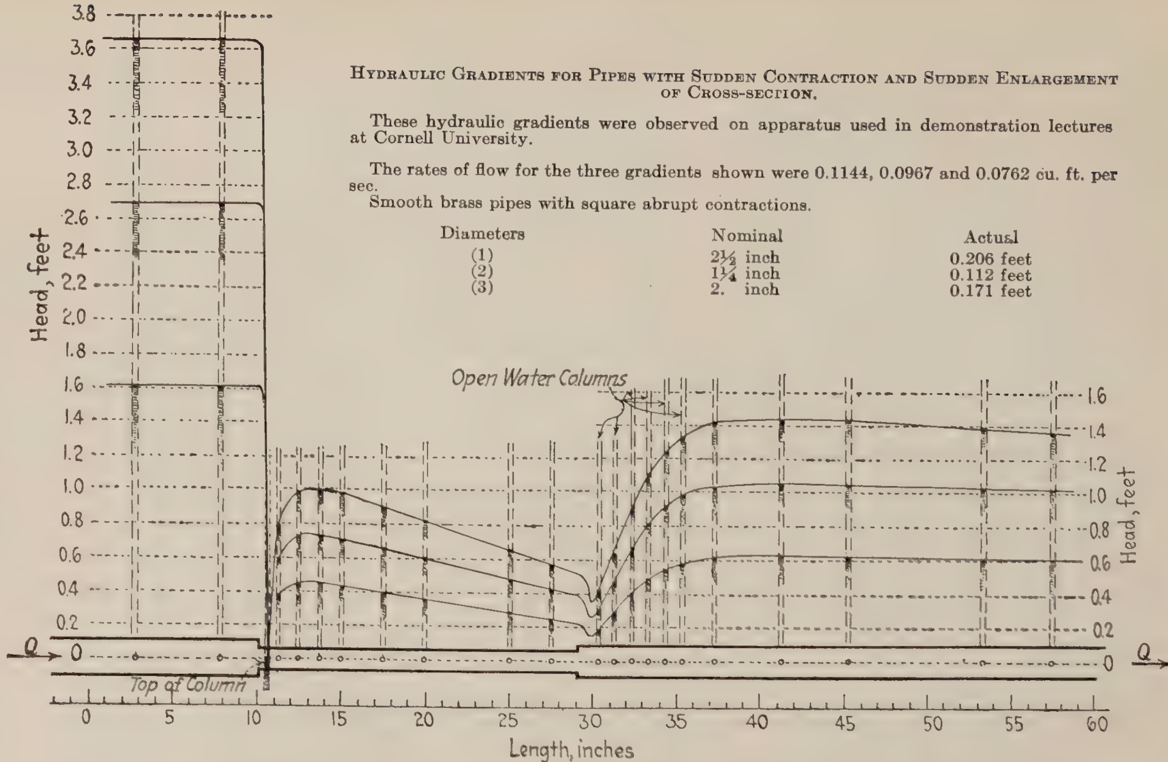


FIG. 115.

(See Table VIII.) Thus far the friction loss between 1 and 2 has been neglected. Even if the pipe were all of the larger size d_2 with no enlargement, there would be some loss at velocity v_2 due to plain pipe friction. In interpreting experimental data allowance for this must be made by the usual method for normal friction in the two pipes.

The theory underlying Borda's formula assumes the loss of energy to be due to direct inelastic impact of a fast-moving body of water against a slower-moving body of water (see Fig. 114).

For various ratios of d_1 to d_2, the following table shows the decimal fraction of the upstream velocity-head lost due to the sudden enlargement of cross-section, by Borda's formula.

Table VIII

(See Eq. (66))

Diameter ratio $\frac{d_1}{d_2} =$	0 90	0.80	$\frac{3}{4}$ 0.75	0.70	$\frac{2}{3}$ 0.67	0.60	$\frac{1}{2}$ 0.50	0.40	$\frac{1}{3}$ 0.33	0.30	$\frac{1}{4}$ 0.25	$\frac{1}{5}$ 0.20	0.15	$\frac{1}{10}$ 0.10	0
$\left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2 =$	0.036	0.13	0.19	0.26	0.31	0.41	0.56	0.71	0.79	0.83	0.88	0.92	0.955	0.98	1.00

T. J. Rodhouse found (see *Cornell Civil Engineer*, November, 1916), that the loss is 0.97 of values by Borda's formula for a 1- to 2-in. and for a 1.5- to 2-in. sudden enlargement in smooth brass pipes, showing an excellent verification.

With brass pipes of 1, 1.75, 2.5, and 3 in. in diameter, and values of v_1 ranging from 3.8 to 31.6 ft. per sec. and $\frac{A_1}{A_2}$ from 1.45 to 9.32, W. A. Archer (in *Trans. Am. Soc. Civil Eng.*, vol. 76, p. 1025, 1913; also *Eng. News*, Dec. 1, 1921, pp. 886–887) gives for sudden enlargement

$$h_L = \frac{1.098 v^{1.919}}{2g}\left(1 - \frac{A_1}{A_2}\right)^{1.919}.$$

This is not far different from Borda's ideal formula, the 10 per cent increase in the multiplier (above unity) tending to offset the 4 per cent decrease in the exponent.

Problems

1. A certain kind of pipe requires a length of 40 diameters in order to have the lost head equal to the velocity-head. Considering a total length L of this pipe equal to 360 diameters, show that the velocity of the issuing jet will be equal to $0.316\sqrt{2gh}$, where h is the difference in elevation between water in the reservoir and end of pipe. Entry loss may be neglected.

2. A new 8-in. cast-iron pipe 150 ft. long has its discharge end 9 ft. below the level of the water in the reservoir supplying it. The entry to the 8-in. pipe is made well rounded. Compute the discharge. If you neglect any items state your reasons for doing so. Compute by the $\frac{L}{d}$ method and check by the so-called "approximate" pipe-friction formula.

3. (*a*) Find the velocity and the discharge through a circular well-rounded orifice (making allowance for friction) 4 ft. in diameter under a head of 6 ft. (*b*) Same for same orifice but sharp edge instead of rounded. (*c*) Same for a 4-ft. (rough) pipe, square-edge entry, 200 ft. long. (*c*) Same for a 4-ft. (rough) pipe, square-edge entry, 10,000 ft. long. (*e*) Calling the discharge in (*a*) 100 per cent what are the percentage discharges in (*b*), (*c*), and (*d*)?

Ans. (*a*) 19.25 ft. per sec., 242 c.f.s.
(*b*) 19.25 ft. per sec., 150 c.f.s.
(*c*) 12.43 ft. per sec., 156 c.f.s.
(*d*) 2.66 ft. per sec., 33.4 c.f.s.
(*e*) 100 per cent, 62 per cent, 64.5 per cent, 13.8 per cent.

4. For a pipe 50 diameters long as in Fig. P.55, and with f equal to 0.025, how far below the water surface is the hydraulic gradient at A, five diameters length from E?

Ans. 5.90 ft. below surface.

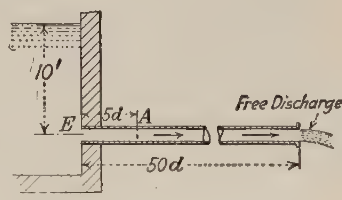


Fig. P.55.

5. Find size of rough square culvert, square-edged entry, 150 ft. long, axis of culvert 10 ft. under a railroad track, necessary to discharge 10 c.f.s. Water on one side 8 ft. below base of rail, and water on other side 5 ft. below base of rail. *Ans.* 1.25 ft.

6. A single circular culvert of a canal lock is 350 ft. long (see Appendix B) and has a beveled entry causing half as much loss of head as a square-edged entry. There are three 90-deg. elbows or short-turn bends, each of which causes a loss of 0.50 × velocity-head. The difference of levels is 6 ft., the exit being submerged. The friction is midway between that of "fairly smooth" and "rough" pipes. It is desired to have an initial rate of discharge of 400 c.f.s. Compute the diameter.

7. Between points 29.2 and 57.45 in. on the scale in Fig. 115, compute the excess loss of head over and above that which would be caused by normal friction in the pipe with a flow of 0.1144 c.f.s., the hydraulic gradient being as shown. Compare with Borda's formula loss.

8. Under the railroad fill is a circular culvert of corrugated iron with exterior casing of concrete. During construction, before the abutments are built, and while the ends of the pipe are projecting out beyond the fill, there is a flood during which the water levels on the two sides of the track stand as shown in the sketch (Fig. P.55*A*). Compute the discharge in c.f.s. if the entry loss for the projecting pipe is 0.8 times the velocity-head, and if

it requires (for this size and roughness of pipe and at this rate of flow) 45 diameters length to cause a loss of head equal to the velocity-head.

Ans. 143.8 c.f.s.

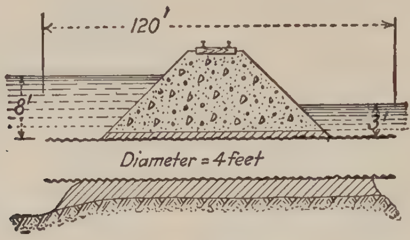


Fig. P.55*A*.

9. (*a*) The rough square culvert in Fig. P.55(*B*) is constructed under a railroad fill to carry off concentration of rainfall from the drainage area of 0.5 sq. mile above the culvert with the difference of levels as shown. If the rate of rainfall is 2 in. per hour compute the necessary size of the culvert, assuming square-edged entry and 1 in. per hour per acre equal to 1 c.f.s. One square mile = 640 acres. (Culvert is 100 ft. long.)

(*b*) If both ends of the culvert are flared as shown by the dashed lines in the sketch so that the sides of the entry and exit openings are each = $1.19d$, how much more (per cent) water will the culvert carry with the same difference of level?

Ans. (*a*) 5.25 ft.
(*b*) 30 per cent.

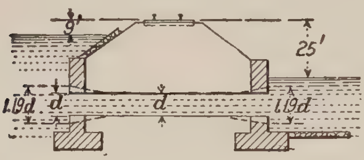


Fig. P.55*B*.

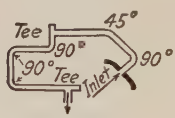


Fig. P.56.

10. At the Cornell Hydraulic Laboratory there is a 10-in. W. I. pipe line that has been in service over 15 years. It contains four 90-deg. curves ($r = d$), two tees, one 45-deg. curve ($r = d$), take loss = one-fourth that of 90-deg. curve), and a square-edged entry. The length is 110 ft. and the head is 75 ft. (see Fig. P.56). *Compute the discharge,* using Fig. 107 and data in Table VII. *Do not neglect any items that require or consume head.* (Actually the line is not all in one plane.) The curves may be regarded as fairly smooth, same inside diameter as pipe; the tees taken as common screw-type; and the pipe assumed in the "rough pipes" category. *Ans.* 11.66 c.f.s.

11. A 48-in. riveted steel pipe leading from Beebe Lake Dam to the laboratory is 240 ft. long, and feeds into a 30-in. pipe 40 ft. long, also of riveted steel. Lake water level is 9 ft. above canal level, discharge under water

(see Fig. P.57 for details). The loss in screen may be taken as $\frac{v^2}{2g}$. *Compute the discharge*, using table of coefficients. *Ans.* 74.8 c.f.s.

12. Compute the proper diameter for two spiral riveted steel culverts, laid side by side, 92 ft. long through a railroad embankment, if the discharge is 800 c.f.s. The head on the upstream end is 10 ft., the discharge taking place into the atmosphere (free discharge). Assume square-edged entry.

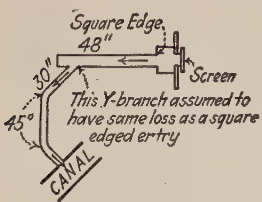


FIG. P.57.

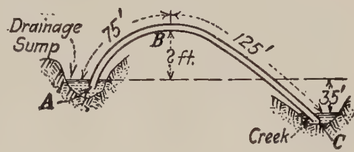


FIG. P.58.

13. Drainage siphon (Fig. P.58). All losses and transformations of head should be considered in the solution. Entry loss of head at A is one-half the velocity-head in the pipe. Length of each 3-in. siphon is 200 ft. (*a*) How many 3-in., wrought-iron pipes must the contracting engineer provide to drain 2 c.f.s. from the sump into the creek? Each of the siphon pipes arranged as shown. (*b*) What is the maximum permissible height of summit B above water level in sump A so that the water pressure at summit B during flow may not be less than 3 lb. per sq. in. absolute, the barometer reading 29 in.? (Actually a suction pump would be required at the summit to keep the syphon working continuously. Why?) *Ans.* (*a*) 4.

(*b*) 11.4 ft.

CHAPTER XIII

EXACT FORMULAS AND DIAGRAMS FOR FLOW OF WATER IN PIPES

116. Exact Formulas.—The fact that the value of f *changes* with the velocity and also with the diameter (see Fig. 107) indicates that the formula $h_F = f\frac{L}{d}\frac{v^2}{2g}$ does not have the proper exponents for v and d, namely, 2 and -1 (if d be regarded as a multiplier), but that both these values should be somewhat smaller, because f *decreases* as d and v *increase.*

Experiments on any one particular pipe (see Fig. 104) show that

$$h_F = mv^n. \tag{67}$$

This is most readily evidenced by plotting the experimentally found values of h_F and v on logarithmic paper.[1] The value of the exponent n depends on the degree of roughness of the interior of the pipe. For the very smoothest and most regular surfaces like drawn brass and glass, n = about 1.75, and it increases for rougher surfaces until, for those regarded in hydraulics as very rough, n = about 2.00. (These statements assume "turbulent" flow, not viscous flow. See footnote, p. 178.)

For any one category of roughness (see below Fig. 107), it is found that $m = \frac{K}{d^p}$, found by plotting values of m with the corresponding diameter d on logarithmic paper (Fig. 116). In this formula the exponent p has a value somewhat *greater* than 1.00. Substituting this found value of m in the formula $h_F = mv^n$, then

$$h_F = K\frac{v^n}{d^p}, \tag{68}$$

as the general form of the exact, or so-called "exponential," form of the equation for uniform flow of fluids in pipes, each degree of roughness having a different set of values for K and n.

[1] See Fig. 119B, p. 228.

As to the exact value of the exponent p of the diameter, authorities are not entirely in agreement as to its constancy or as to the law of its small variation, if any. This is chiefly because there is no accepted standard of roughness except for the very smoothest pipes (which are not common in practice).

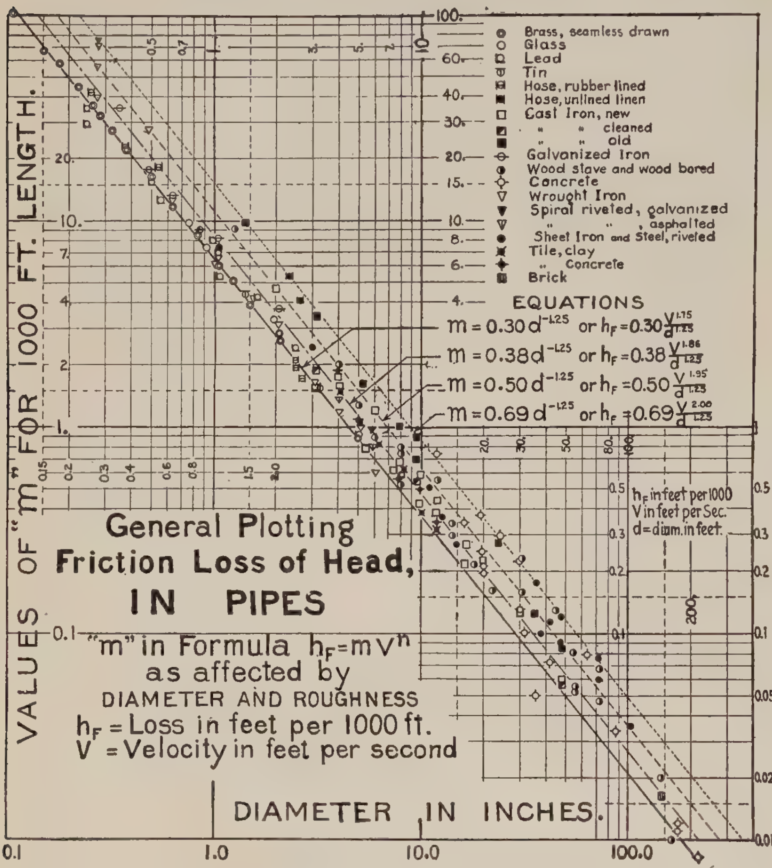


Fig. 116.—(See par. 116 and Appendix E.)

The value 1.25 for p, the exponent of d, is correct for the smoothest pipes,[1] and the preponderance of evidence indicates that this value 1.25 may be used for the rougher pipes also.[2]

[1] See *Trans. Am. Soc. Civil Eng.*, vol. 51, p. 306, 1903; or *Cornell Civil Engineer*, May, 1910. Experiments by others check all values in Eq. (69).

[2] The five formulas stated in par. 119 (when expressed like the first forms on page 216) show exponents of d ranging from 1.17 to 1.33, averaging 1.25, same as given by the lines drawn on Fig. 116.

Figure 116 is a comprehensive plotting of the whole field of friction in pipes due to turbulent flow of water. The diameters range from 0.1 to 216 in. (18 ft.), and the materials of which the pipes are made include practically all types of construction. The data and references are tabulated in Appendix E. To avoid confusion of points some of the tabulated data are not plotted.

The following four equations (written in several forms) cover fairly well the range from the very smoothest to the roughest pipes. Pipes abnormally deteriorated with heavy interior incrustations encroaching considerably on the cross-sectional area are excluded from consideration.

For detailed descriptions of the kinds of pipes in these several categories of roughness, see Fig. 107.

For extremely smooth pipes[1] (**per 1000 ft.**),

$$h_F = 0.30\frac{v^{1.75}}{d^{1.25}};\quad v = 1.99\,d^{0.71}\,h_F{}^{0.57};\quad v = 103\,d^{0.71}\,s^{0.57};$$
$$v = 277R^{0.71}\,s^{0.57}. \qquad (69)$$

For fairly smooth pipes,

$$h_F = 0.38\frac{v^{1.86}}{d^{1.25}};\quad v = 1.68\,d^{0.67}\,h_F{}^{0.54};\quad v = 69\,d^{0.67}\,s^{0.54};$$
$$v = 175R^{0.67}\,s^{0.54}. \qquad (70)$$

For rough pipes,

$$h_F = 0.50\frac{v^{1.95}}{d^{1.25}};\quad v = 1.43\,d^{0.64}\,h_F{}^{0.51};\quad v = 49\,d^{0.64}\,s^{0.51};$$
$$v = 120\,R^{0.64}\,s^{0.51}. \qquad (71)$$

For extremely rough pipes,

$$h = 0.69\frac{v^{2}}{d^{1.25}};\quad v = 1.20\,d^{0.62}\,h_F{}^{0.50};\quad v = 38\,d^{0.62}\,s^{0.50};$$
$$v = 91\,R^{0.62}\,s^{0.50}. \qquad (72)$$

In these equations h_F is in feet per 1000 ft., so that strictly the first equation should be $h_F = \frac{0.00030Lv^{1.75}}{d^{1.25}}$. The foot-second system of units is used for d and v. In the third and fourth forms, s is the actual hydraulic slope; for example, if h_F is 1.5 ft. per 1000 ft., $s = 0.0015$. In the fourth form of the equations, R is the *hydraulic radius*,[2] or *hydraulic mean depth*, in feet $\left(= \frac{d}{4}\text{ for a circular pipe or square pipe flowing full}\right)$.

[1] For temperature effect in extremely smooth pipes with *turbulent* flow, see par. 122*a*, p. 227.

[2] See pp. 181 and 255.

The fourth form of the equations may be compared with the Chezy formula, $v = C\sqrt{Rs}$, or $v = CR^{0.50}s^{0.50}$, for which C has values as shown on Fig. 107.

117. Comparison of Exact and Approximate Formulas.—Obviously, it makes no difference in the final result whether the approximate or the exact type of formula is used, provided the coefficients used correspond in each case to the experimentally found facts. Such a formula as h_F (per 1000 ft.) $= 0.38\,\frac{v^{1.86}}{d^{1.25}}$ is complete in itself, and, for the particular degree of roughness here called "fairly smooth," no table of coefficients is necessary. But, if it is desired (for certain conveniences of calculation) to involve the expression for the velocity-head $\frac{v^2}{2g}$ and to let $h_F = f\,\frac{L}{d}\,\frac{v^2}{2g}$, then $0.00038L\,\frac{v^{1.86}}{d^{1.25}} = f\,\frac{L}{d}\,\frac{v^2}{2g}$, whence $f = \frac{0.0245}{d^{0.25}v^{0.14}}$. The diagram (Fig. 107), giving f values is made up on this basis, so that in any event the equivalent of the exact formulas with non-even decimal exponents is really being used.

Problem: Express the values for f in terms of d and v for the first, third, and fourth equations on page 216, as it is done above for the second equation.

Also make the calculations necessary to check the last three forms of each of the four equations, both by using a table of logarithms and with the ordinary slide rule.

118. Pipe Diagrams.—For the ready solution of these so-called "exponential formulas" there are presented the logarithmic diagrams of Figs. 117*A*, 117*B*, and 118.

Any pipe problem may be solved by using either (1) the approximate type formula with its accompanying table or diagram of f values, Fig. 107, p. 182; or (2) the appropriate formula of the exact type, making use of diagrams giving a rapid graphical solution.

Choice between Use of Formula and Diagram.—If the problem involves the velocity-head of the issuing stream as an item too large to be neglected, or if a non-negligible velocity-head item comes into the problem elsewhere than at the exit point, then it is a question more or less of taste as to which formula is most

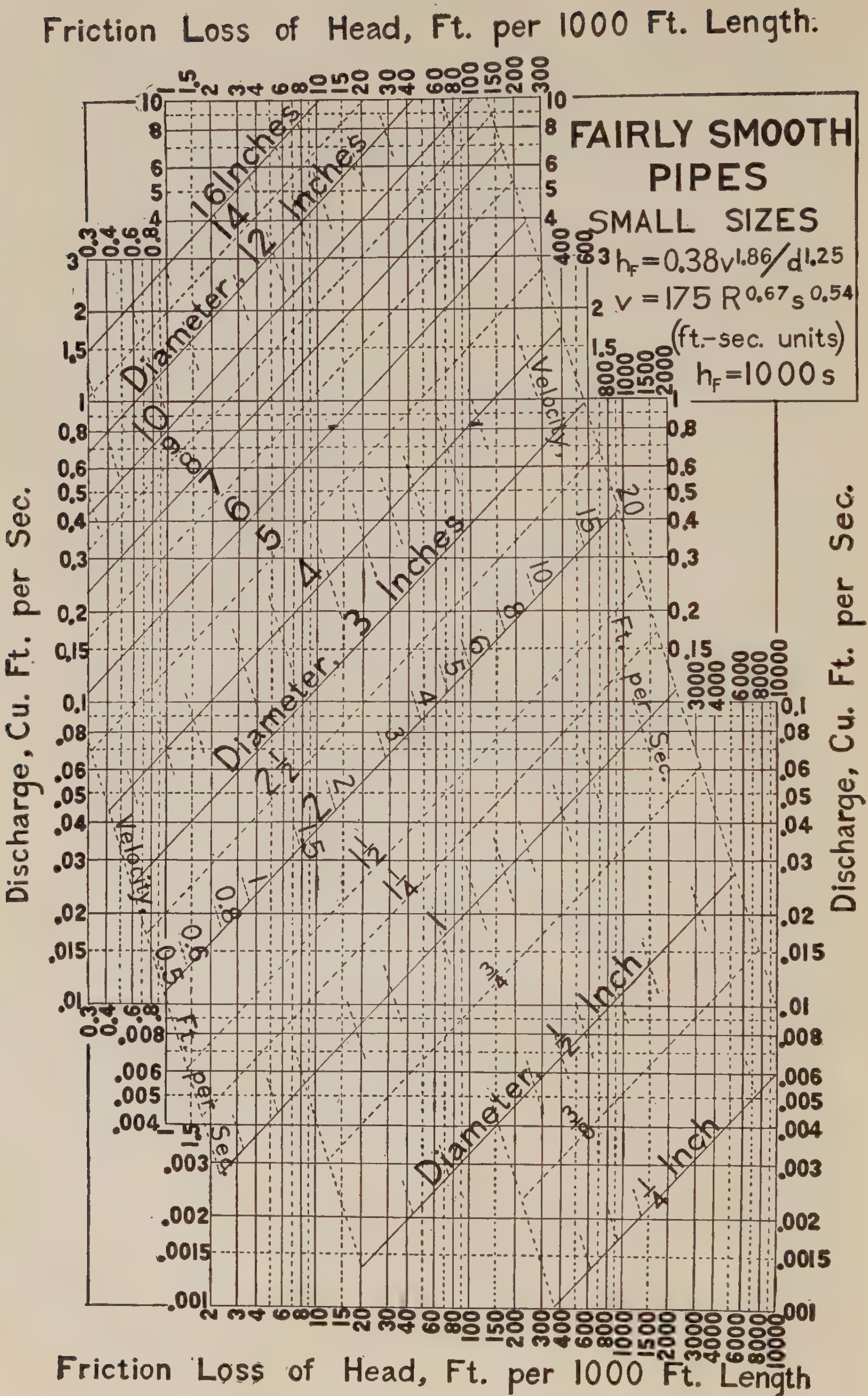


FIG. 117A.—Fairly smooth pipes (small sizes).

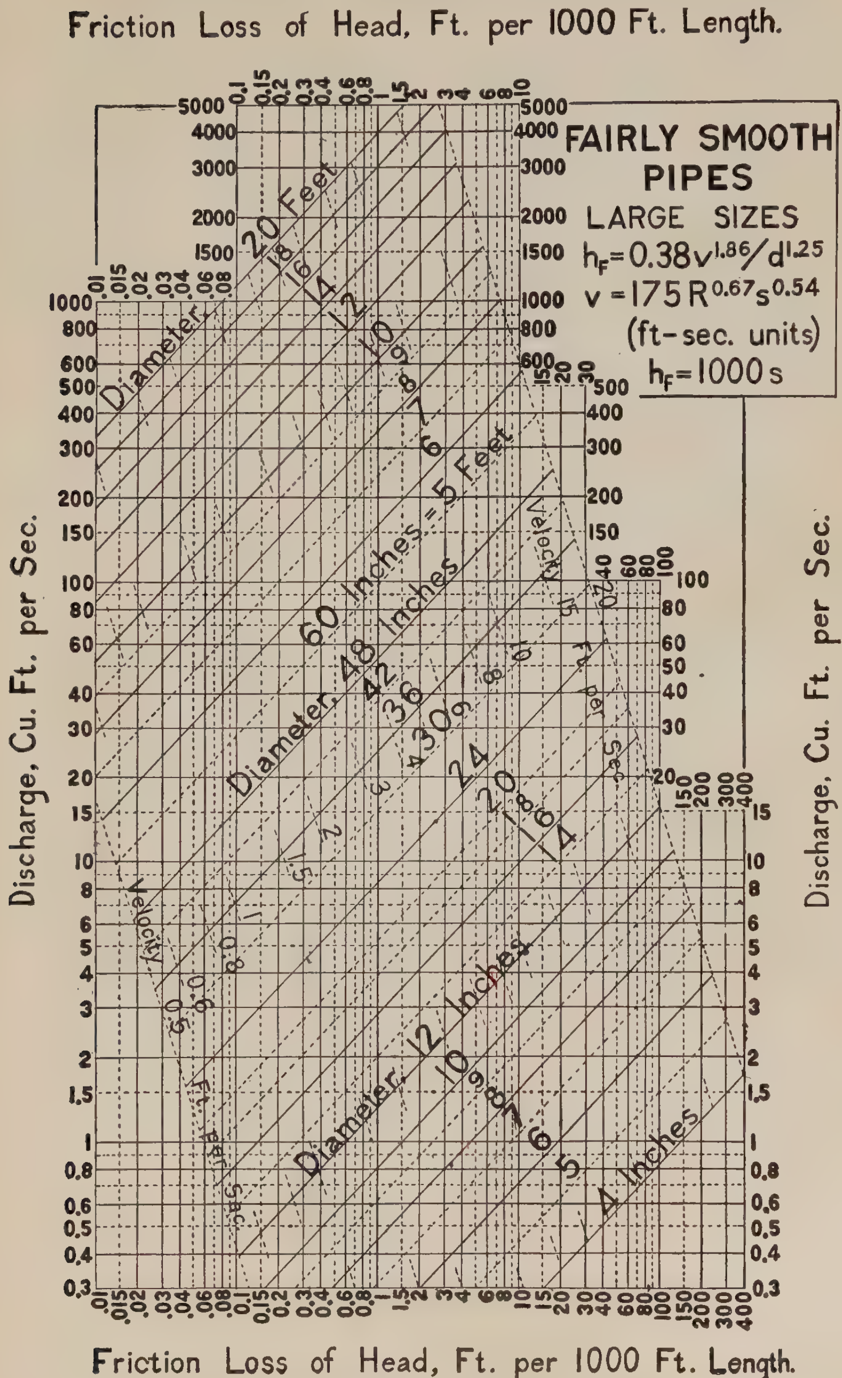


FIG. 117*B*.—Fairly smooth pipes (large sizes).

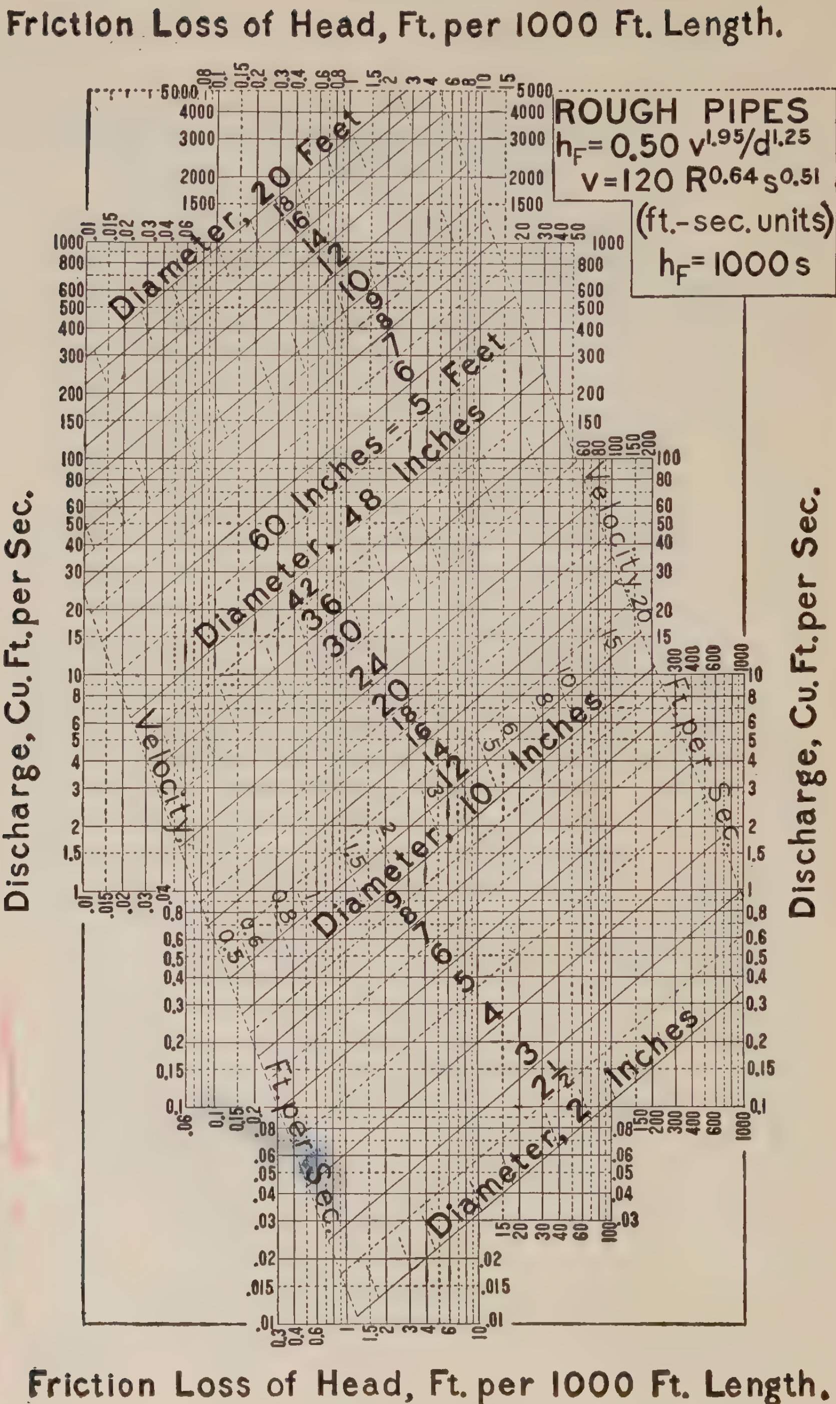


FIG. 118.—Rough pipes.

convenient. If the approximate type of formula is used, $h_F = f\frac{L}{d}\frac{v^2}{2g}$, which already contains the velocity-head expression, the only inconvenience comes in looking up or approximating the value of f.

If the diagrams are used to solve the relation between h_F, d, and Q, there still remains the problem of handling the $\frac{v^2}{2g}$ item by trial or otherwise, since the diagrams (Figs. 117*A*, 117*B*, and 118) handle only that part of the total head which is used up in friction. On page 223 are examples of the use of pipe flow diagrams.

119. Other "Exponential Formulas."—(Only a few representative formulas are given.) From experiments made in 1869–1871, Prof. C. J. H. Lampe suggested[1] the formula $v = 203.3R^{0.694}s^{0.555}$. This is seen to be between the text categories of "extremely smooth" and "fairly smooth," which, indeed, is true of most fairly new cast-iron and wrought-iron pipes.

For woodstave pipes, Fred S. Scobey[2] gives $v = 1.62d^{0.65}h_F{}^{0.555}$. This is very close to fairly smooth pipes category. In this, h_F means loss of head per 1000 ft.

For flow in tile drains, based on elaborate tests on 4- to 12-in. clay and concrete pipes,[3] $v = 138R^{2/3}s^{1/2}$. This is seen to be between the fairly smooth and rough categories, and closer to the latter.

The so-called **Manning formula** (primarily intended for open channels), suggested as a near-equivalent to the Kutter formula (see p. 261),[4] is $v = \frac{1.486}{n}R^{2/3}s^{1/2}$, where n is practically the same as Kutter's n (see Table XI).

If $n = 0.009$, the multiplier of Manning's formula becomes 165, close to text's *fairly smooth;* for $n = 0.012$, the equivalent

[1] See "Hydraulics," by HAMILTON SMITH, pp. 231, 272; also *Der Civilingenieur*, vol. 19, 1873.

[2] *U. S. Dept. Agr., Bull.* 376, 1916.

[3] *U. S. Dept. of Agr., Bull.* 854; and *Eng. News-Record*, p. 902, May 26, 1921.

[4] *Trans. Inst. Civil Eng.* of Ireland, vol. 20, 1890.

is 124, a value close to *rough;* for $n = 0.017$, the equivalent is 87.5, or close to *extremely rough.*

The Hazen-Williams formula[1] is $v = 1.32C_1R^{0.63}s^{0.54}$, where C_1 has values ranging from about 100 for old riveted pipes to 130 for new cast-iron pipe. These latter values agree with the text's *fairly smooth* category.

These comparisons will serve to assist the engineer's judgment in interpreting the roughness categories, and to check statements on page 182.

120. Notes on the Use of Pipe-flow Diagrams.—Those interested in the method of constructing the diagrams (Figs. 117*A*, 117*B*, and 118) are referred to the *Engineering Record,* Sept. 3, 1904, and to the *Cornell Civil Engineer*, May, 1910.

Limitations.—In "long-pipe" problems it is commonly assumed that all of the head (or rather all of the fall in the hydraulic gradient) is used up in overcoming friction; but if this is not practically permissible, *e.g.*, as in example 3, page 201, or in pipe and nozzle problems, it must be remembered that a *friction-head diagram does not provide for other items that consume part of the total head.* Separate or supplementary calculations or allowances must be made to cover losses of head or conversions of head that occur in a problem in addition to the plain pipe *friction-head,* the h_F of Fig. 104, or the $f\frac{L}{d}\frac{v^2}{2g}$ of examples 1 to 4 (pp. 197 to 204). (See pipe with nozzle, p. 235.)

An uncertainty in general easily as great as from 5 to 10 per cent *in the friction-head* for a fixed velocity and discharge *is to be expected* for commercial pipes. Or, if the loss of head is fixed, the percentage uncertainty in velocity and discharge for a pipe of a certain nominal or specified diameter may easily be one-half these values, and probably in most cases it may be fully 5 to 10 per cent (corresponding to errors of about 10 to 20 per cent in the friction-head).

Consequently, if the requirements for friction-head are some ten or more times as great as the *sum* of the velocity-head plus minor losses, the latter items may usually be neglected and the pipe flow diagrams may then be used as though the *total head*

[1] "Hydraulic Tables," J. Wiley & Sons.

and the *friction-head* were identical (see comments on examples 1 and 3, pp. 197 to 202). Thus the Q for example 2 could be picked from Fig. 118 directly, after noting that h_F (per 1000 ft.) $= \dfrac{20}{1.5} = 13.3$ ft.

The *paradox* involved in neglecting the *velocity-head* and yet not calling the velocity itself zero is, of course, explained by the fact that the expression for the friction-head itself involves the velocity term. It is not stated that, for long pipes, the velocity-head is zero $\left(\dfrac{v^2}{2g} = 0\right)$, but merely that 1 (or 1.5 or so) $\times \dfrac{v^2}{2g}$ is negligible in its effect in comparison with $f\dfrac{L}{d} \times \dfrac{v^2}{2g}$ or with some other expression for the friction-head, as already discussed on page 199.

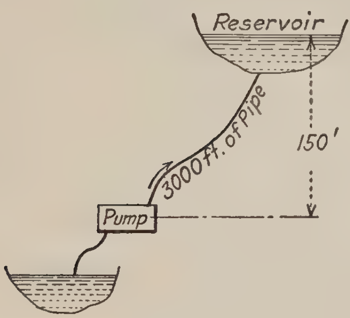


Fig. 119.

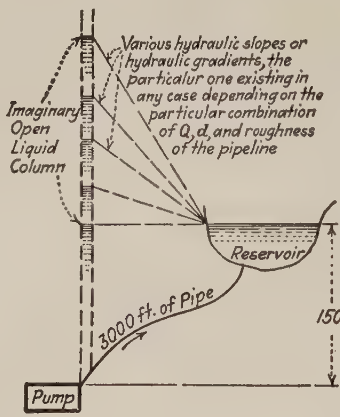


Fig. 119A.

Example 5: It is necessary to pump 1 million g.p.d. to an elevation 150 ft. above the pumps through a pipe line 3000 ft. long. What size of pipe is best (Fig. 119)? This is a very common type of problem confronting the engineer.

General Analysis.—The head in the pipe at the pump (with pipe line full of water but with no flow) is 150 ft. As soon as the pump starts to force water into the reservoir the head (pressure-head) at the pumps is greater than 150 ft.; or, in the phraseology of hydraulics, the *hydraulic gradient must slope*

downwards from the pump to the reservoir. Water may be pumped so as to flow uphill through a pipe, but the *slope* (of the *hydraulic grade line*) is always downwards in the direction of the flow, due to the loss of head by friction (Fig. 119*A*). To force more water through a given pipe requires more head and a steeper hydraulic slope; as also to force a given Q through a smaller pipe, or through a rougher pipe of the same diameter.

The *total* head pumped against is equal to 150 ft. *plus* the suction lift *plus* all losses in the suction-pipe, in the pump, and in the discharge pipe. In this problem the losses in the discharge pipe 3000 ft. long are being investigated.

Solution: On each of the pipe diagrams (Figs. 117*A*, 117*B*, and 118) it appears that there are a number of diameters satisfying each value of the discharge. But the friction-head increases rapidly as the diameter of the pipe becomes smaller.

In order to have a comprehensive view of the situation, Table IX is prepared by starting with the h_F values obtained from the diagrams for the given Q = 1 million g.p.d. = 1.55 c.f.s. (The student should check these tabulated values so as to acquire familiarity with the diagrams.)

Table IX

	Fairly smooth pipes			Rough pipes		
Diameter of pipe, inches	Diagram value for h_F, feet per 1000 ft.	Friction-head, feet in 3000 ft.	Total head, pumped against, feet	Diagram value for h_F, feet per 1000 ft.	Friction-head, feet in 3000 ft.	Total head pumped against, feet
4	330	990	1140	550	1650	1800
5	110	330	480	175	525	675
6	43	129	279	68	204	354
8	10.5	31.5	181.5	15.5	46.5	196.5
10	3.5	10.5	160.5	4.7	14.1	164.1
12	1.4	4.2	154.2	1.9	5.7	155.7
16	0.32	0.96	151.0	0.43	1.29	151.3
20	0.11	0.33	150.3	0.13	0.39	150.4
24	0.044	0.13	150.1	0.055	0.16	150.2

These tabulated values give a bird's-eye view of the problem before the engineer in selecting the proper size of pipe. Several important facts appear clearly, *viz.:*

1. The friction-head in all sizes of pipes is some 50 per cent greater for the *rough* pipes than for the so-called *fairly smooth* pipes. (*Vice versa,* it would be found that the rough pipes discharge some 25 per cent less than the fairly smooth pipes for the same head.) The student should remember that these particular categories (of degree of roughness) are somewhat arbitrarily chosen, and that there is no reason to prevent selection of an intermediate category if personal experience justifies.

2. The pressure-head in the pipe at the pump, and against which the pump must force the water (static head + friction-head) is very high for the small diameters, but approaches within a few per cent to the difference of elevation, here 150 ft. (the lower limit in this problem), at a moderate diameter, say 10 or 12 in. Consequently, there is no practical object in adopting a still larger diameter in order to relieve the pump of work, since the 150-ft. elevation exists in any event.

At a moderate diameter, moreover, the *difference* between the *total pressure-heads* for the smooth and the rough pipes (which difference for the small diameters is actually greater than the 150 ft.-lift) also becomes relatively very small. Consequently, if it is desired to provide for effects of age on the roughness of the pipes the smallest sizes should be avoided on this account also.

3. Since horsepower $= \frac{Qwh}{550} \times$ efficiency, and since it requires 1 hp. extra at the pump for each 1 c.f.s. forced against each 6 to 7 ft. extra pressure-head (at 68 to 80 per cent efficiency of pump), the smallest sizes of pipes are ruled out of consideration because of the large cost of operation in pumping water through them at the required rate.

On the other hand, the larger sizes are ruled out because the lost interest on the excess first cost over the cost for moderate sizes becomes greater than the saving in cost of pumping.

In view of all these facts, a 10- or 12-in. pipe is the proper size. The final selection of size depends on estimated increase in future

demand, cost of pumping, and probable rate of deterioration of pipe.

121. Laws of Variation, Flow of Water in Long Pipes, also Percentage Effects Due to Small Changes in Any One Factor.—In the formula, $h_F = f\frac{L}{d}\frac{v^2}{2g}$, substituting for v its equivalent $\frac{Q}{A} = \frac{Q}{\left(\frac{\pi}{4}\right)d^2}$, for a circular pipe, $h_F = \left(\frac{f}{\left(\frac{\pi}{4}\right)^2 2g}\right)\frac{LQ^2}{d^5}$. Hence, if f remains practically constant (as it does for a moderate range in values of d and v (see Fig. 107), it may be stated that h_F *varies as* $\frac{LQ^2}{d^5}$, and from this law it is seen that the

Head h_F varies as L if Q and d stay the same.
Head h_F varies as Q^2 if L and d stay the same.
Head h_F varies inversely as d^5 if Q and L stay the same.
Discharge Q varies inversely as $\sqrt{L}$ if h_F and d stay the same.
Discharge Q varies directly as $\sqrt{h_F}$ if L and d stay the same.
Discharge Q varies directly as $d^{5/2}$ if L and h_F stay the same.
Diameter d varies inversely as $h_F^{1/5}$ if L and Q stay the same.
Diameter d varies directly as $L^{1/5}$ if h_F and Q stay the same.
Diameter d varies directly as $Q^{2/5}$ if h_F and L stay the same.

Problem: Compare the corresponding *Orifice Flow Laws*, as far as they are comparable, with the above-stated laws, and work out a few illustrative numerical examples such as the following:

To triple the discharge of an orifice, how much must its area be increased? To triple the discharge of a long pipe, how much must its area be increased? If a long pipe is replaced by one with 50 per cent greater diameter, how much is the discharge increased; and what is the increase for a similar enlargement of an orifice? *Etc.*

If the more exact types of exponential formulas are used, *viz.:* $h_F = 0.00038, L\frac{v^{1.86}}{d^{1.25}}$ (fairly smooth pipes), and $h_F = 0.00050, L\frac{v^{1.95}}{d^{1.25}}$ (rough pipes), the above-stated laws of variation would be changed only slightly, *viz.:*

For fairly smooth pipes, h_F varies as $\frac{LQ^{1.86}}{d^{4.97}}$, and for rough pipes, h_F varies as $\frac{LQ^{1.95}}{d^{5.15}}$, each of which is close to the approximate $\frac{LQ^2}{d^5}$.

122. Percentage effects due to small changes, uncertainties, or errors in any one factor may now be examined, by the principles stated on page 170, in accord with the above-stated laws of variation.

For a pipe line where friction-head and total head may be regarded as practically the same, or for any case where plain pipe friction only is under consideration, it may be stated, if the other factors remain the same, that:

A small percentage change in the available head causes one-half that percentage change in the discharge Q, and corresponds to one-fifth that percentage change in opposite way in the diameter d. *A small percentage change in the diameter* d corresponds to 2.5 times that percentage change in Q, or an opposite change of five times the percentage change in h_F. *A small percentage change in the discharge* Q corresponds to two times that percentage change in h_F, or two-fifths that percentage change in d. *A small percentage change in the length* L corresponds to one-fifth the percentage change in d, or to an opposite change of one-half the percentage change in Q.

The *considerable effect of* small variations in *diameter* is thus particularly emphasized. To illustrate, if the diameters of two nominal 6-in. pipe lines are actually 5⅞ and 6⅛ in., a difference of 4 per cent, the discharge of the larger pipe under the same head (or, perhaps, pump pressure) will be 10 *per cent more*, and the friction-head with the same discharge flowing will be 20 *per cent less* than for the smaller pipe, due solely to this small variation in diameter.

If the smaller diameter is due, in whole or part, to rusting, tuberculation or incrustation, or any form of lime or slime deposits (these all being very common happenings to pipes in service), then the increased roughness will still further decrease the discharge for a fixed head.

Problem: Work out the corresponding laws for errors in estimating the value of the friction factor f.

122a. Effect of Temperature.—Experiments on very smooth seamless-drawn brass pipes from 1/10 to 5 in. in diameter show that a 10°F. rise in the temperature of the water decreases the loss of head about 4 per cent *for turbulent flow*. The formula given

for extremely smooth pipes is for a temperature of 55°F. On rougher pipes, such as galvanized iron, there seems to be very little effect of temperature.

Figure 119*B* shows the results of tests on water in smooth brass pipes both above and below the critical velocity (see lines 3–5, p. 278).

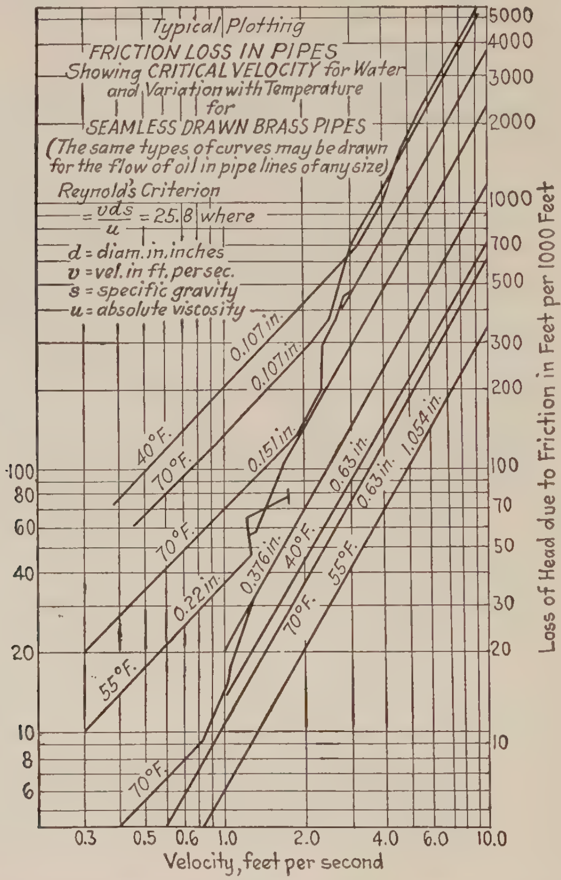


FIG. 119*B*.—(*From Trans. A.S.C.E., Plate* X, *Vol.* LI, 1903. SAPH and SCHODER, *Flow of Water in Pipes.*)

Problems

1. Using assumed values, verify by pipe diagrams the law that discharge varies as the five-halves power of the diameter where h_F is constant. Show agreement. State the law showing how loss of head varies with the diameter for same Q. Verify it by pipe diagrams. Show the agreement.

2. The Manning formula expresses the velocity for steady flow in terms of the hydraulic radius, the slope, and a constant n as follows: $v = \frac{1.486}{n} R^{2/3} s^{1/2}$. Transform this equation to the loss of head form involving loss of head per 1000 ft., velocity, and diameter. If n for a particular condition is 0.010, what is the category of "roughness" into which this pipe would fall?

3. The loss of head per 1000-ft. length of fairly smooth pipe is given by the formula, $h_F = \frac{0.38\, v^{1.86}}{d^{1.25}}$. (*a*) For a given fairly smooth pipe line, what would be the percentage error in the calculated discharge if an error of plus or minus 8 per cent is made in the determination of h_F?

4. The computed Q for a *nominal* 12-in. pipe was determined to be 2.68 c.f.s. The pipe was, however, found to have an actual diameter of 11.52 instead of the nominal 12 in. (*a*) Find, by the laws of variation (not diagrams) the actual quantity which will flow. (*b*) What head in terms of original h would be required to force the Q of 2.68 c.f.s. through the actual smaller pipe?

5. In a pumping problem the line AB is 828 ft. long and the lift to a reservoir is 60 ft. The cost of pipe line is limited to $2500. The cost of a pipe line is given by the formula $c = 20 + 2ad^{1.55}$, in which c is the cost in cents per foot of length, a is the cost of iron in cents per pound, and d is the diameter of the pipe in inches. Use the following data: cost of iron (cast into pipes), $60 per ton; amount of water pumped, 4 c.f.s.; efficiency of pumps, 80 per cent. (*a*) Draw hydraulic grade line and compute the pressure at the pumps in pounds per square inch. (*b*) Compute the horsepower required of the engines running the pumps. (*c*) Compute (*a*) and (*b*) for conditions after 15 years in service.

6. If $h_F = 0.38 \frac{v^{1.86}}{d^{1.25}}$ (foot-pound-second system) properly expresses conditions accompanying flow in fairly smooth pipes: (*a*) Derive the law of variation connecting h_F, L, Q, and d. (*b*) If, in the comparison of two cases, the total friction-head (loss by friction) in the second case is exactly twice as great as in the first case, the second length is exactly five times the first, and the second Q is exactly three times the first, what is the ratio between the second diameter and the first $\left(\frac{d_2}{d_1} = ?\right)$? Work this out by formula found in (*a*). (*c*) Taking two specific cases and using the proper pipe-flow diagram, check the results obtained in (*b*). Assume for first case, total $h_F = 50$ ft., $L = 2000$ ft., $Q = 5$ c.f.s. *Ans.* (*b*) 1.81.

CHAPTER XIV

EQUIVALENT, COMPOUND, LOOPING, AND BRANCHING PIPES

123. Equivalent Pipes.—Occasions often arise in engineering where correct answers are wanted to such questions as, for example: "How large a single pipe is required to discharge as much water as three parallel 8-in. pipes, two 12-in. pipes, a 20-in. and a 16-in., etc."; or *vice versa*. It is understood that the distance is the same, and that the same head or hydraulic slope prevails.

The approximate relations stated on page 226 make it possible to compute the desired results, but the pipe-flow diagrams give the information more simply and correctly.

As an example, a series of values such as shown in Table X is readily obtainable from either the rough or fairly smooth pipe diagram by first finding the discharges for various diameters for h_F = 1.0 ft. per 1000 ft., and then working out the ratios stated in the body of the table. Any other value of h_F will give the same ratios. (The ratios obtained from the two diagrams differ slightly.)

The student by actual trial should verify these last statements as well as the values given in Table X. Although the area ratios are shown in the table only for the 48-in. pipe, the student may readily write in values for any other of the sizes shown. The table may also readily be extended to cover additional sizes.

Table X makes it clear that **discharges of different pipes do not vary as the area of cross-section** (which is the law for orifices). For example, the table shows that a 48-in. pipe is equivalent to eighteen 16-in. pipes as against an area ratio 9. As another example, it requires *six* 6-in. plus *two* 2½-in. pipes to give the same discharge as *one* 12-in. pipe, where the ratio of areas would call for only *four* 6-in. pipes. *One* 36-*in. pipe* is seen to be *equivalent to nearly three* 24-*in. pipes* in discharging capacity.

TABLE X.—ILLUSTRATIVE EQUIVALENT PIPE RATIOS
(See Page 230)

Diameter of pipe	Q, c.f.s., for h_F = 1 ft. per 1000 ft.	Number of pipes, of diameter stated in left column, necessary to give same discharge as single pipes of the following diameters. (Same distance, same hydraulic slope.)											
		Area ratio	48 in.	36 in.	30 in.	24 in.	20 in.	16 in.	12 in.	10 in.	8 in.	6 in.	4 in.
48	43.6	(1)	1										
36	20.4	(1.78)	2.14	1									
30	12.6	(2.56)	3.46	1.62	1								
24	6.99	(4.0)	6.24	2.92	1.80	1							
20	4.32	(5.76)	10.1	4.72	2.92	1.62	1						
16	2.40	(9.0)	18.2	8.50	5.26	2.92	1.80	1					
12	1.121	(16.0)	38.9	18.2	11.2	6.24	3.85	2.14	1				
10	0.692	(23.0)	63.0	29.5	18.2	10.1	6.24	3.46	1.62	1			
8	0.384	(36.0)	113	53.1	32.8	18.2	11.2	6.24	2.92	1.80	1		
6	0.180	(64.0)	241	113	70.0	38.9	24.0	13.3	6.24	3.85	2.14	1	
4	0.0616	(144)	708	331	205	113	70.1	38.9	18.2	11.2	6.24	2.92	1

On page 226 it has already been shown that by the approximate formula Q varies as $d^{5/2}$, and not as d^2. Using the exact exponential formulas, Q varies as $d^{2.67}$ and as $d^{2.64}$ for the fairly smooth and the rough pipes, respectively. The table above is in agreement with the $d^{2.64}$ variation.[1]

124. Compound Pipes.—Problems involving a pipe line compounded of different sizes *in series*, the same discharge flowing through each (Fig. 120), may be solved either by the algebraic method indicated on page 203, or by using the pipe diagrams, as shown in the following solutions of a typical problem:

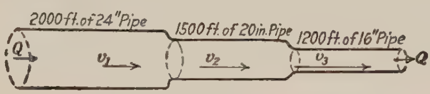


FIG. 120.

Example 6: What is the rate of discharge Q through the fairly smooth compound pipe shown in Fig. 120 for a total loss of head h_F = 20 ft.? Minor head items may be neglected.

A. Solution by Diagram by Reduction of the Compound System to an Equivalent Length of a Pipe Line of Any Desired Single Diameter.—(The hydraulic gradients involved are shown in Fig. 120*A*.) Assume any Q at random, provided only that on the diagram the line intersects the lines for the diameters of the problem, say a value 3 c.f.s. For this Q the fairly smooth dia-

[1] Values in Table X are stated more precisely than obtainable directly from diagram, Fig. 118, p. 220, in order to permit checking by slide-rule calculation from the "Rough Pipes" formula, and to show internal consistency for equal ratios between any two diameters under comparison.

gram (Fig. 117*B*) shows the values of h_F in the second column of the following tabulation, where the further computations of the reduction (here to 16 in.) are also shown in the third and fourth columns.

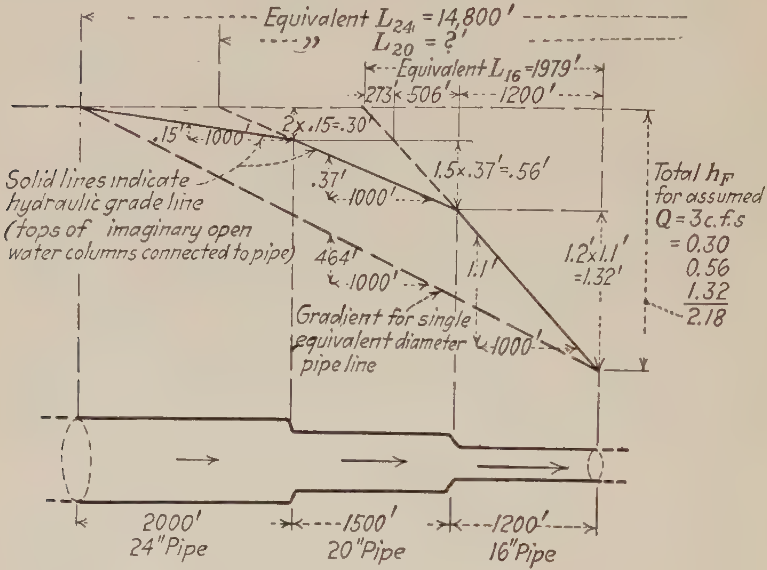


Fig. 120*A*.

Diameters of pipes in the problem, inches	h_F per 1000 ft. for the assumed Q = 3c.f.s. (from diagram), feet	Computations to find lengths of 16-in. pipe to give same h_F as the sizes in the left column and of lengths as in problem	Equivalent length of 16-in. pipe, feet
24	0.15	$2000 \times \frac{0.15}{1.1}$	273
20	0.37	$1500 \times \frac{0.37}{1.1}$	506
16	1.1	....	1200
Total equivalent length of 16-in. pipe..................			1979

Now 20-ft. loss in 1979 ft. of 16-in. pipe is 10.1 ft. per 1000 ft., for which the diagram shows, for 16-in. pipe, Q = 10 c.f.s. *Ans.*

As a check, reduce to an equivalent length of 24-in. pipe, changing the headings of the third and fourth columns correspondingly:

24 in.	0.15 ft.	....	2000 ft.
20 in.	0.37 ft.	$1500 \times \frac{0.37}{0.15} =$	3700 ft.
16 in.	1.1 ft.	$1200 \times \frac{1.1}{0.15} =$	8800 ft.
Total equivalent L of 24 in.			14,500 ft.

Then 20-ft. loss in 14,500 ft. of 24-in. pipe is 1.38 ft. per 1000 ft., for which the diagram shows, for 24-in. pipe, Q = 10 c.f.s. *Ans.*

Likewise, the same Q is obtainable from the equivalent lengths of any other selected size.

(The student should select some other value of Q and work out the equivalent length of 20-in. pipe, and also the length for a size not represented in Fig. 120, say 36 or 12 in.)

B. Solution by Reduction of the Compound Pipe Line to a Single Pipe of Equivalent Diameter and Same Total Length (see Fig. 120*A*).—As in solution A, assume a Q at random, say the same 3 c.f.s. as before. The total h_F for the three pipes is first computed as follows:

$$\begin{aligned}
&24 \text{ in.}, 0.15 \times 2 &&= 0.30 \text{ ft.}\\
&20 \text{ in.}, 0.37 \times 1.5 &&= 0.56 \text{ ft.}\\
&16 \text{ in.}, 1.1 \times 1.2 &&= \underline{1.32 \text{ ft.}}\\
&\quad\text{Total } h_F \ldots\ldots\ldots &&\ 2.18 \text{ ft. (for the assumed } Q = 3 \text{ c.f.s.)}
\end{aligned}$$

The total length of the compound pipe is 4700 ft., and a loss of 2.18 ft. uniformly along 4700 ft. is 0.464 ft. per 1000 ft. The fairly smooth diagram shows for this value of h_F and Q = 3 c.f.s. a corresponding diameter of 19 in. This is the diameter of a single-sized pipe that would give the same discharge as the compound pipe. But in the problem the available head is 20 ft., a rate of loss of 4.25 ft. per 1000 ft. for a single pipe, which value, with d = 19 in., shows on the diagram a Q = 10 c.f.s *Ans.*

(The equivalent diameter is always intermediate between the extreme sizes involved in any problem, for the same total length.)

C. Solution by Diagram by Apportioning the Total Head Properly among the Several Pipes.—Starting as in solution B, divide the available 20-ft. head into three parts of the same relative

magnitudes as the values 0.30, 0.56, and 1.32 bear to their sum 2.18, *i.e.:*

For the 24-in., $\frac{0.30}{2.18} \times 20 = 0.137 \times 20 = 2.75$ ft., or 1.38 ft. per 1000 ft.

For the 20-in., $\frac{0.56}{2.18} \times 20 = 0.257 \times 20 = 5.14$ ft., or 3.43 ft. per 1000 ft.

For the 16-in., $\frac{1.32}{2.18} \times 20 = 0.606 \times 20 = 12.11$ ft., or 10.1 ft. per 1000 ft.

Any one of these three values for h_F per 1000 ft. is seen to give on the diagram, for the appropriate diameter, the discharge $Q =$ 10 c.f.s. *Ans.*

If any one of the component pipes shows a Q differing considerably from the others, an error somewhere is indicated. Consequently, a check on the solution is given by carrying the work through for more than one size.

D. Solution by Trial, by Diagram.—Trying $Q = 3$ c.f.s., it is found that the total h_F adds up to 2.18 ft., as in solution C. But in the problem the available head is 20 ft., and evidently the trial Q is too small. One could now proceed by "hit-or-miss," guessing for the right Q to cause 20-ft. loss, but to make further guesses for Q somewhat scientifically, proceed as follows: Q varies nearly as $\sqrt{h_F}$ for fixed diameter. Hence approximately the trial 3 c.f.s. must be multiplied by $\sqrt{\frac{20}{2.18}}$, or by 3.03, *i.e.*, the adjusted trial Q is now 9.1 c.f.s.

Actually, since h_F varies with a power of v and Q a little *less than* 2, the adjustment should be by a power for h_F a little *more than* ½. Hence for the new trial a value a little greater than the above calculated 9.1 c.f.s. should be used.

However, trying 9.1 c.f.s., then $h_F = (2 \times 1.2 + 1.5 \times 2.9 + 1.2 \times 8.2) = 16.6$ ft., still somewhat too small, since 20 ft. is available.

Trying $\sqrt{\frac{20}{16.6}} \times 9.1 = 10$c.f.s., then $h_F = (2 \times 1.4 + 1.5 \times 3.4 + 1.2 \times 10) = 19.9$ ft. This is practically 20 ft., and hence the last assumption that $Q = 10$ c.f.s. is correct.

E. Solution by Formula (See Example 4, p. 203).—Letting the subscripts 1, 2, and 3 refer to the 24-, 20-, and 16-in. pipes, respectively,

$$h_F = f_1\frac{L_1}{d_1}\frac{v_1^2}{2g} + f_2\frac{L_2}{d_2}\frac{v_2^2}{2g} + f_3\frac{L_3}{d_3}\frac{v_3^2}{2g} \text{ (see Fig. 119}a\text{).}$$

$$20 = \left[f_1 \times 1000 + f_2 \times 900\left(\frac{24}{20}\right)^4 + f_3 \times 900\left(\frac{24}{16}\right)^4\right]\frac{v_1^2}{2g}$$

$$= [1000f_1 + 1870f_2 + 4560f_3]\frac{v_1^2}{2g}.$$

To find the exactly correct values of f the velocity should be known; but it appears from Fig. 107 that $f = 0.018$ is a rough average for fairly smooth pipes, 16 to 24 in., at a medium velocity of about 5 ft. per sec.

Using this value 0.018 for f_1, f_2, and f_3 for a first approximation to the correct velocity, then

$$20 = (7430 \times 0.018)\frac{v_1^2}{2g} = 134\frac{v_1^2}{2g}, \text{ whence } \frac{v_1^2}{2g} = 0.149 \text{ ft.,}$$

and $v_1 = 3.10$ ft. per sec. Then $v_2 = \frac{A_1}{a_2}v_1 = \left(\frac{6}{5}\right)^2 v_1 = 4.46$ ft. per sec.; and $v_3 = 6.98$ ft. per sec.

Using these approximate values of v_1, v_2, and v_3 to find values of f_1, f_2, and f_3 from Fig. 107, then

$$20 = [1000 \times 0.018 + 1870 \times 0.017 + 4560 \times 0.017]\frac{v_1^2}{2g}, \text{ or}$$

$$20 = 127.3\frac{v_1^2}{2g}, \text{ and } \frac{v_1^2}{2g} = 0.157 \text{ ft., whence } v_1 = 3.18 \text{ ft. per sec.}$$

(It is seen that the values of f need no further revision.) Hence $Q = A_1v_1(= A_2v_2 = A_3v_3) = 3.14 \times 3.18 = 10$ c.f.s. *Ans.*

125. Pipe (or Hose) with Nozzle.—In such problems the head is partly used up in overcoming friction and partly in causing the high velocity at the tip of the nozzle. If the nozzle is not at the same elevation as the hydrant (see Fig. 121*a*) or as the upstream end of the pipe or hose, part of the head may be necessary to overcome elevation (if the nozzle be the higher), or the head may be augmented by the difference of elevation (if the nozzle be the lower).

Example 7: What is the greatest permissible length of fire hose so that a stream of 250 g.p.m. may flow through the hose

(d = 2½ in.) and nozzle (d = 1⅛ in.). The nozzle is 35 ft. higher than the hydrant where the pressure is 115 lb. per sq. in. (Fig. 121*a*).

Comments: A single so-called "first-class fire stream" has a discharge of 250 U. S. g.p.m. through a nozzle 1⅛ in. in diameter. The best quality of rubber-lined hose has about *one-third less* loss by friction than the tables and diagrams for fairly smooth pipes show. The common unlined fabric mill hose causes about *one-third greater loss* than indicated for fairly smooth pipes.

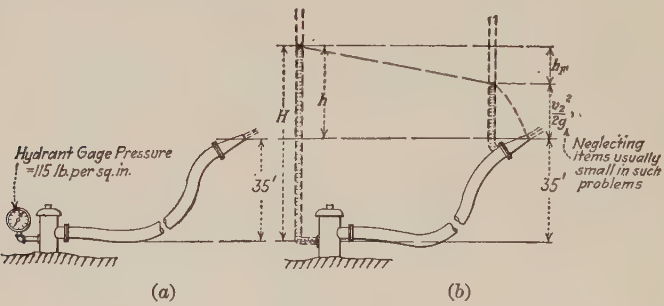


FIG. 121.

A. Solution by Diagram.—Figure 121*b* shows the hydraulic gradient from hydrant to the issuing stream.

$$H \text{ (equals } 115 \times 2.31 = 266 \text{ ft.)} = h + 35.$$

$$h = h_F + \frac{v_2^2}{2g}, \text{ where } v_2 \text{ is the tip velocity.}$$

In this problem Q and d are known: $Q = \dfrac{250}{7.48 \times 60} = 0.557$ c.f.s.; and v_2, the tip velocity, $= \dfrac{Q}{A_2} = \dfrac{0.557}{0.00690} = 80.7$ ft. per sec. and $\dfrac{v_2^2}{2g} = 101$ ft.

In this the minor head items, such as entry loss of head and loss in nozzle, are neglected (see comments, p. 237). Hence *the head available for friction in the hose,* h_F, = (266 − 35) − 101 = 130 ft.

On the fairly smooth pipes diagram, for Q = 0.557 c.f.s. and d = 2½ in., the h_F per 1000 ft. = 490 ft. Take two-thirds of this for the loss in the best-quality rubber-lined hose, or 327 ft. per 1000 ft.; while for the unlined fabric mill hose take (4⁄3) × 490 = 653 ft. per 1000 ft.

Hence the greatest permissible lengths are:

$^{130}\!/_{327} \times 1000 = 398$ ft. for the best-quality lined hose, and $^{130}\!/_{653} \times 1000 = 199$ ft. for the unlined mill hose, or, in round numbers, 400 and 200 ft. respectively. *Ans.*

B. Solution by Formula.—As in solution *A* above $(266 - 35) = 231 = f\frac{L}{d}\frac{v_1^2}{2g} + \frac{v_2^2}{2g}$. But $v_1 = \frac{0.557}{0.0341} = 16.35$ ft. per sec., and $\frac{v_1^2}{2g} = 4.15$ ft. As in solution *A*, $\frac{v_2^2}{2g} = 101$ ft. From Fig. 107 the value of f for $2\frac{1}{2}$-in. diameter and for v about 16 ft. per sec., *fairly smooth pipe*, is 0.025, and two-thirds of this is 0.017. Hence $\frac{L}{d} = \frac{130}{0.017 \times 4.15} = 1840$ and L (in feet) $= 1840 \times \frac{2.5}{12} = 383$ ft. This is for the best-quality rubber-lined hose.

Comments: If $\frac{v_1^2}{2g}$ is, as above, about 4 ft., the loss of head at entry to hose is about 2 ft. (see p. 206). The loss in the nozzle is about $0.05 \times 101 = 5$ ft. The sum of these, 7 ft., although for many problems quite large and not at all negligible, is in this case not enough to warrant making allowance in view of uncertainties as to the exact friction loss, exact size of hose, etc.

Loss of Pressure in Fire Hose.—The drops in pressure corresponding to the losses of head found in solution *A* above, *viz.*, 327 and 653 ft. per 1000 ft., are 14 and 28 *lb. per sq. in. per 100 ft.*

Now 115 lb. per sq. in., the hydrant pressure used in the example above, is rather high as compared with the street-main pressure in most towns. In fact, pressures no greater than 50 to 75 lb. per sq. in. are very common.

But it has just been found that for a hose line 400 ft. long (only 200 ft. for mill hose) fully 100 lb. pressure at the hydrant is necessary for a flow of 250 g.p.m. thrown out at the nozzle with a velocity-head $\frac{v_2^2}{2g} = 101$ ft., which means a pressure of 44 *lb. per sq. in. at the base of the nozzle, in the hose.*

For fire protection, such a "stiff stream" from the nozzle is essential, in order to carry to the blaze as a fairly compact stream, so as to cool the burning material below the temperature of ignition.

Evidently the hydrant pressure must be enough greater than the nozzle requirement to allow for friction loss in several hundred feet of hose (42 lb. per sq. in. for 300 ft.). But if a city or an industrial plant has a pressure of only 50 or 60 lb. per sq. in. available, the alternatives are to depend on less volume of flow through the hose, or on somewhat less "throwing" distance and height for the jet, or both (see example 8 below).

For low main pressures or small mains, or both, however, there is quite general use of fire engines to "boost" the hydrant pressure from a low to a high value at the entry to the hose. In such cases the flow available through the street mains is increased, because the hydrant pressure can be allowed to drop to a very much lower value than when no fire engine is available. Theoretically, the pressure on the hydrant side of the fire engine may be allowed to approach a vacuum pressure, but, practically, the main pressure during a fire should still be great enough to supply water to the upper stories of buildings for domestic, manufacturing, and sanitation purposes. This requirement for sufficient capacity in the mains to supply both domestic and fire supplies simultaneously is fundamental, except for small villages or manufacturing plants.

Example 8: With hydrant pressure at 60 lb. per sq. in., nozzle at same level as hydrant, what is the volume of flow through 300 ft. of high-quality rubber-lined 2½-in. hose having a nozzle with tip ⅞ in. in diameter (see comments, p. 237)?

A. Solution by Formula.

$$h = h_F + \frac{v_2^2}{2g} = f\frac{L}{d}\frac{v_1^2}{2g} + \frac{v_2^2}{2g}.$$

To approximate the correct value of f, it is assumed that v_1 in the hose is (of the order of) 10 ft. per sec., and from Fig. 107, taking two-thirds of the value for fairly smooth pipes, f = two-thirds of $0.027 = 0.018$. Also total $h = 60 \times 2.31 = 138.6$ ft.

$$138.6 = \left[0.018\,\frac{300 \times 12}{2.5}\left(\frac{7/8}{5/2}\right)^4\right]\frac{v_2^2}{2g} + \frac{v_2^2}{2g} = [0.018 \times 1440 \times 0.0150 + 1]\frac{v_2^2}{2g}$$

$$= [0.389 + 1]\frac{v_2^2}{2g}, \text{ or } \frac{v_2^2}{2g} = \frac{138.6}{1.389} = 99.8 \textit{ ft.}$$

(= 43 lb. per sq. in.).

Hence

$v_2 = 80.1$ ft. per sec., and

$$v_1 = \left(\frac{7/8}{5/2}\right)^2 \times 80.1 = 0.1225 \times 80.1 = 9.81 \text{ ft. per sec.}$$

This is close enough to the assumed value for v_1 so that the used value for f does not change perceptibly (see Fig. 107). Hence $Q = A_2v_2 = 0.00417 \times 80.1 = 0.334$ c.f.s., or 150 g.p.m. This is 60 per cent of the discharge of a so-called first-class fire stream (250 g.p.m.). By using the $\frac{7}{8}$-in. nozzle tip instead of the $1\frac{1}{8}$-in., however (a reduction of about 40 per cent in cross-section of the issuing jet), the velocity-head and the throw of the nozzle stream have been kept up to an adequate value for fire-extinguishing purposes for small houses.

Problem

Show that by increasing the size of the nozzle tip from $\frac{7}{8}$ to 1 in., the *discharge* is *increased* about 20 per cent and the head is decreased about 15 per cent in the nozzle stream.

B. Solution of Example 8 *by Diagram.*—(By Trial.)

The friction-head must, of course, be less than the head at the hydrant, otherwise there would be no head left for the nozzle stream. The velocity-head, moreover, varies inversely as the fourth power of the diameter, thus in this case the velocity-head in the hose is only 0.015 of that in the jet (see (*A*), p. 238). Hence, with $f = 0.018$ and $\frac{1}{f} = 55.5$ $\left(\text{or } \frac{L}{d} = 55.5 \text{ for } h_F = \frac{v_1^2}{2g}\right)$, it would take a length of $\frac{55.5}{0.015} = 3700$ diameters ($= 770$ ft.) of hose to cause a loss of head as great as the velocity-head in the jet.

This illustration makes it clear that whenever the nozzle tip diameter is small compared to the hose or pipe (say with diameter one-third or less that of the hose) the major part of the head goes not into friction loss in the hose but into the velocity-head of the issuing jet. In this particular case about one-fourth of the head goes into friction loss, the rest into jet velocity-head.

Try Q (through the hose) $= 0.3$ c.f.s., then $h_F = \frac{2}{3} \times 155$ (from Fig. 117) $= 103$ ft. per 1000 ft., or 31 ft. in 300 ft. This leaves $139 - 31 = 108$ ft. for the nozzle stream. But if $108 = \frac{v_2^2}{2g}$, $v_2 = 83.5$ ft. per sec., and the Q (through the nozzle) would be $A_2v_2 = 0.00417 \times 83.5 = 0.348$ c.f.s.

This shows that the trial Q, 0.3 c.f.s., through the hose is a little too small, since the same Q must flow through hose and nozzle.

Note that a small percentage increase in Q through the hose increases the friction-head about twice this percentage, and to this extent cuts down the remaining head at the nozzle, thereby causing the nozzle discharge to decrease.

Trying Q (through the hose) = 0.33 c.f.s., the $h_F = \frac{2}{3} \times 180 = 120$ ft. per 1000 ft., or 36 ft. in 300 ft. This leaves $139 - 36 = 103$ ft. for $\frac{v_2^2}{2g}$, or $v_2 = 81.4$ ft. per sec., and Q (through the nozzle) = 0.338 c.f.s. This brings the two values of Q in sufficiently close agreement. Hence $Q = 0.33$ c.f.s., as in solution A. *Ans.*

126. "Looping" Pipes and Gridiron Systems of Pipes.—For a simple case as shown in Fig. 122, but typical in its general

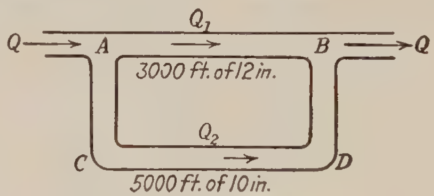


FIG. 122.

aspects, note that *some water will flow through each branch* whenever the pressure at A is greater than that at B, or *vice versa.* The greater volume will flow through the branch offering the lesser resistance. Evidently, $Q = Q_1 + Q_2$. Moreover, *each branch has the same total loss of head from A to B.* If the lengths are not the same, h_F per 1000 ft. is different in the two pipes.

Several types of problems may arise:

1. If h_F from A to B is known, Q_1 and Q_2 are found just as if there were two separate pipes connecting two reservoirs with the difference of elevation known.

2. Another type of problem is illustrated in the example following.

Example 9: The total discharge Q in Fig. 122 is to be 6 c.f.s. Find the loss of head between A and B, and the values of Q_1 and Q_2. Pipes are to be considered as rough (after, say, a dozen years of service).

A. Solution by Diagram by Properly Apportioning the Discharge among the Several Branches of the Loop.—Assume $Q_1 = 1$ c.f.s. (or any other convenient value shown on the diagram). For the 12-in. pipe, Fig. 118 shows $h_F = 0.80$ ft. per 1000 ft., or a total in 3000 ft. of 2.4 ft. But *this must also be the total* h_F for the 10-in. branch, whence the h_F per 1000 ft. $= \frac{2.4}{5} = 0.48$ ft. For this h_F and 10-in. diameter the diagram shows $Q_2 = 0.47$ c.f.s. The total Q for both branches is $1 + 0.47 = 1.47$ c.f.s.

Having found that the ratio $\frac{Q_1}{Q_2} = \frac{1}{0.47}$, the given 6 c.f.s. is divided into two parts having this same ratio, *i.e.*, $\frac{1}{1.47} \times 6 =$ 4.1 c.f.s. for Q_1, and $\frac{0.47}{1.47} \times 6 = 1.9$ c.f.s. for Q_2. These values should now be checked as follows:

If the values of Q_1 and Q_2 are correct, the total h_F in each branch should be the same. For $Q_1 = 4.1$ c.f.s. and 12-in. pipe, the *rough* diagram shows $h_F = 13$ ft. per 1000 ft., or 39 ft. in 3000 ft., and for 1.9 c.f.s. and 10-in. pipe, $h_F = 7.5$ ft. per 1000 ft., or 37.5 ft. in 5000 ft. These values, 39 and 37.5 ft., show an agreement as close as the diagrams permit, and as close as one is justified in expecting a check in a practical problem *in view of the inherent uncertainties.* Hence $h_F = 38$ ft., or about 16 lb. per sq. in. loss from A to B; $Q_1 = 4.1$, and $Q_2 = 1.9$ c.f.s. *Ans.*

B. Solution by Formula, Otherwise as Above in Solution A.—Another way to find the ratio between Q_1 and Q_2 is to use the general relations developed on page 226. Since Q varies approximately as $\sqrt{\frac{h_F d^5}{L}}$, or, when h_F is the same in two pipes under comparison, as $\sqrt{\frac{d^5}{L}}$, then $\frac{Q_2}{Q_1} = \sqrt{\frac{(d_2/d_1)^5}{L_2/L_1}}$ as a general relation. For the present example, $\frac{Q_2}{Q_1} = \sqrt{\frac{(10/12)^5}{5/3}} = \sqrt{\frac{0.402}{1.667}} = 0.49$, a fair check on the value 0.47 from the diagrams in solution A above.

After apportioning the total Q among the several pipes, as in solution A, the solution may be checked and completed by apply-

ing the formula $h_F = f\frac{L}{d}\frac{v^2}{2g}$ to each of the branches of the loop, using the proper v_1 and v_2 values and the corresponding values of f_1 and f_2. As in solution *A* above, the values of the total h_F from *A* to *B* should check each other.

C. Solution by Diagram by Reduction of the Loop to an Equivalent Length of a Single Pipe of Some Chosen Diameter.

In certain extensive compound pipe systems where one or more of the sections are composed of a loop or a gridiron of parallel pipes, it simplifies the solution of the general problem to deal with a single equivalent pipe imagined to replace such sections.

Starting as in solution *A* it is found that for $Q = 1 + 0.47 = 1.47$ c.f.s., the total h_F from *A* to *B* is 2.4 ft., *i.e.*, for these temporarily assumed rates of flow. Assuming a single 12-in. pipe to exist instead of the loop, it is found on the diagram that, for 1.47 c.f.s., $h_F = 1.75$ ft. per 1000 ft. Hence the equivalent length is $\frac{2.4}{1.75} \times 1000 = 1370$ ft., *i.e.*, a single 12-in. pipe 1370 ft. long will discharge as much water as the two pipes of Fig. 122 with the same loss of head.

With the whole 6 c.f.s. in a 12-in. pipe, the diagram shows h_F per 1000 ft. to be 27 ft. In the equivalent 1370 ft. the loss is $1.37 \times 27 = 37$ ft. *Ans.*

Or, assuming a single 10-in. pipe to replace the loop, the diagram shows, for $Q = 1.47$ c.f.s., $h_F = 4.4.$ ft. per 1000 ft. Hence the equivalent length of 10-in. pipe is $\frac{2.4}{4.4} \times 1000 = 545$ ft.

With the whole 6 c.f.s. through a 10-in. pipe, the diagram shows $h_F = 68$ ft. per 1000 ft. In the equivalent 545 ft. the loss is $0.545 \times 68 = 37$ ft. *Ans.*

Problem

At the start, assume $Q_1 = 2$ c.f.s. instead of 1 c.f.s., and then replace the loop by an equivalent length of 20-in. pipe, and solve.

D. Solution by Diagram by Reducing the Loop to the Equivalent Diameter of a Single Pipe of Some Chosen Length.—(See comment in small type at beginning of solution *C* above.)

Starting as in solution A, h_F from A to B = 2.4 ft. for total $Q = 1 + 0.47 = 1.47$ c.f.s. Assuming that a single pipe replaces the loop and is laid where the 12-in. pipe is shown, *i.e.*, with a length = 3000 ft., the h_F per 1000 ft. $= \frac{2.4}{3} = 0.80$ ft. For this h_F and $Q = 1.47$ c.f.s., the diagram shows a diameter of about 14 in., *i.e.*, 3000 ft. of 14-in. pipe would replace the loop in discharging capacity. With the whole 6 c.f.s. through the 14-in. pipe, the diagram shows $h_F = 12.5$ ft. per 1000 ft., or 37.5 ft. in 3000 ft. *Ans.*

Or if the single pipe be imagined laid where the 10-in. pipe is, *i.e.*, with the length = 5000 ft., then, starting as before, $h_F = \frac{2.4}{5} = 0.48$ ft. per 1000 ft., and for this the diagram shows a diameter of about 15½ in. With the whole 6 c.f.s. through such a pipe, the diagram shows $h_F = 7.2$ ft. per 1000 ft., or 36 ft. in 5000 ft. *Ans.*

E. Solution by Trial, Either by Diagram or by Formula.—For each assumed Q_1, the value of $Q_2 = Q - Q_1$. Trials must be continued until the computed friction-head is the same for each branch.

127. Gridiron Systems.—If a loop has more than two branches, the same arguments and methods as used in Par. 126 are applicable. For extensive interconnected systems of pipes, as in the streets of a large city, repeated applications of reduction methods must be made, with arbitrary approximations as to division and direction of flows. Usually it is desirable to use the method of reducing the whole of a simple system of pipes, or portions of a complex system to a single pipe. It is optional whether to deal with the equivalent length of a chosen diameter or the equivalent diameter of a chosen length. In simple cases the method of apportionment may be used to advantage.

This subject forms an important phase of water supply engineering. It will not be pursued farther in this text.

Loops with Compound Branches.—If any of the branches of a loop or gridiron system are composed of several sizes of pipe in series, such branches may be reduced to a single pipe of equivalent length, or of diameter, by methods given on pages 231 to 235, example 6.

Symmetry.—Where the two or more branches are of the same size and length, the flow will, of course, be the same through each.

Reservoir Effect of Large Main.—Where several pipes start some distance apart from a much larger main and are interconnected, the loss of head in the large main may often be neglected if the distance between take-off points is not excessive. The effect is practically as if the two pipes proceeded from a single reservoir.

128. Branching Pipes.—The term *branching pipes* is here applied to the case of a pipe that subdivides into two or more

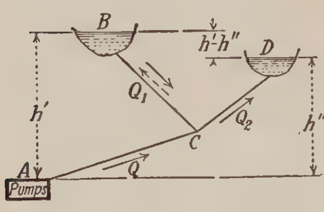


FIG. 123.—Case I (pumping system).

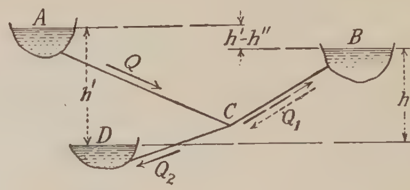


FIG. 124.—Case II (gravity system).

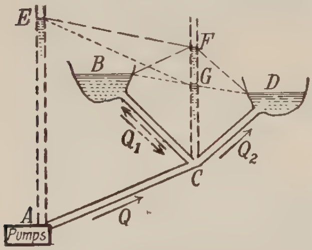


FIG. 125.—Case I (pumping system) showing hydraulic gradients.

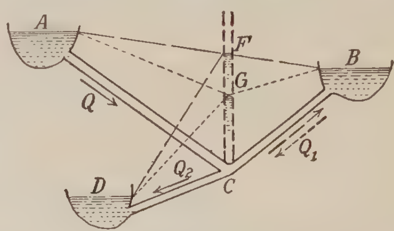


FIG. 126.—Case II (gravity system) showing hydraulic gradients.

pipes which are not cross-connected farther downstream but which supply separate reservoirs or water supply services. Figures 123 and 124 illustrate, respectively, a pumping system and a gravity system with simple cases of branching pipes.

Figures 125 and 126 illustrate the corresponding cases with the addition of the hydraulic grade lines for each of the two directions of flow that may prevail, the criteria of which will now be discussed.

If the pressure-head at the *point of branching* C is greater than the difference of elevation between C and B, water will flow

into B as well as into D. This applies to each of the two cases illustrated. See dashed lines (Figs. 125 and 126).

But if the head at C is less than this difference of elevation between C and B, the water will flow *from B*, as well as from A, into D. This condition is shown by the *dotted* hydraulic gradients and dotted arrows in Figs. 125 and 126.

The head at C depends (for a fixed Q), on the size, length, and roughness of the A-C pipe line. In case I it also depends on the head (or pressure) maintained by the pumps. (At present all regulating valves in the pipe lines are supposed to be wide open.)

But if the pressure at the pumps is constant (Fig. 125), and the length, size, and roughness of line A-C are fixed, then for both cases I and II the head at the branching point C depends on the resistance to flow in lines C-D and C-B.

Example 10 (Fig. 127): Find values of Q, Q_1, and Q_2, and directions of flow for each of two cases, the length A-C being: (1) L = 1000 ft.; (2) L = 5000 ft.; other lengths as shown in

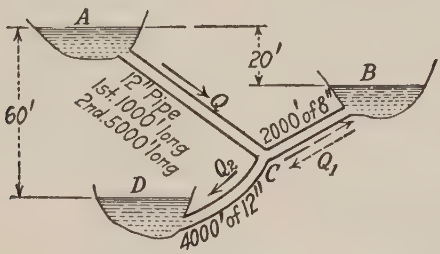


FIG. 127.

Fig. 127, which also shows sizes and elevations. Assume pipe as fairly smooth.

Solution by Diagram, by Trial: 1. Length A-C = 1000 ft. Try h_F, A to C, = 10 ft., then Q by diagram = 4.5 c.f.s. This assumption leaves 20 − 10 = 10 ft. for the h_F from C to B, the conditions being as shown in Fig. 126 by the *dashed* lines for hydraulic gradients. This is 5 ft. per 1000 ft., for which, with 8-in. pipe, the diagram shows Q_1 = 1.05 c.f.s. Also the h_F

remaining for C-D is $\frac{(60 - 10)}{4} = 12.5$ ft. per 1000 ft., for which the diagram shows $Q_2 = 5.1$ c.f.s.

But this would make $Q = Q_1 + Q_2 = 1.05 + 5.1 = 6.15$ c.f.s., a value larger than the assumed 4.5 c.f.s. corresponding to the assumed 10 ft. for the h_F in A-C.

If the h_F, A to C, is assumed larger, the effect is to *increase* Q and to *decrease* both Q_1 and Q_2 by making the remaining available friction-head smaller.

Try h_F, A to C, $= 15$ ft., thus leaving 5 ft. total h_F for C-B and 45 ft. for C-D. The diagram shows, for 15 ft. per 1000 ft. of 12-in. pipe in A-C, $Q = 5.7$ c.f.s. Also, for C-B, with 2.5 ft. per 1000 ft. in 8-in. pipe, $Q_1 = 0.72$ c.f.s.; and for C-D with 11.25 ft. per 1000 ft. of 12-in. pipe, $Q_2 = 4.9$ c.f.s. This gives a total $Q = 5.62$ c.f.s., which is close enough to the 5.7 c.f.s. corresponding to the assumption for h_F in this second trial.

Hence $Q_1 = 0.7$ c.f.s. *flowing from C to B;* and Q_2 is 4.9 c.f.s. *flowing from C towards D.*

2. Length A-C = 5000 ft. If any h_F, A to C, *less than* 20 ft. is tried, so that water flows *into* B, then Q from the diagram is less than Q_2, let alone the sum of Q_1 and Q_2.

But, if *more than* 20 ft. is allowed for the loss in A-C, the head at C will not be enough to cause flow *into* B. In fact the hydraulic conditions will be as shown by the *dotted* hydraulic gradients in Fig. 126, and consequently the flow will be *from B towards C.*

Try h_F, A to C, $= 25$ ft., or 5 ft., per 1000 ft. Then $Q = 3$ c.f.s. This leaves $\frac{(60 - 25)}{4} = 8.75$ ft. per 1000 ft. for C-D, for which $Q_2 = 4.2$ c.f.s. Then, also h_F *from B to C* equals $\frac{(25 - 20)}{2} =$ 2.5 ft. per 1000 ft., for which $Q_1 = 0.72$ c.f.s. *out of reservoir B.*

The sum of Q and Q_1 should $= Q_2$. But 3.72 c.f.s. falls short of 4.2 c.f.s. Hence more h_F must be allowed from A to C, and simultaneously more from B to C.

Try h_F, A to C, $= 27$ ft., making the drop from B to $C = 7$ ft. For the corresponding losses per 1000 ft., the diagram gives $Q =$ 3.2 and $Q_1 = 0.88$ c.f.s., making a total of 4.08 c.f.s. For Q_2 there remains an $h_F = \frac{(60 - 27)}{4} = 8.25$ ft. per 1000 ft., for

which the diagram shows, for 12-in. pipe, 4.1 c.f.s. This is a fair check for the sum of Q and Q_1.

Hence Q = 3.2 c.f.s., and Q_1 *from B towards C* = 0.9 c.f.s., and Q_2 = 4.1 c.f.s. *Ans.*

Problems: 1. Work out the two cases of example 10 for *rough pipes.*

2. With conditions as in Figs. 123 and 125, the pipe lengths and sizes being

A-C, . . . (1), 1000 ft.; (2), 5000 ft., of 12-in. pipe;
B-C, 2000 ft. of 8 in. pipe;
C-D, 4000 ft. of 12-in. pipe;

and the h' of Fig. 123 = 60 ft., and $h'—h''$ = 20 ft.; what pressures must be maintained by the pumps for (*a*), Q = 5.7; and (*b*), Q = 3.2 c.f.s.?

3. With length *A-C* (Fig. 123) = 5000 ft., other conditions as in the previous problem, Q = 4.1 c.f.s., except that now a valve in line *C-D* located close to *C is partially closed* so that Q_2 is reduced to 1 c.f.s., what is the pressure at the pumps, and how many horsepower must be supplied to the pumps with 60 per cent efficiency?

Hint.—Make a sketch and draw the hydraulic gradients in their approximate locations, remembering that there is a *sudden drop in pressure* where water flows through a partially closed valve. Hence draw open water columns not only at *A* and *C*, but also one to the right of the valve.

4. How many horsepower are required to force the water through the valve opening?

129. Special Losses of Head in Terms of Equivalent Length of Straight Pipe.—On page 184, it is shown that the reciprocal of f is the value of $\frac{L}{d}$ (or number of diameters in length) necessary to cause $h_F = \frac{v^2}{2g}$.

Since the losses of head due to the presence of special fittings in a pipe line are expressed in Table VII, p. 206, as fractions of $\frac{v^2}{2g}$, another column for the corresponding $\frac{L}{d}$ values might be added. Since, however, these $\frac{L}{d}$ values depend (as does f) on the diameter,

velocity, and degree of roughness of the pipe, it is better to compute them for each problem as it arises.

Pipe with Bends, Etc., Reduced to a Straight Pipe. Example 11: A 6-in., fairly smooth pipe line 150 ft. long has a square-edged entry and contains five short-turn elbows and four tees (each 90-deg. deflection). It conveys water from one tank to another with a difference of water levels = 3 ft. Compute the discharge.

Solution: Assuming a medium velocity, say 8 ft. per sec., Fig. 107 shows $f = 0.022$. On the reciprocal scale alongside, the value for $\frac{L}{d} = 45 \left(\text{for } h_F = \frac{v^2}{2g}\right)$, *i.e.*, since $h_F = f\frac{v^2}{2g}$ for $\frac{L}{d} = 1$ (or for one-diameter length of pipe), it takes $\frac{1}{f}$ diameters length for h_F to equal $\frac{v^2}{2g}$.

Hence the sum of all the heads except plain pipe friction, *viz.*, loss at entry, loss in elbows, loss in tees, terminal velocity-head, which totals $(0.50 + 5 \times 0.50 + 4 \times 1.25 + 1)\frac{v^2}{2g}$, equals $9\frac{v^2}{2g}$, equals the friction in $9 \times 45 = 405$ diameter lengths of plain, straight pipe, or, in this case of $d = 6$ in., 202 ft.

Consider now the equivalent simple straight pipe, with plain friction loss only, $150 + 202 = 352$ ft. long.

The pipe-flow diagram shows, for h_F per 1000 ft. $= \frac{3}{0.352} = 8.5$ ft., and $d = 6$ in., that $Q = 0.65$ c.f.s.

But this means that $v = \frac{Q}{A} = 3.3$ ft. per sec. Then $f = 0.024+$ and the $\frac{L}{d}$ value is 41. Hence a slight revision is necessary. More exactly, $9 \times 41 = 369$ diameters, or 185 ft. to be added to 150 ft., making a total of 335 ft. This makes h_F per 1000 ft. $= \frac{3}{0.335} = 8.95$ ft. and for this, $Q = 0.69$ c.f.s. *Ans.*

Example 12: Suppose that the diameter of the pipe in example 11 is unknown and to be found, the Q required being 1.4 c.f.s.

Solution: To find a nearly correct value for f it is necessary to have an approximate idea of the diameter. Momentarily assuming that the whole 3 ft. is lost by plain friction in 150 ft. of pipe, this would be $\frac{3}{0.150} = 20$ ft. per 1000 ft. corresponding on the pipe-flow diagram, for $Q = 1.4$ c.f.s., to $d = 6.7$ in. Actually, some of the 3 ft. is lost in overcoming entry, elbow, and tee resistance and some is required to create the velocity. Therefore the friction-head would be less than 20 ft. per 1000 ft. and the diameter would be somewhat larger than 6.7 in. But the student will note that the pipe-flow diagram shows little change in d for a considerable change in h_F per 1000 ft. Hence for purposes of finding f assume $d = 7$ in. The value of $v = \frac{Q}{A}$ $= \frac{1.4}{0.267} = 5.25$ ft. per sec. On Fig. 107 the corresponding f equals 0.024 and $\frac{L}{d} = 42$ for $h_F = \frac{v^2}{2g}$. Hence the equivalent L $= 150 \text{ ft.} + (0.5 + 5 \times 0.5 + 4 \times 1.25 + 1)42d = 9 \times 42d + 150 \text{ ft.} = 378d + 150$ ft. Using the assumed $d = 7$ in., the total equivalent $L = 150 \text{ ft.} + 220 \text{ ft.} = 370$ ft.; or h_F per 1000 ft. ft. $= \frac{3}{0.37} = 8.1$ ft., for which $Q = 0.95$ c.f.s. (This is too small.)

Trying $d = 8$ in., $L = 150 + 252 = 402$ ft.; or $h_F = 7.5$ ft. per 1000 ft. for which $Q = 1.3$ c.f.s. (Not quite enough.)

If the given requirement for Q ($= 1.4$ c.f.s.) is fixed, a pipe larger than 8 in. must be used. The next standard size is 10 in.

Problems

1. A compound pipe line ABC "rough pipe," consists of $AB = 2200$ ft. of 12-in. and $BC = 1800$ ft. of 10-in. The total loss of head A to $C = 100$ ft. Compute the discharge through the pipe line.

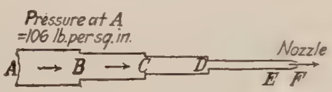


Fig. P.59.

2. In the sketch (Fig. P.59), AB is 1600 ft. of 12-in. fairly smooth pipe. BC is 1200 ft. of 8-in. rough pipe. CD is 600 ft. of 6-in. rough pipe. DE is 400 ft. of 3-in., high-grade rubber-lined hose. EF is a 3- by 1-in. nozzle. The gage pressure at A is 106 lb. per sq. in. (*a*) Compute Q, first reducing system

including nozzle to a single rough pipe of equivalent diameter 3800 ft. long, allowing $\frac{1}{20}$ of velocity-head in issuing jet for the loss in the nozzle. (*b*) Reduce line *ABCDEF* to an equivalent length of 3-in., high-grade rubber-lined hose. (*c*) Compute the flow through the system, using your result of (*b*).

Ans. (*a*) Diameter = 4.2 in. Q = 0.58 c.f.s.
(*b*) 1827 ft.
(*c*) 0.58 c.f.s.

3. A fire hydrant is supplied through 500 ft. of 6-in. pipe extending from the end of 16,000 ft. of 12-in. pipe. The total drop in pressure due to friction in both pipes is to be limited to 35 lb. per sq. in. What discharge in c.f.s. is available for the hydrant? Solve by trial. Use diagram (Fig. 118). *Ans.* 1.7 c.f.s.

4. Gage shows 90 lb. per sq. in. when there is no flow, hydrant outlet capped. The old (rough) 4-in. pipe 800 ft. long conveys water from the large main to a hydrant (Fig. P.60). The pressure in the large main is

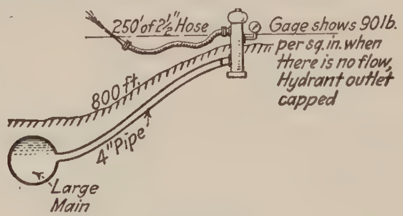


Fig. P.60.

practically constant. Under static (no-flow) conditions, the pressure at the hydrant is 90 lb. per sq. in. To reach a fraternity house, 250 ft. of hose (2½-in.) are needed. The rough-pipes diagram applies to the 4-in. pipe and the friction in the hose is seven-tenths of that given by the diagram for fairly smooth pipes. Loss of head in the hydrant and other minor head items may be neglected. (*a*) (1) With a single "good fire stream" of 250 g.p.m. (= 0.56 c.f.s.) flowing through the pipe and hose, what is the pressure at the hydrant and what is it in the hose close to the nozzle? (2) For this rate of flow what is the proper diameter for the tip of the nozzle? (*b*) If a nozzle tip of ¾-in. diameter is used, what is Q in cubic feet per second? What is the pressure in the hose close to the nozzle?

Ans. (*a*) (1) 62 lb. per sq. in., 25 lb. per sq. in.
(2) 1.29 in.
(*b*) 0.3 c.f.s., 64 lb. per sq. in.

5. What pressure in pounds per square inch is required at a fire hydrant to maintain a flow of 250 g.p.m. through 300 ft. of best-quality, rubber-lined, 2½-in. hose, terminated by a nozzle with a tip diameter of 1¼ in.?

6. (1) (*a*) Reduce the compound pipe line (Fig. P.61) to an equivalent length of 16-in. pipe. (*b*) Reduce the compound pipe line to an equivalent diameter with the same total length. (2) In the compound pipe line of part 1, what total head between points *A* and *D* would be required to furnish

a flow of 2 c.f.s. at point D with 3 c.f.s. drawn off the line at point C and 5 c.f.s. drawn off the line at point B?

7. For the compound pipe system shown in Fig. P.62 with 15-ft. drop in head from A to D, compute the discharge in cubic feet per second (neglecting all minor losses and head items), by the methods indicated. Assume pipes "rough." Make a sketch showing hydraulic gradient. First: (*a*) By the approximate law of variation apportion the loss of head among the several pipes, *i.e.*, *by formula, not diagrams.* (*b*) Using the pipe flow diagram, check this apportionment. (*c*) Calculate the Q in the 16-, 20-, and 12-in. pipes. Second: *By formula* for approximate law of variation, *not by diagram*,

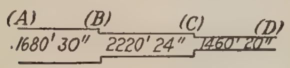


Fig. P.61.

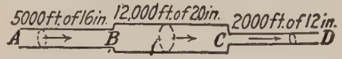


Fig. P.62.

reduce the system to equivalent length of a single 12-in. pipe. Then, for 10-ft. drop in head from A to D, find the discharge *by diagram.* Third: *By diagram* find the *exact diameter* of a single pipe 10,000 ft. long equivalent to the system; and for total h_F of 30 ft. find Q in c.f.s. by diagram.

Ans. (1) (*c*) 2.2 c.f.s.
(2) 4124 ft., 1.7 c.f.s.
(3) 14½ in., 3.1 c.f.s.

8. Solve by fairly smooth pipe diagrams, assumed applicable to both pipe and hose. A 24-in. main, with water pressure practically constant at 150 lb. per sq. in., supplies a hydrant through 3500 ft. of 6-in. pipe. Three 3-in. fire hose lines (of length to be determined) are supplied from the hydrant, there being a loss of 7 lb. per sq. in. in hydrant and connections between the 6-in. pipe and entrance to hose lines. Each hose supplies a stream from a nozzle of 1⅛ in. tip diameter with 45 lb. per sq. in. pressure in the hose just upstream from the nozzle. This is a "first-class fire stream." Find maximum permissible length of hose lines for such fire streams.

Ans. 256 ft.

9. A pipe line ABC consists of 3500 ft. of 16-in. pipe A to B and 2500 ft. of 12-in. pipe B to C. By means of the diagram for rough pipes, *reduce* the pipe line ABC to: (*a*) an equivalent length of 16-in. pipe, (*b*) to an equivalent length of 12-in. pipe. (*c*) Using result in (*a*), find the discharge for a fall of 5 ft. in the hydraulic grade line in the total length, *i.e.*, in 6000 ft. (*d*) Check this result by using value from (*b*).

Ans. (*a*) 14,500 ft.
(*b*) 3300 ft.
(*c*) 1.4 c.f.s.

10. Three 2½-in. hose lines, each 300 ft. long (laid side by side), convey water from a hydrant to a single "Siamese" nozzle with a tip diameter = 2 in. The friction head in the hose is seven-tenths of the value shown by the diagram for fairly smooth pipes. The nozzle discharges 750 g.p.m.

Compute the necessary pressure at the hydrant, neglecting minor losses of head. *Ans.* 84 lb. per sq. in.

11. Points B, C, D, and E (Fig. P.63) are on a flat 300 ft. lower than the reservoir. At first, line AFD (dotted) does not exist. (*a*) Assuming all pipes 8 in. (rough), compute the discharge Q ($= Q_1 + Q_2$) that may be drawn at E with a pressure of 100 lb. per sq. in. in the mains at E. Neglect minor losses. (*b*) By what percentage is this Q increased if line AFD (1100 ft. long) is connected as shown? Query: In this case, in what direction will the water in BD flow? *Ans.* (*a*) 3.5 c.f.s.

(*b*) Increase 48.5 per cent.

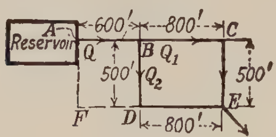


Fig. P.63.

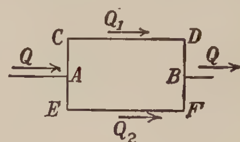


Fig. P.64.

12. $ACDB$ is 2000 ft. of 12-in. fairly smooth pipe. $AEFB$ is 2500 ft. of 6-in. fairly smooth pipe. Q is 4.0 c.f.s. (see Fig. P.64). How does the Q divide in the two parts of the loop? (*a*) Work the problem by the approximate law of flow. (*b*) Work the problem by diagrams.

13. $ACDB$ is 1272 ft. of 12-in. fairly smooth pipe. AB is 434 ft. of 16-in. old cast-iron pipe. $AEFB$ is 1643 ft. of 8-in. old cast-iron pipe (see Fig. P.65). It is proposed to replace the three lines shown by a single line of new cast-iron pipe 434 ft. long between A and B, keeping the pressure at point X the same as now. What size of line should be installed? (State your answer in actual computed diameter and then state what commercial size you would install.)

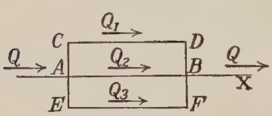


Fig. P.65.

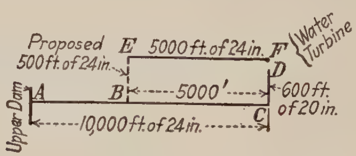


Fig. P.66.

14. It is proposed to increase the discharge of a 24-in. gravity line 12,000 ft. long, carrying 12.6 c.f.s. by paralleling it for two-thirds of its length by a second similar 24-in. pipe. (*a*) How many per cent, approximately, is the discharge increased? Explain. (*b*) What is the new Q?

15. It has been proposed to connect the upper and lower 24-in. pipes of the Ithaca city water supply system by a cross line B—E (see Fig. P.66). At present all the water to the filtration plant at C and to the water turbine in the pump station at F and D flows through the line $ABCD$. (*a*) Compute the loss of head in ABC with 12 c.f.s. flowing to C, and also the total loss in

ABCD with 4 c.f.s. taken off at *C* and the remaining 8 c.f.s. flowing to *D*. Assume rough pipes. Neglect all minor losses of head. (*b*) If, now, the cross connection *B—E* be put in, with ends *F* and *D* discharging into a common terminal pipe under pressure (to supply a turbine), compute the new total loss of head from *A* to *D*. Query: How much water will flow in each of the two branches of the loop for 12 c.f.s. total discharge? *N. B.:* 4.c.f.s. is to be taken off at *C*. As a matter of interest, but not data necessary for the solution of this problem, it may be stated that the difference of elevation between *A* and *C* is 80 ft., and between *A* and *F* is 240 ft.

Ans. (*a*) 30 ft.
(*b*) 18.15 ft.

16. The system of cast-iron pipes shown in Fig. P.67 has been in service 20 years. The discharge through the system is 3 c.f.s. What is the pressure drop between points *A* and *D*?

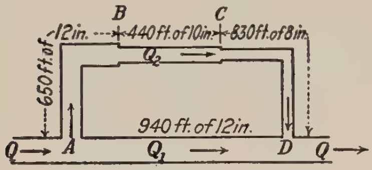


Fig. P.67.

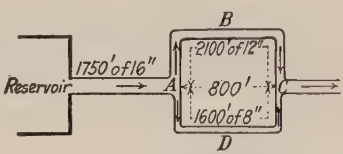


Fig. P.68.

17. It is desired to replace the loop *ABCD* shown in Fig. P.68 by a single pipe from *A* to *C*, a distance of 800 ft. All pipe fairly smooth. (*a*) What sized pipe should be installed? (*b*) A discharge of 6 c.f.s. is required at *C* with the pressure at *C* equal to 15 lb. per sq. in. How high above *C* must the level of the water in the reservoir be held?

18. Between reservoir (Fig. P.69) and point *G* there is a loop with two branches *BCF* and *BDF*. (At first the dotted line of pipe *BEF* does not exist.) At the start all pipes may be classed as fairly smooth. (*a*) How many 250-g.p.m. fire streams can be taken off at *G* if the pressure in the main at that point is not to fall below 80 lb. per sq. in.? (*Q* is what in c.f.s., and number of fire streams equals what?) (*b*) For conditions of flow as in (*a*) what are the elevations of the hydraulic grade line at points *B* and *F*? (*c*) A number of years after installation it is found that with 80 lb. per sq. in. at *G* the available draft (flow) is only eight-tenths of what it was at first. Assuming that all pipes have lost capacity at the same percentage rate, what per cent increase in loss of head over original conditions does this decrease in *Q* represent? For simplicity, base calculations on data for the *FG* 12-in. pipe 1000 ft. long. (*d*) It is proposed to restore the original avail-

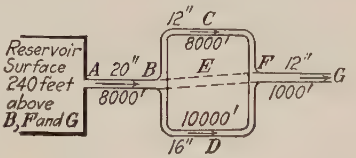


Fig. P.69.

able draft at G by connecting B and F with a third branch BEF of new fairly smooth pipe 4000 ft. long. Find the proper diameter for this branch.

Ans. (*a*) 6.5 c.f.s.; 11.6 streams.
(*b*) 13 ft., 32 ft. below reservoir.
(*c*) 50 per cent.
(*d*) Approximately 18 in.

19. Refer to Fig. P.70. *All pipes are rough.* (*a*) Reduce ADC to an equivalent length of 16-in. pipe. (*b*) For a total flow of 4 c.f.s. find approximately the flow in each line of the loop. (Work this out without tables or diagrams.) (*c*) For a flow of 4 c.f.s. (total), a gage at A reads 50 lb. per sq. in. What would a gage read at C? (Both are at the same elevation.) (*d*) Check your results of (*b*) by using rough-pipe diagrams. (*e*) Reduce the entire loop to an equivalent diameter pipe with a length of 5000 ft.

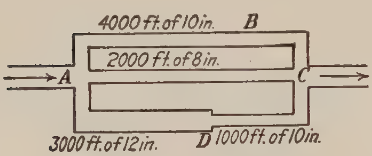


FIG. P.70.

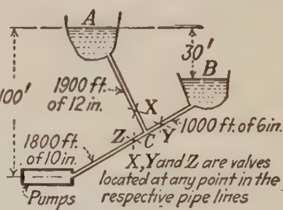


FIG. P.71.

20. Fairly smooth pipes. In each case draw the hydraulic gradient (see Fig. P.71). (*a*) Pumps deliver 3 c.f.s. All valves wide open. Find the flows into A and B. Find pump pressure. (*b*) Given pressure at the pumps 90 lb. per sq. in. With all valves wide open find the flows into A and B. (*c*) Pumps are pumping 3 c.f.s. with valve Y partly closed and valves X and Z wide open. Flow into B is 0.8 c.f.s. Find the pressure at the pumps. How many feet of head are lost at valve Y?

21. Figure P.72 shows three reservoirs connected by fairly smooth pipe, all pipes wide open. BC is 4000 ft. of 10-in. pipe and the loss of head in AC is 20 ft. with 10 c.f.s. flowing. Compute how much water in c.f.s. will flow in branch BC and in what direction?

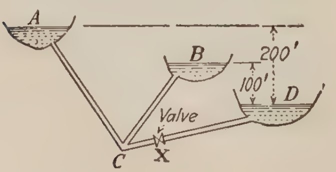


FIG. P.72.

22. In a rough-pipe system similar to that shown in Fig. P.72, reservoir B is 20 ft. and D is 60 ft. lower than A. AC consists of 5000 ft. of 12-in. pipe, BC 2000 ft. of 8-in., and CD 4000 ft. of 10-in. (*a*) With pipe diagram find the c.f.s. flowing into, or out of, each reservoir. State direction of flow. (*b*) To what rate of flow in cubic feet per second must the flow in CD be throttled (by valve at X) so that 1 c.f.s. will flow into reservoir B? (*c*) Assuming valve X quite close to point C, what head (feet of water) is lost through the valve under conditions of (*c*)?

CHAPTER XV

UNIFORM FLOW OF WATER IN OPEN CHANNELS[1]

130. General Considerations.—In Chap. XI general comments on the flow of water in open channels are given, and the Chézy formula, $v = C\sqrt{Rs}$, is derived. Also on page 216, the so-called "exponential" pipe formulas are stated so as to involve R and s; and on page 221 still other formulas containing R and s are stated. The student should review this matter.

Note that on page 181 the wetted perimeter, $w.p.$, refers to the *waterway*, and does not include the portion of the cross-section

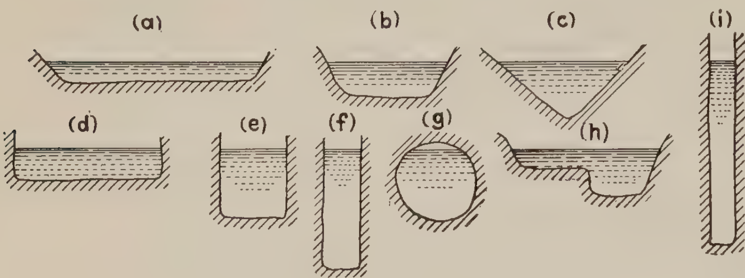


FIG. 128.

in contact with the atmosphere. At all ordinary velocities of water, the friction against the air is negligible, violent wind effects excepted.

The Hydraulic Radius.—It appears from the basic formula (Eq. (59), p. 181, $h_F = f_1 L \frac{w.p.}{A} v^2$, that the smaller the ratio of the wetted perimeter to the area of cross-section of the flowing stream the less is the loss of head by friction; or, reversing the formula, the greater the ratio of the cross-sectional area to the wetted perimeter the greater is the velocity of the water.

[1] For the *backwater* type of non-uniform flow see appendix C.

This latter form of the statement is condensed in the parlance of hydraulics by stating that, slope and roughness remaining the same, *the greater the hydraulic radius* (or the hydraulic mean depth) *the greater is the velocity;* and this is symbolized in the Chézy formula, $v = C\sqrt{Rs}$, and in the other formulas like the last form of the several exponential formulas on page 216.

In Fig. 128, all of the cross-sections shown have equal areas, but a glance shows that the amounts of rubbing surface differ widely or, in the parlance of hydraulics, the hydraulic radii are different.

A wide, shallow stream, as in *a*, or a narrow, deep ditch or gorge, as in *i*, offers great resistance to flow, shape *i* even more than shape *a*. *Compact* areas of cross-section, such as *e* and *g*, offer least resistance.

An index of such different conditions is furnished by the value of the hydraulic radius because it is actually the area of stream cross-section per unit of wetted (rubbing) perimeter or, taking unity length along the stream, *the hydraulic radius is the volume of water per unit of rubbing area.*

131. Oddities and Paradoxes of Open Channels.—Such a cross-section as *h* consists, in flow effect, of two streams side by side with the same slope, but with the water flowing slower in the shallow portion, a matter of common observation. Such a case should be treated in computations not as a simple single stream, but as a combination of two streams side by side with different velocities and hydraulic radii, but with the same slope (see example 8, p. 269).

For a culvert or sewer of square cross-section and flowing almost, but not quite, full, so that the water surface does not touch the flat top, then (Fig. 128*e*) the value of the hydraulic radius R is $\left(\frac{A}{w.p.}\right) = \frac{d^2}{3d} = \frac{d}{3}$. But if, for any reason, the water level rises a trifle so that the square is "plump" full, the flowing water rubs against the top, and $R = \frac{d^2}{4d} = \frac{d}{4}$, a decrease of 25 per cent from $\frac{d}{3}$, involving a decrease in velocity and discharge of some 17 per cent (as the student will be able to calculate for

himself presently). Such a reduction of velocity and discharge will result if the head cannot change rapidly. An example of this is a railroad culvert in flat country with large ponds of water on both sides of the track. For a sewer, the water would rise in manholes until the hydraulic slope had increased over 40 per cent to maintain the same velocity with the greater rubbing surface, *i.e.*, with the smaller hydraulic radius.

Likewise, interesting and practically important comparisons may be made for a circular section when flowing not quite full, for instance, with a depth of water $= 0.93d$. In this case the area A is 97 per cent of the full circle value, but the wetted perimeter is only 83 per cent of the full circumference. Hence the hydraulic radius R is 17 per cent *larger* than for the filled circle. This results in a velocity about 11 per cent greater and a discharge about 9 per cent greater than when the circular section runs full.

Of course, if a culvert or sewer runs full "under pressure," strictly open channel conditions no longer prevail, with surface slope parallel to the bed of the waterway, but the hydraulic gradient is steeper (or sometimes gentler) than the grade of the sewer. Such gradient is indicated by the heights to which water rises in manholes of the sewer.

132. Kutter's Formula and Kutter's n.—The discussion on pp. 214–7 applies to open channels as well as to pipes if a few changes in symbols are made. The value of C in the Chézy formula $= 2\sqrt{\frac{2g}{f}}$ (see p. 185), and the comments on f apply also to C. (The variations in values for C, with changes in R and s, are shown in Table XI.) The exact formula becomes, for open channels, $s = K'\frac{v^n}{R^p}$, or $V = K''R^{\frac{p}{n}}s^{\frac{1}{n}}$, neither of the exponents being exactly $\frac{1}{2}$. **(See for comparison the Manning formula stated on p. 221.)**

There has been, since 1869, however, a widespread acceptance and use of an expression for the value of C in the Chézy formula proposed by Ganguillet and Kutter. The Chézy formula con-

Table XI.—Kutter's Formula

(This table gives values of C in the Chézy formula, $v = C\sqrt{Rs}$, computed according to the formula of Ganguillet and Kutter (foot-second units).)

Hydraulic radius R, in feet	1000 × slope or feet per 1000 ft.	Values of C for the following values of Kutter's coefficient index of roughness, n													Slope s
		.009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	.060	
0.1	1	110	94	83	73	65	54	45	36	27	21	17	14	8	.001
	.1	90	78	68	60	54	44	37	30	22	17	14	12	7	.0001
	.05	78	67	59	52	47	39	33	26	20	16	13	11	7	.00005
	.025	65	57	50	44	40	33	28	23	17	14	12	10	6	.000025
0.2	1	129	113	99	89	81	66	57	45	34	27	22	18	11	.001
	.1	112	98	86	76	69	57	48	39	29	23	19	16	10	.0001
	.05	100	87	77	68	62	51	44	35	26	21	18	15	9	.00005
	.025	87	75	67	59	53	45	38	31	24	19	16	14	7	.000025
0.3	1	141	124	109	98	89	74	63	51	39	30	25	21	13	.001
	.1	125	109	97	87	78	65	56	45	34	27	22	19	12	.0001
	.05	114	99	88	79	71	59	50	41	31	25	21	18	11	.00005
	.025	100	88	78	70	61	53	45	37	29	23	19	16	10	.000025
0.4	1	150	131	117	105	96	80	69	56	43	34	28	24	14	.001
	.1	136	119	106	95	86	72	62	50	38	31	25	22	13	.0001
	.05	124	109	97	88	79	66	57	46	35	28	24	20	12	.00005
	.025	111	97	87	78	70	59	51	42	32	26	22	19	12	.000025
0.6	1	161	142	127	115	104	88	76	63	48	39	32	27	17	.001
	.1	149	131	118	105	96	81	70	57	44	35	30	25	16	.0001
	.05	139	122	109	98	90	76	65	53	41	33	28	24	15	.00005
	.025	127	112	100	90	81	69	60	49	38	31	26	22	14	.000025
0.8	1	169	150	134	122	111	94	82	68	52	42	35	30	19	.001
	.1	158	140	126	114	103	88	76	63	48	39	33	28	18	.0001
	.05	150	133	119	107	98	83	71	59	46	37	31	27	17	.00005
	.025	138	122	109	99	90	77	66	55	43	35	30	25	16	.000025
1.0	1	175	155	139	127	116	99	86	71	56	45	38	33	20	.001
	.1	166	147	132	120	109	93	81	67	52	42	35	31	19	.0001
	.05	158	140	126	114	104	89	77	64	49	40	34	29	19	.00005
	.025	148	131	118	106	97	83	72	60	47	38	32	28	18	.000025
1.5	1	184	165	149	136	124	108	93	78	62	50	43	37	24	.001
	.1	178	159	144	130	120	103	89	75	59	48	41	35	23	.0001
	.05	173	154	139	126	116	99	87	72	57	47	40	34	22	.00005
	.025	166	148	133	121	111	95	83	69	55	45	38	33	22	.000025
2	1	191	171	155	142	130	112	98	83	66	54	46	40	26	.001
	.1	187	168	151	138	127	109	96	81	64	53	45	39	25	.0001
	.05	184	164	148	135	124	107	94	79	62	51	44	38	25	.00005
	.025	179	160	144	131	121	104	91	77	61	50	43	37	24	.000025
3	1	199	179	163	149	138	119	105	89	71	59	51	45	29	.001
	.1	198	178	162	149	137	119	104	89	71	59	51	45	29	.0001
	.05	198	178	161	148	136	118	104	88	71	59	50	44	29	.00005
	.025	197	177	160	147	135	117	103	88	70	59	50	44	29	.000025
4	1	204	184	168	154	142	124	110	93	75	63	54	48	32	.001
	.1	206	186	169	155	143	125	111	94	76	64	55	49	32	.0001
	.05	207	187	170	156	145	126	111	95	77	64	56	49	33	.00005
	.025	209	188	172	158	146	127	113	96	78	65	56	49	33	.000025
6	1	...	190	174	160	149	130	116	99	81	68	59	52	36	.001
	.1	...	195	178	164	152	134	119	102	84	71	61	54	37	.0001
	.05	...	199	182	168	156	137	122	105	85	72	63	56	38	.00005
	.025	...	206	188	174	161	142	126	108	88	74	64	57	39	.000025
10	1	...	...	181	167	155	136	122	105	87	74	65	58	40	.001
	.1	...	...	188	174	162	143	128	111	92	78	69	62	42	.0001
	.05	...	...	195	181	169	149	134	116	96	82	72	64	45	.00005
	.025	...	...	207	192	179	159	142	124	102	87	76	68	47	.000025
20	1	...	...	...	175	163	144	129	113	94	81	72	65	47	.001
	.1	...	...	...	185	173	154	139	122	102	89	79	71	52	.0001
	.05	...	...	...	196	184	165	149	131	110	96	85	77	55	.00005
	.025	...	...	...	215	202	181	164	144	121	106	94	84	60	.000025
50	1	...	...	...	...	170	151	137	120	101	89	79	72	54	.001
	.1	...	...	...	...	185	166	151	134	114	100	91	83	63	.0001
	.05	...	...	...	...	201	181	165	148	127	112	101	93	70	.00005
	.025	...	...	...	...	228	207	190	170	147	130	117	107	80	.000025
100	1	...	...	...	...	...	155	141	124	105	94	85	77	59	.001
	.1	...	...	...	...	...	172	158	140	121	108	98	91	70	.0001
	.05	...	...	...	...	...	190	175	158	137	123	112	104	80	.00005
	.025	...	...	...	...	...	223	207	187	163	147	134	124	96	.000025
3.28 ft. (=1 meter)	For all slopes	201	181	164	151	139	121	106	91	72	60	52	46	30	

For greater slopes than s = 0.001 (1 ft. per 1000 ft.), use values of C given for s = 0.001.

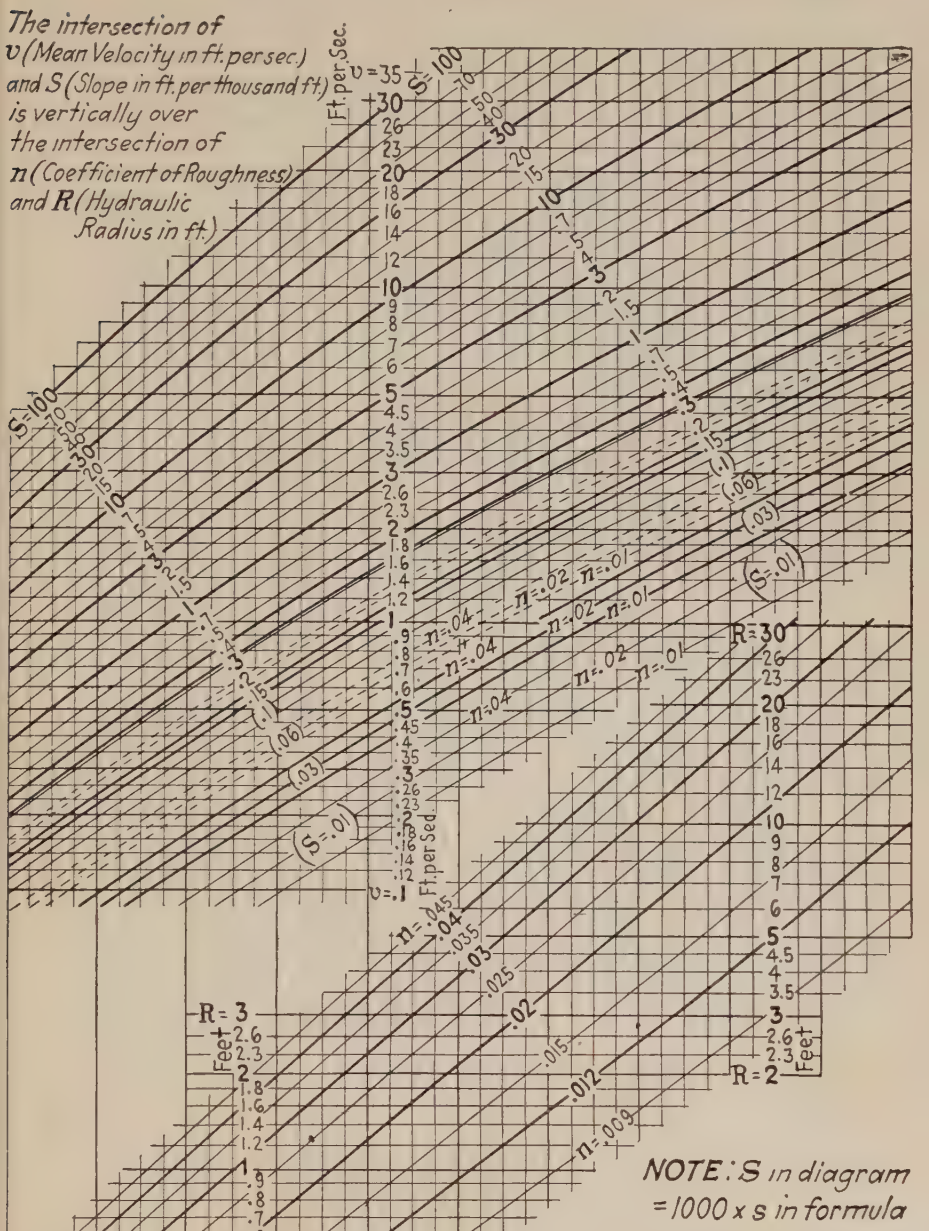


FIG. 129.—Kennison's diagram. Flow of water by the Kutter formula for ft.-sec. units. (*Courtesy of Karl R. Kennison, Consulting Engineer, Boston, Mass.*)

$$v = \frac{41.6 + \frac{.0028}{s} + \frac{1.81}{n}}{1 + \left(41.6 + \frac{.0028}{s}\right)\frac{n}{\sqrt{R}}}\sqrt{Rs}$$

taining this expression for C is commonly called *Kutter's formula*, which is:

$$v = \left[\frac{\left(41.6 + \frac{0.0028}{s}\right) + \frac{1.81}{n}}{1 + \left(41.6 + \frac{0.0028}{s}\right)\frac{n}{\sqrt{R}}}\right]\sqrt{Rs}, \text{ (foot-second units).} \quad (73)$$

This seemingly complicated expression has been worked out in the form of tables and diagrams, so that there is seldom any necessity of direct substitution in the formula itself. Table XI gives values of the Chézy C for various values of Kutter's roughness index n.

Kennison's diagram (Fig. 129) gives a ready solution for any one of the "hydraulic elements" v, R, s, and n when the other three are known.

From experimental measurements of A and *w.p.* to get R, and of the mean velocity v by some one of several available field methods, and of the surface slope s, on all sorts of natural and artificial water courses ranging from small ditches, sewers, and pipes, to the Mississippi River, a number of categories of roughness have been selected, and for each of these there is assigned the appropriate numerical value of Kutter's roughness index n, as stated in Table XII.

Comments on Kutter's n.—The velocity of the stream varies inversely as n, approximately. This is not readily seen by casual inspection of Kutter's formula, but appears by examining values of the Chézy C in Table XI. Hence, for example, an uncertainty whether to use, say, $n = 0.015$ or 0.017, a variation of about 6 per cent either way from the mean value 0.016, means an uncertainty of 6 per cent also in v in opposite direction on this account alone. If the slope s were unknown and to be calculated for a fixed v and R (*i.e.*, for a certain shape and size of channel with fixed Q), the uncertainty in its value would be twice 6 per cent = 12 per cent, since s varies nearly as n^2. This is so because, from the Chézy formula $s = \frac{v^2}{C^2R}$, and if R and v are constant, s varies as $\frac{1}{C^2}$ or as n^2.

TABLE XII

Descriptions of categories of roughess	Corresponding values of Kutter's n
I. *Artificial channels of uniform cross-section:*	
Sides and bottom lined with well-planed timber evènly laid.....	0.009
Lining of neat cement plaster; also smoothest pipes............	0.010
Cement plaster (3 cement to 1 sand); also fairly smooth iron pipes	0.011
Unplaned timber evenly laid; also iron pipes of intermediate roughness..	0.012
Ashlar masonry; best brickwork; well-laid new sewer pipe; Riveted steel pipes; rough pipes..........................	0.013
(*N. B.*: The value 0.013 should be used for the foregoing smoother categories if there is any doubt as to the excellence of construction, or as to the maintenance free from deterioration of surface, and free from slime, rust, or other deposits and growths.)	
Average brickwork; foul planks; foul iron pipes; ordinary sewer pipes after average uneven settlement and average fouling...	0.015
Good rubble masonry; concrete laid in rough forms; poor brickwork; heavily incrusted iron pipes (clear opening not seriously reduced)..	0.017
II. *Channels subject to non-uniformity of cross-section:*	
Excellent, clean canals dug in firm gravel, of fairly uniform cross-section with good alignment; also linings of rough rubble or dry paving..	0.020
Ordinary earth canals, and rivers in good order, free from large stones and heavy growths of weeds, fair alignment...........	0.025
Same with stones and weeds occasionally......................	0.030
Canals and rivers of rough appearance, or with irregular bottom; or with many stones and weeds, or strewn with detritus, with poor alignment..	0.035–0.040
Still higher values of n are necessary for tortuous channels, but for such cases the value of R is very uncertain; for example, mountain torrents and destructive floods in forested or settled valleys..	0.050–0.070

133. Other Formulas.—If certain simplifications are made in Kutter's formula[1] the so-called *Bazin's formula* is obtained. This may be written

$$v = \frac{158}{1 + \frac{N}{\sqrt{R}}}\sqrt{Rs}, \text{ for foot-second units,}$$

in which values of N compare with Kutter's n as follows:

n = 0.010, 0.013, 0.017, 0.020, 0.025, 0.035.

N = 0.11, 0.29, 0.83, 1.54, 2.35, 3.2.

For the *Manning formula*, $v = \frac{1.486}{n}R^{2/3}s^{1/2}$, (ft. sec. units), the n values differ but little from Kutter's values for n.

[1] See *Eng. News*, p. 351, Aug. 22, 1912.

Comparison of Values of f and n.—It will be seen that the range of values of n in Table XII is nearly the same as the range of values of f in Fig. 107, each extending from about 0.009 to 0.050. Note, however, that the value of f for any particular pipe or channel is, in general, different from the value of n for the same case.

134. Hydraulic Elements.—Table XIII shows the values of various hydraulic elements for a few of the more common types of artificial open channels. Such values are usually calculated as the need arises, and this table is not so much for reference as to furnish the student an opportunity for becoming familiar, by checking, with the calculations necessary.

TABLE XIII.—SAMPLE HYDRAULIC ELEMENTS

Shape of cross-section	Depth of water	Width of free surface	Area of cross-section	Wetted perimeter	Hydraulic radius, $R = \frac{A}{w.p.}$	Remarks
Circle	d	Zero	$3.14r^2$	$6.28r$	$\frac{r}{2} = \frac{d}{4}$	
Circle	$0.9d$	$1.2r$	$2.98r^2$	$5.00r$	$0.596r$	A is 5.2 and $w.p.$ is 20.5 per cent less than full circle.
Circle	$0.75d$	$1.73r$	$2.53r^2$	$4.19r$	$0.604r$	A, 19.6 and $w.p.$ 33.3 per cent less than full circle.
Circle	$0.5d$	$2r = d$	$1.57r^2$	$3.14r$	$\frac{r}{2} = \frac{d}{4}$	R same as full circle.
Circle	$0.25d$	$1.73r$	$0.614r^2$	$2.09r$	$0.293r$	
Square	d	Zero	d^2	$4d$	$\frac{d}{4}$	R same as for inscribed circle.
Square	d	d	d^2	$3d$	$\frac{d}{3}$	
Half square	$\frac{d}{2}$	d	$\frac{d^2}{2}$	$2d$	$\frac{d}{4}$	R same as for full square.
Rectangle	d	b	bd	$b + 2d$	$\frac{bd}{(b + 2d)}$	
V-flume, 60 deg.	d	$1.154d$	$0.577d^2$	$2.309d$	$\frac{d}{4}$	R same as for inscribed semicircle.
V-flume, 90 deg.	d	$2d$	d^2	$2.83d$	$\frac{d}{2.83}$	

Trapezoidal Channels (bottom width $= b$; depth $= d$) (Fig. 131):

Half of regular hexagon, side slopes 60 deg. with horizontal, $R = \frac{d}{2}$.

Side slopes 45 deg., $R = \frac{(bd + d^2)}{(b + 2.83d)}$.

Side slopes $1\frac{1}{2}$ horizontal to 1 vertical, $R = \frac{(bd + 1.5d^2)}{(b + 3.61d)}$.

Side slopes 2 horizontal to 1 vertical, $R = \frac{(bd + 2d^2)}{(b + 4.47d)}$.

For a wide stream, many times wider than deep, $R =$ depth (approximately). Hence the term "hydraulic mean depth" used interchangeably with "hydraulic radius."

Note that the value of the hydraulic radius is *always less* than the *depth of water* at the deepest part of the cross-section.

135. Open-channel Problems.—Among common problems confronting the engineer are:

	Given	To find
I	Q, shape and size of cross-section, and n	Necessary slope s
II	Q, shape, but not size of cross-section, s, and n. (Sometimes the width is known but not the depth, and *vice versa.*)	Necessary size
III	Shape and size of cross-section, s and n	v and Q
IV	Field measurements of R, v and s	n

A problem belonging to any of the four types mentioned above may be solved either by formula or by diagram. Certain preliminary calculations are common to both methods.

Example 1: Required to find the grade, on which a semicircular steel flume should be laid, to carry off 25 c.f.s., the flume being 42 in. wide, depth of water = radius. Take Kutter's n = 0.012.

Solution: Since both the formula and the diagram involve v, R, s, and C (or n), preliminary calculations must be made to find the values of such of these as are implied in the data of the problem.

$$\text{Mean } v = \frac{Q}{A} = \frac{25}{\frac{1}{2}\frac{\pi}{4} \times (3.5)^2} = \frac{25}{4.81} = 5.20 \text{ ft. per sec.}$$

$$\text{Hydraulic radius } R = \frac{A}{w.p.} = \frac{\frac{1}{2} \times \frac{\pi}{4}d^2}{\frac{1}{2}\pi d} = \frac{d}{4} = \frac{3.5}{4} = 0.875 \text{ ft.}$$

Now, completing the solution by *formula and table,* note that Table XI shows sets of values for C opposite R = 0.8 ft. and R = 1.0 ft. in the column headed n = 0.012. It is seen that the value of C is different for different slopes. Assume that, for finding a trial value for C, s is of the order of magnitude of 0.001 (1 ft. in 1000 ft.) but not necessarily close to this value. By interpolation to nearest whole number, C = 124.

Then, for $v = C\sqrt{Rs}$, $5.20 = 124 \times \sqrt{0.875 \times s}$, and $s = \dfrac{\left(\frac{5.20}{124}\right)^2}{0.875} = 0.0020$ (or 2.0 ft. in 1000 ft.). Now looking back in Table XI, the value of C for this slope is the same as that

assumed. Hence the slope is 0.0020, or 2.0 ft. per 1000 ft. *Ans.*

By Kennison's diagram (Fig. 129), the above-stated values of R and n give an intersection which locates a vertical line. This vertical line intersects the 5.2 velocity line in the upper portion of the diagram. The slope at this upper intersection is found to be 2.0 ft. per 1000 ft. *Ans.*

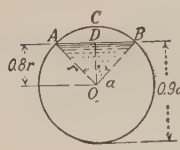


FIG. 129A.

Example 2: It is desired to carry off 150 c.f.s. in a circular sewer (average brickwork) of 7-ft. diameter when flowing with depth = $0.9d$. Find the grade on which the sewer should be laid (see Fig. 129A).[1]

Solution:

Mean $v = \dfrac{Q}{A}$. But A = circle − segment

$\therefore A = \pi r^2 -$ (sector $ACBO$ − triangle ABO)

$\therefore A = 36.5$ sq. ft.

Hence $v = \dfrac{150}{36.5} = 4.11$ ft. per sec. Also $R = \dfrac{A}{w.p.}$.

But $w.p. = \dfrac{360 \text{ deg.} - 73.8 \text{ deg.}}{360 \text{ deg.}} \pi d = 0.795\pi d = 17.5$ ft.

Hence $R = \dfrac{36.5}{17.5} = 2.09$ ft.

Now, completing the solution by *formula and table* as in example 1, interpolate between values of C opposite $R = 2$ and $R = 3$ ft. in the column headed $n = 0.015$ (the value given for average brickwork). For trial, $s = 0.001$, $C = 113$. Then from $v = C\sqrt{Rs}$, $4.11 = 113\sqrt{2.09s}$, and $s = 0.00063$ (or 0.63 ft. in 1000 ft.). Now looking back in the table the value of C for this slope is not enough different to change the trial value 113. Hence the slope is 0.00063, or 0.63 ft. per 1000 ft. *Ans.*

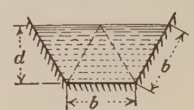


FIG. 130.

By diagram (Fig. 129) for open channels, with the values $n = 0.015$, $v = 4.11$ ft. per sec., and $R = 2.09$ ft., the slope is found to be 0.64 ft. per 1000 ft. *Ans.*

Example 3: Find the proper size for a channel to be excavated in firm gravel, with uniform cross-section and good alignment,

[1] If the water surface approaches much closer to the top, A increases slowly, R decreases rapidly, and with it goes a decrease in v. Thus the Q flowing ($= Av$) actually decreases. See p. 257, last half of Par. 131.

the shape being half of a regular hexagon (Fig. 130) for a discharge of 200 c.f.s. with a slope of 1.5 ft. per mile.

Preliminary Calculations: Since the unknowns include the size and the velocity (which is dependent on the size for a given Q), and since R of the formula and diagram depends on the size, it is convenient to express the Q in terms of R and v.

Then $R = \dfrac{A}{w.p.} = \dfrac{1.299b^2}{3b} = 0.433b$, or

$b = 2.31R$. (Such a relation always exists for a *fixed shape* of cross-section.)

But $Q = 200 = Av = 1.299b^2v = 6.93R^2v$, or $28.85 = R^2v$, for this case.

This expression may be solved for R (whence b and d are obtainable) either by formula and table or by diagram.

Solution by Formula: Substituting for v its value $C\sqrt{Rs}$,

$$28.85 = R^2C\sqrt{Rs}, \text{ or } R^5 = \frac{28.85^2}{0.000284C^2}.$$

The proper value for C in Table XI cannot be found until a roughly approximate value for R is known. Letting $C = 100$ (the usual offhand value), then $R^5 = 293$, or $R =$ (by slide rule) 3.11 ft. For this value Table XI gives (for $s = \dfrac{1.5}{5280} = 0.000284$ or 0.28 ft. per 1000 ft.; and $n = 0.20$) $C = 89$. Then $R^5 = 369$, and $R = 3.26$ ft. This revised value does not change C from 89, and further revision is therefore unnecessary. Hence $b = 2.31 \times 3.26 = 7.53$ ft., and $d = 6.52$ ft. The surface width $= 2b = 15.06$ ft. The area $A = 73.7$ sq. ft. *Ans.*

Solution by Diagram (Fig. 129): For a particular slope, each value of n has a different v for each R. In this problem $R^2v = 28.85$. With $n = 0.020$, and $1000s = 0.284$, start by trying, say, $R = 5$ ft. The corresponding v on the diagram $= 3.6$ ft. per sec. Then $R^2v = 25 \times 3.6$ (and this by inspection is obviously a great deal too much, as R^2v must equal 28.85).

(Try $R = 3$ ft., then $v = 2.6$ ft. per sec., and $R^2v = 23.4$, somewhat too low.)

Now the easy process of guessing may be continued until the desired value for R^2v is closely approximated; or the known laws of variation may be used for guidance as follows.

Since $Q = Av$, and since A varies exactly as d^2 and as R^2, while v varies nearly as $R^{1/2}$, therefore Q varies as $R^{5/2}$ just as for pipes Q varies as $d^{5/2}$ (see p. 226).

Hence to produce a certain small percentage change in Q, or in a constant $\times Q$, R must be changed by two-fifths of that percentage. By slide rule, 28.85 is 23 per cent greater than 23.4. Therefore, in order to increase R^2v from 23.4 to 28.85, increase R by $2/5 \times 23$ per cent $= 9.2$ per cent. Now, try $R = 1.092 \times 3 = 3.28$ ft., for which the *open-channel diagram* shows $v = 2.7$ ft. per sec., and $R^2v = 29.1$ is obtained, which is close enough to 28.85. Hence $R = 3.28$ ft. and $b = 2.31 \times 3.28 = 7.57$ ft. and $2b = 15.14$ ft. *Ans.*

Example 4 (see Fig. 131): Instead of a half hexagon, as in example 3, taking a case where the depth is fixed by engineering considerations at 5 ft., it is required to find the necessary widths at water surface and bottom, the side slopes being 1½ horizontal to 1 vertical. (Note that in this case the *shape* of the cross-section is not fixed, but changes as the width b changes.)

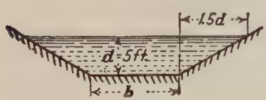


FIG. 131.

Solution: $A = (b + 1.5d)d = 5b + 37.5.$

$$R = \frac{A}{w.p.} = \frac{(b + 1.5d)d}{b + 3.61d} = \frac{5b + 37.5}{b + 18.05}.$$

Try $b = 10$ ft., then $A = 87.5$ sq. ft. and $R = 3.12$ ft. By diagram, $v = 2.65$ ft. per sec. Then $Q = 87.5 \times 2.65 = 232$ c.f.s., about 16 per cent too much.

Here the law of variation of Q with b is not as simple as in example 3. There, b represents two dimensions, but here b is less than one dimension as far as influence on area is concerned. Thus, if b is doubled, the area A is less than doubled. Also, in influencing R, b acts far differently from a linear dimension of a fixed geometric shape. Even if $b = 0$, R still has a value. The upshot of these considerations is that, instead of changing b only two-fifths as many per cent as the desired change in Q, the change in b must be fully as many per cent and possibly more than the desired per cent effect on Q.

Trying b 20 per cent less than the previously assumed 10 ft., or 8 ft., $A = 77.5$ sq. ft., and $R = 2.98$ ft. Then v, from the diagram (Fig. 129), $= 2.6$ ft. per sec., and $Q = 77.5 \times 2.6 = 201$ c.f.s., practically the same as the desired value. Then $b = 8$

ft., and the surface width at water level $= 8 + 15 = 23$ ft. At ground level the width of the cut, of course, would be greater.

Comparing the area of cross-section in examples 3 and 4, it is seen that the latter is about 5 per cent greater for the same discharge.

FIG. 132.

Example 5: Find how much water flows in an unplaned timber 90-deg. V-flume laid on a grade of 2 ft. per 1000 ft., for depths of 3, 3.5, and 4 ft. (See Fig. 132.)

Solution: For a fair job on the timber lining, $n = 0.013$

$A = d^2$. $R = \dfrac{d^2}{2.83d} = \dfrac{d}{2.83}$.

	For $d = 3$ ft.	3.5 ft.	4 ft.
	$A = 9$ sq. ft.	12.25	16 sq. ft.
	$R = 1.06$ ft.	1.24	1.41
(From Table XI)	$C = 117$	120	123
$v = C\sqrt{Rs}$,	$v = 5.39$ ft. per sec.	5.98	6.53 (by slide rule)
$Q = Av$.	$Q = 48.5$ c.f.s.	73.5	104.5 c.f.s.

Using the diagram (Fig. 129) the values of v may be picked off at once for the intersection of $n = 0.013$; and the several computed values of R, for $s = 0.002$ (2 ft. per 1000 ft.). Then the values of Q are computed by $Q = Av$.

Example 6: Observations in the concrete-lined canal of the Cornell University Hydraulic Laboratory showed a depth of 2.80 ft. in the canal for a flow of 398 c.f.s. The canal is rectangular, 16 ft. wide, and the slope of the bottom is 1 ft. in 500 ft. What is the indicated value of Kutter's coefficient of roughness, n?

Solution: First compute the values of v, R, and s from the data. $v = \dfrac{Q}{A} = \dfrac{398}{2.8 \times 16} = \dfrac{398}{44.8} = 8.88$ ft. per sec., $R = \dfrac{A}{w.p.}$ $= \dfrac{44.8}{21.6} = 2.07$ ft., $s = \dfrac{1}{500} = 0.002$, (2 ft. per 1000 ft.).

By formula, $v = C\sqrt{Rs}$, or $8.88 = C\sqrt{2.07 \times 0.002}$, whence $C = 138$. To find n, first locate the columns in Table XI having values of C above and below 138 for $R = 2.07$ ft. and $s = 0.002$ (2 ft. per 1000 ft.) (C being the same as for 1 ft. per

1000 ft. as per note at the bottom of the table). For $n = 0.012$, and $R = 2.07$ ft., $C = 142$ by interpolation between values for $R = 2$ and 3 ft. For $n = 0.013$, same R and s, $C = 131$. By interpolation, for $C = 138$, the desired $n = 0.0124$.

By Kennison's diagram (Fig. 129), for $v = 8.88$ ft. per sec., $s = 0.002$, and $R = 2.07$ ft., it is seen that $n =$ about 0.0123. *Ans.*

(The canal had a smooth concrete bottom with quite rough sides due to the deterioration of the concrete).

Example 7: A stretch of the St. Lawrence River about midway between Montreal and Quebec, just downstream from Lake St. Peter, was found to have surface elevations as follows for a flow of 250,000 c.f.s.: Station Three Rivers, 10.35 ft.; Station Batiscan, 7.99 ft. This is a fall of 2.36 ft. in the 17.48 miles, or 92,300 ft., making $s = 0.0000256$ (or 0.0256 ft. per 1000 ft.). The average cross-sectional area is 158,400 sq. ft., with average width = 5602 ft. This gives an average depth = 28.28 ft., and this may be taken as the value of R for so wide a river. What are the indicated values of C and n? (Tentative data used in 1916 by International Waterways Commission.)

Solution: Average $v = \dfrac{Q}{A} = \dfrac{250{,}000}{158{,}400} = 1.58$ ft. per sec. Hence $1.58 = C\sqrt{28.28 \times 0.0000256}$, whence $C = 58.8$. In Table XI, the corresponding value of n is seen to be greater than the value 0.060 in the last column. But by several trial substitutions in the Kutter's formula itself a value 0.071 yields a check value for $C = 58.8$.

Comments: This very high value for n is probably due to the fact that the river bed is too wide a departure from a channel of reasonably uniform cross-section, containing, as it does, not merely local irregularities, but many "deeps," shallows, bars, etc., so that the average cross-section (from a few measurements) and the average width are not fair measures for the equivalent average R and the average v.

Merely as an indication of possible departure, a value of $R = 12$ ft. goes with $n = 0.030$ for the given v and s. But if R and the depth, proper in a hydraulic sense, are reduced, v is increased. On the diagram, for $n = 0.030$, a value of $R =$ about 20 ft. goes with the given slope and the

velocity corresponding to this value of R and depth. Such a value for n agrees with values found on numerous large rivers.

It must be borne in mind that a river with locally smooth banks and bed may nevertheless be rough in a hydraulic sense if the body of water between any two cross-sections departs too widely from the *prismatic shape* implied in the term "uniform flow."

Example 8: A river, after being improved for navigation, has a reasonably uniform cross-section averaging as shown in Fig. 133. What depth of water may be expected for a flood flow of 150,000 c.f.s. and a slope of 1 ft. in 20,000 ft.? It is estimated that $n = 0.030$ for the deep channel, and 0.040 for the shallow (unimproved) overflow portions.

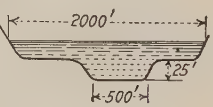


FIG. 133.

Solution: Assuming for a first trial (lacking specific information to the contrary) that all of the water will flow in the deep part, or that $d = 25$ ft., and that $R = d$ closely enough, then (for $n = 0.030$, $s = 0.00005$ (0.05 ft. per 1000 ft.), and $R = 25$ ft.) $v = 3.5$ ft. per sec.

Therefore, $Q = Av = (500 \times 25)3.5 = 43{,}800$ c.f.s., a value considerably too small. Therefore the river will overflow its regular channel.

Trying $d = 35$ ft., then for the deep portion $v = 4.4$ ft. per sec. (by extrapolation on the diagram); and for the shallow portion with $R = 10$ ft., $n = 0.040$, the diagram shows $v = 1.35$ ft. per sec. Then $Q = Q_1 + Q_2 = (500 \times 35)4.4 + (1500 \times 10)1.35 = 77{,}000 + 20{,}000 = 97{,}000$ c.f.s. (still about 35 per cent too small).

Trying $d = 42$ ft. for the deep portion, or 17 ft. for the shallow parts, and resorting to Table XI (since the diagram (Fig. 129) does not extend beyond $R = 30$ ft.), it is found that $C = 108$ for the deep portion and 73 for the shallow parts. Then $v_1 = 108\sqrt{42 \times 0.00005} = 4.95$ ft. per sec., making $Q_1 = 21{,}000 \times 4.95 = 104{,}000$ c.f.s. Also $v_2 = 2.1$ ft. per sec., and $Q_2 = 25{,}500 \times 2.1 = 53{,}000$ c.f.s., so that $Q = 157{,}000$ c.f.s., which is about 5 per cent greater than the given value.

Hence the water will be about 41 ft. deep in the "ship channel" and about 16 ft. deep in the shallow "overflow" parts of the river.

N. B.: In such problems as example 8 it is highly important that the values for n be obtained, if possible; from measurements made on the river in question, and at several stages of the water level. Otherwise, calculations may be far astray from the truth, because n for rivers is not as easily estimated as for artificial canals of smaller size.

Problems

1. For a wide river, such that the hydraulic radius may be taken equal to the depth of water, derive the relation between slope and depth for constant discharge, assuming that the Chézy C remains constant. Then, as a numerical example, if the depth be doubled, what happens to the slope?

Ans. s = one-eighth first value.

2. What size 90-deg., V-shaped flume laid on a grade of 0.4 ft. per 1000 ft. is necessary to carry 100 c.f.s. if $n = 0.013$? *Ans.* Depth 5.2 ft.

3. For an open channel with cross-section of a given *shape* (such as a semicircle, square, 60-deg. V-shape, etc.), how and how much must the linear size of flume and depth of water change if the roughness increases from $n = 0.014$ to 0.016, the slope and discharge remaining the same, assuming that the Chézy $C = \dfrac{(\text{A constant})}{n}$? *Ans.* Increase 5.7 per cent.

4. As an engineer, suddenly called on to give a roughly approximate estimate without reference to tables or diagrams, compute the discharge of a circular sewer 2 ft. in diameter flowing *half full* when laid on a grade of 3.6 ft. per 1000 ft. *Ans.* 6.66 c.f.s.

5. What is the discharge of a circular vitrified pipe sewer, 12 in. in diameter, laid on a uniform slope of 1 ft. in 2000 ft.: (*a*) When flowing full (but not under pressure)? (*b*) When flowing to a depth of 0.933 times diameter? (*c*) When flowing to a depth of 0.25 times diameter?

Ans. (*a*) 0.605 c.f.s.
(*b*) 0.664 c.f.s.
(*c*) 0.0738 c.f.s.

6. (*a*) What size and shape (for minimum amount of lumber) would you make a flume of planed lumber to carry 5 cu. ft. per sec. on a slope of 0.05 per cent? (*b*) If the slope were in error by 5 per cent what would be the approximate error in the discharge for the flume as designed? (*c*) If you are uncertain as to the value of n in Kutter's formula between 0.010 and 0.012, what is the percentage uncertainty in the value of the computed discharge?

Ans. (*a*) Half square, side = 2.04 ft.
(*b*) 2.5 per cent.
(*c*) ?

7. A rectangular flume 6 ft. wide with water 2 ft. deep is to convey 60 c.f.s. Kutter's $n = 0.015$. (*a*) On what slope must the flume be laid? Solve by

table; also by diagram. (*b*) If the slope is one-half the above value, how deep will the water run in the flume for same discharge? Solve as for (*a*).

Ans. (*a*) 2 ft. per 1000 ft.
(*b*) 2.6 ft.

8. (Use open-channel diagram.) In the cross-section of an open channel lined with concrete equivalent to well-laid brickwork, the bottom width of the *trapezoid* is to be double the depth; side slopes 1 to 1. (*a*) Compute the proper value for depth x if 400 c.f.s. are to be carried with "uniform motion" (define it) with a fall of 2 ft. to the mile. (*b*) How much more area of concrete lining (per cent) would be required if for the *one* trapezoidal channel *two* channels, each a half square in section were used, to secure the same flow with the same fall? Let x denote depth in second case.

Ans. (*a*) 5.25 ft.
(*b*) 59 per cent excess.

9. Use diagram concrete-lined flume $n = 0.013$. (*a*) Find proper radius of rounding at bottom so that velocity shall be 2.5 ft. per sec. with water surface as shown in Fig. P.73, for minimum discharge of 7.5 c.f.s. (*b*) On what grade must the flume be laid to secure this result? (*c*) What will be the depth of flow for $Q = 75$ c.f.s. for this flume laid on a grade of 1 ft. per 1000 ft. with radius of bottom rounding = 2 ft.?

Ans. (*a*) 1.36 ft.
(*b*) 0.83 ft. per 1000 ft.
(*c*) 3.6 ft.

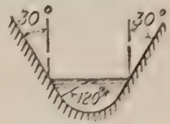


FIG. P.73.

10. (From New York State Service Examination for Junior Engineers, December, 1915.) Find the proper diameter for circular pipes such that *two of them* will just carry the water delivered by an open channel of half-square section 6 ft. wide and 3 ft. deep. Given for both channel and pipes, $v = C\sqrt{Rs}$, $s = 0.0009$, $C = 120$. *Ans.* 3.8 ft.

11. Storage feed canal of Umatilla project, Oregon. (See *Eng. Record*, Oct. 21, 1911, and *Cornell Civil Engineer*, December, 1911.) Semicircular new concrete canal, $d = 9.8$ ft., 2055 ft. long including one 100-ft. radius curve (120 ft. long) and two 50-ft. radius reverse curves. $s = 0.0018$, mean depth = 3.95 ft., $Q = 205$ c.f.s. Check the following values also given in the article: mean area = 28.47 sq. ft.; $R = 2.11$ ft.; $v = 7.20$ ft. per sec.; $C = 117$; $n = 0.0146$.

12. (*a*) A drain of "ordinary sewer pipe" is to be 300 ft. long of 12-in. sewer tile. It is to be laid to such a grade that it will carry off the waste from a manufacturing plant amounting to 175 g.p.m. To allow for future growth, the grade is to be such that the drain will flow just half full with the above discharge. What should be the difference in levels between point of beginning and end of sewer? Solve by formula. Check by diagram. (*b*) What would be the discharge of the drain in (*a*) if it ran "just full" and were laid on a grade of 0.08 ft. in 100 ft.? (*c*) What would

be the discharge of the drain in (a) if it ran with a depth of 10.8 in., laid to a grade of 0.08 ft. in 100 ft.?

13. If water flows 6 in. deep in a stretch of a rectangular canal, 20 ft. wide ($n = 0.013$) where the slope is 1 in 100, how deep does the same stream of water flow where the slope is 1 in 1000? *Ans.* 1.0 ft.

14. Figure P.74 shows a cross-section of a river in moderately good order having occasional stones and weeds. The fall of the river is 0.422 ft. in 2 miles. (A stream of this cross-section should be considered as two streams side by side.) (a) Find the flow of the river in c.f.s. by diagram. (b) Find the flow of the river in c.f.s. by tables. (c) Find the flow of the river in c.f.s. by diagram, considering the stream as *one* channel of the shape shown.

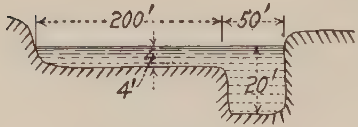


Fig. P.74.

15. (a) (Figure P.75.) With $x = 5$ ft., $y = 5$ ft., $d = 4$ ft., and $b = 10$ ft., on what slope should the canal be dug to carry a discharge of 67.2 c.f.s.? Work by diagram. (b) Work (a) by formula and table.

16. (a) (Figure P.75.) With $x = y$ and $d = b$ and the slope of the canal 0.00025 ft. per ft., compute the size of the canal to discharge 100 c.f.s. Work by diagram. (b) Work (a) by formula and table.

17. (a) (Figure P.75.) With $x = 5$ ft., $y = 7.5$ ft., $b = 5$ ft., and $d = 2$ ft. and canal dug on a slope of 0.5 ft. per 1000 ft., what would be the discharge? Work by diagram. (b) Work (a) by formula and table.

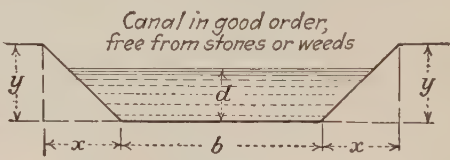


Fig. P.75.

18. An open channel of indefinite length has a bottom slope of 1 ft. in 1000 ft. and is rectangular in section, being of depth 2 ft. and of width 5 ft. It is constructed of unplaned timber. Compute the hydraulic radius, the velocity, and the rate at which water must be supplied at the upper end to keep up a steady flow with all cross-sections full, making no allowance for leakage (or seepage).

19. A canal of trapezoidal section (8 ft. wide on the bottom, 1 on 1 side slopes) is to carry a flow of 300 c.f.s. The canal is to be maintained in moderately good order and regimen. Determine the necessary slope of the canal for a depth of flow of 6 ft.

20. A flume to discharge 40 cu. ft. per sec. is to be built of rough planks on a slope of ½ ft. per 1000 ft. Section is to be rectangular. What should the dimensions be to give the minimum wetted perimeter?

21. For a canal 8000 ft. long with rectangular cross-section of 600 sq. ft., depth 8 ft., average velocity 3 ft. per sec., the coefficient C for Chézy's formula is 60, which can be increased to 100 by improved alignment of the bottom and sides. With net return of developed water power $10 per H.P. per annum, what amount can be expended to advantage on this improvement if interest is figured at 7 per cent per annum?

It is assumed for this problem that the water level at the upper end of the canal remains unchanged and that the discharge is constant. Hence the improvement will *decrease* the necessary surface slope, and the depth at the lower end of the canal will be *increased*, thus giving more head on a power plant located there. *Hint:* First compute the original slope, then the first approximation to the improved slope based on original values of R and v. Next use averages of end values of R and v, and compute the revised improved slope.

Ans. 2.20 ft. head saved; $64,300.

22. Figure P.76 shows a section of a river upon which the following measurements were made: The slope was found to be 0.07 ft. in 1000 ft. The discharge was found to be 3325 c.f.s. (a) Find the coefficient of roughness n for this river. (b) Compute the elevation of the water surface for a flood

Fig. P.76.

condition of 8000 c.f.s., assuming the same value of n as found in (a) and that the surface slope is uniform.

23. Figure P.77 shows the river of Prob. 22, with a center channel dredged as shown. (a) Compute the elevation of the water surface if the flood condition of 8000 c.f.s. should occur after dredging.

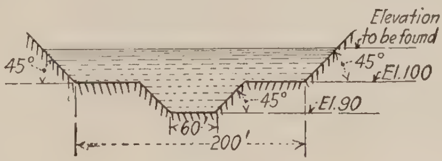


Fig. P.77.

24. In a circular sewer of 10 ft. diameter, flowing nearly full, the water surface is assumed to be of width $AB = 5$ ft. (a) Compute the hydraulic radius. (b) The sewer is laid on a grade of 1 ft. per mile. The mean velocity as measured by current meters is found to be 2.90 ft. per sec. Compute the discharge in cubic feet per second. (c) For flow as in (b), compute the value of C in the Chézy formula. (d) For flow as in (b) compute by table and check by diagram the value of Kutter's n, interpolating as necessary. (e) For depth of flow and grade as in (b), what would the velocity be if n were

0.017? (*f*) Compute velocity and discharge for sewer flowing just flush full. (*g*) If another circular sewer of the same diameter and laid on the same grade flows nearly empty with surface width of water again = 5 ft., and the masonry be of "best" or "well-laid" brickwork, compute the discharge, using diagrams.

Ans. (*a*) 2.91 ft.
(*b*) 222 c.f.s.
(*c*) 123.5.
(*d*) 0.0143.
(*e*) 2.4 ft. per sec.
(*f*) 2.6 ft. per sec., 204 c.f.s.
(*g*) 1.88 c.f.s.

25. (Figure P.78.) The vertical distances given are from water surface to water surface. (*a*) *A-B* is a trapezoidal canal dug in firm gravel with side slopes of 1.5 horizontal to 1 vertical. It is to carry 500 c.f.s. when the depth of water in the canal is 4 ft. Distance *A-B* is 6670 ft. Compute the width of the bottom of canal using Chézy's formula and table of *C*. (*b*) *B-C* is a half-square flume (depth = one-half width) made of well-planed timber

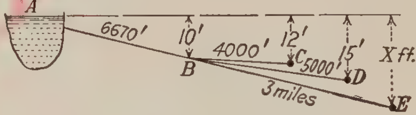


FIG. P.78.

evenly laid with a length of 4000 ft. carrying 100 c.f.s. Compute the size of flume using diagram. (*c*) *B-D* is a line of ordinary sewer pipe which is to run with the depth of water = 0.95 of the diameter when delivering 30 c.f.s. Its length is 5000 ft. Compute the diameter by diagram. (*d*) *B-E* is a semicircular flume of riveted steel with a radius of 4 ft. and carries 370 c.f.s. How far below *A* (X = ?) will be the point *E* distant 3 miles from *B*?

26. If water flows 3 ft. deep in a stretch of a rectangular canal, 20 ft. wide (n = 0.013) where the slope is 1 in 100, how deep does the same stream of water flow where the slope is 1 in 1000? Solve by diagram. *Ans.* 6.5 ft.

27. If a slope of 1 in 2000 is proper for a certain canal when n = 0.017, how many feet error in elevation of canal grade would result in a length of 6 miles if the correct value of n is 0.020 instead of assumed 0.017? Will lower end of canal be too high or too low? (Answer on approximate basis without involving R, table, or diagram. Value of R may be taken = 5 ft., if a value is deemed necessary.) *Ans.* 6.78 ft. too high.

28. (Solve by formula.) What size 90-deg., *V*-flume, n = 0.015, is necessary to carry 1000 c.f.s. on a grade of 0.15 ft. per 1000 ft.?

Ans. 16 ft. deep, 32 ft. top width.

29. What is the value of Kutter's n if the mean velocity in a rectangular canal 50 ft. wide, water 12 ft. deep, with a fall of 15 ft. in 100,000 ft. (= 18.9 miles) is found to be 3.3 ft. per sec. Solve by formula and table of values of Chézy's *C*. Show clearly values used in interpolation. *Ans.* 0.0232.

CHAPTER XVI

VISCOUS FLOW OF OIL AND WATER; ALSO FLOW OF WATER-SAND MIXTURES AND OF GASES IN PIPES

136. Influence of Viscosity on Flow of Oils and Water in Pipes.—*The coefficient of viscosity* (or of internal friction) is the resistance offered by, or the "drag" transmitted through the liquid by, a layer of the liquid of unit area to the motion parallel to this area of another layer of the liquid, at unit distance, moving with unit velocity relatively to the first layer. For centimeter-gram-second units, this coefficient is known as the *absolute viscosity,* being the force in dynes per square centimeter at a velocity of 1 cm. per sec., at a distance of 1 cm. The unit of absolute viscosity is called a *poise.* The viscosity of glycerin and of castor oil is about 1000 times, and that of machine oils from 100 to 300 times as great as that of water at ordinary room temperatures. The viscosity of oils changes much more with change of temperature than does water; water itself changes from 0.0179 poise at 32°F. to 0.0131 at 50°, 0.0100 at 68.4°, 0.0069 at 100°, and 0.0040 at 160°F.

The *change* of viscosity with temperature of typical American mineral lubricating oils is as follows:

Saybolt viscosity at 100°F.	380	760	1200	3300	7500
Saybolt viscosity at 212°F.	68	90	120	210	300

For effect of high pressure see footnote.[1]

Kinematic viscosity is the ratio u/s, where u is the absolute viscosity and s is the specific gravity. In viscous flow, the loss

[1] Recent investigations (published by the British Department of Scientific and Industrial Research) show that the viscosity of water increases but slightly with increase of pressure; for animal and vegetable oils at 5000 (10,000) [15,000] lb. per sq. in. pressure the percentage increase of viscosity is about 75 (200) [350] per cent; for mineral oils at the same pressure the increase is 125 (550) [1600] per cent.

of pressure varies directly as the absolute viscosity, while loss of head varies as u/s.

137. Viscous or "Stream-line" Flow.—If a liquid is sticky towards other substances so that it "wets" or adheres to them, as is the case for water or oil adhering to the inside of a pipe, then any forced movement of the liquid through the filled pipe is resisted by this adhesive grip and by the viscosity of the adjacent layers of liquid, the result being a sort of telescopic sliding motion with the central core moving fastest (at about twice the mean velocity), and the outermost layers not moving at all. Such stream-line or viscous flow takes place with any liquid and in any size of pipe as long as the velocity is not forced beyond the *critical velocity*. For such flow the loss of head along the pipe varies directly as the length, the velocity, and the viscosity; and inversely as the cross-sectional area of the pipe.

Poisseuille's formula is:

$$\frac{h_F}{L} = 0.0326\left(\frac{u}{s}\right)\left(\frac{v}{d^2}\right), \text{ c.g.s. units,} \tag{74}$$

where h is the loss of head in length L, u is the absolute viscosity, s the specific gravity, v the velocity, and d the diameter. Consequently, $p = 27.3\,\frac{uQ}{d^4}$, where p is the loss of pressure in pounds per square inch per 1000 ft. of pipe, Q is the volume of flow in U. S. gallons per minute, d the diameter in inches, and u the absolute viscosity. For Q in barrels per hour (1 bbl. = 42 U. S. gal.), the constant is 19.1. The roughness of the pipe has practically no effect on true viscous flow, at least for the range of roughness likely to be met with in oil pipe lines.

138. The Critical Velocity, and Turbulent or Eddy Flow.—When the rate of flow is forced beyond the critical velocity, turbulent or eddy flow sets in. The influence of viscosity is still evident, as is shown by the distribution of the forward velocities (see Fig. 170, p. 336), but there is now also to some extent an actual bodily sliding through the pipe of a turbulent mass of liquid. The loss of head along the pipe for such turbulent flow varies as $v^{1.75}$ for very smooth pipes and up to about v^2 for quite rough pipes; also it no longer varies inversely as the cross-section, as for stream-line flow, but more nearly inversely as the circumference (or the rubbing perimeter), actually as about $\frac{1}{(\pi d)^{1.25}}$; also it

now varies as a power of the viscosity much less than 1, probably about 0.25, although experiments in some cases seem to indicate values as high as 0.50.

The *formula for the turbulent flow of water in fairly smooth pipes*, h_F per 1000 ft. $= \frac{0.38v^{1.86}}{d^{1.25}}$ on which the diagram (Figs. 117*A* and 117*B*) is based, becomes, for an oil pipe,

$$h_F = 2\left(\frac{u}{s}\right)^{0.25}\frac{0.38v^{1.86}}{d^{1.25}},$$

when the hidden viscosity term is exposed (using the value for water at 32°F. = 0.0179, and reducing the constant of the formula about 25 per cent because of better hydraulic conditions in a fairly new or a well-maintained old oil pipe line than in an average 10-year-old water main), *i.e.*, the friction-head values of the diagram are to be multiplied by $2\left(\frac{u}{s}\right)^{0.25}$ or $2\sqrt{\sqrt{\frac{u}{s}}}$, an easy slide-rule operation.

To cover uncertain temperature of highly viscous oils, slight deposits in the pipe, etc., the head values of Figs. 117*A* and 117*B* may be multiplied by $2.5\sqrt{\sqrt{\frac{u}{s}}}$.

139. Reynold's Criterion.—Whether the flow in a pipe takes place above or below the critical velocity is usually judged by Reynold's criterion,

$$\frac{vds}{u},$$

where v is the mean velocity, d the diameter, s the specific gravity, and u the absolute viscosity of the liquid in the pipe. There is viscous or stream-line flow if this criterion has a value in centimeter-gram-second units less than about 2000, or if $\frac{Qs}{du}$ is less than about 1570, where Q is cubic centimeters per second. For customary engineering units $\frac{Qs}{du}$ at the critical velocity has a value about 63, Q being the discharge in U. S. gallons per minute, d the diameter in inches, s and u as above. For Q in barrels per hour (1 bbl. = 42 U. S. gal.), the criterion value is about 90. These values may be too high for oils. Tests reported in *Lubri-*

cation, July, 1920, show the criterion value $R = \frac{700}{u^{0.23}}$, giving 2000 for $u = 0.01$, 1200 for $u = 0.1$, and 700 for $u = 1$, c.g.s. units. The critical velocity is not a definite velocity but a region of transition from stream-line or viscous flow to turbulent or eddy flow (see Fig. 119*B*).

140. Viscosimeters.—Viscosity is measured by observing the time required for a certain volume of the liquid to flow, under stated conditions as to head through a short tube of small bore. The *Saybolt Universal viscosimeter* has a vertical tube 0.483 ± 0.004 in. long and 0.0625 ± 0.0006 in. in diameter. For heavy oils the *Saybolt-Furol viscosimeter* is used; it differs from the Saybolt in having a tube diameter of 0.1240 ± 0.0008 in. The time of efflux of 60 c.c. in seconds is called the viscosity in "seconds Saybolt." The time of the efflux of the Furol is approximately one-tenth that of the Universal.

Viscosities of lubricants are measured at 100 and 210°F.; of fuel oils at 77 and 122°F.

The value of kinematic viscosity u/s can be obtained from the indications in seconds t of various viscosimeters by the following equations:

Saybolt Universal viscosimeter, $\frac{u}{s} = 0.00220t - \frac{1.80}{t}$.

Redwood (English) viscosimeter, $\frac{u}{s} = 0.00260t - \frac{1.715}{t}$.

Engler (German) viscosimeter, $\frac{u}{s} = 0.00147t - \frac{3.74}{t}$.

Redwood Admiralty (English) viscosimeter, $\frac{u}{s} = 0.027t - \frac{20}{t}$.

141. Examples: *To Find Frictional Loss of Pressure.*—It is required to pump 100 bbl. per hr. of an oil of 20°Bé. gravity and 250 sec. Saybolt viscosity through a 3-in. pipe. What pump pressure per 1000 ft. of pipe must be provided?

Solution: The specific gravity for 20°Bé. is 0.933, and by the formula above the absolute viscosity is 0.51. The value of Reynold's criterion $\frac{Qs}{du}$ for these units = 61. This being less than 90, viscous flow is indicated. From $p = 19.1\frac{uQ}{d^4}$ (see

Par. 137), $p = 12.0$ lb. per sq. in. per 1000 ft., or 63 lb. per mile. If, for such a criterion value, the possibility of turbulent flow is suspected (for formula, see Par. 138), it is found from the diagram (Fig. 117) (for $Q = 100 \times 0.00156 = 0.156$ c.f.s. and $d =$ 3 in.), $h_F = 19$ ft. per 1000 ft. and $2\sqrt{\sqrt{\frac{u}{s}}} \times 19 = 1.72 \times 19 =$ 32.7 ft. of oil, or $\frac{(32.7 \times 0.93)}{2.31} = 13.2$ lb. per sq. in. per 1000 ft. *Ans.*

To Find Proper Diameter.—In pumping 1300 bbl. per hr. of an oil of specific gravity 0.9 and absolute viscosity 0.7, what is the minimum diameter of pipe permissible, if a friction loss of pressure of 160 lb. per sq. in. per mile is not to be exceeded?

Solution: Here Reynold's criterion $= \frac{(1300 \times 0.9)}{0.7d} = \frac{1670}{d}$, and for any probable value of d smaller than 20 in., turbulent flow is indicated; 160 lb. per sq. in. per mile or 30.3 lb. per 1000 ft. is $\frac{(30.3 \times 2.31)}{0.9} = 78$ ft. (of oil head) per 1000 ft., and $\frac{78}{2 \times \left(\frac{0.7}{0.9}\right)^{0.25}}$ $= \frac{78}{1.88} = 41.5$, and for this friction loss in feet per 1000 ft. Fig. 117 shows (for $1300 \times 0.00156 = 2.03$ c.f.s.) a 7-in. pipe. *Ans.* Using 2.5 instead of 2, looking on the diagram with 33 ft. per 1000 ft., a 7-in. pipe, or, to be quite safe, an 8-in. pipe is found. *Ans.*

If the oil were quite viscous, say 16,000 sec. Saybolt ($u = 34$), specific gravity being 0.97 (14°Bé.), viscous flow is indicated by Reynold's criterion. From $p = 19.1\frac{uQ}{d^4}$, or $d^4 = 19.1\frac{uQ}{p}$, $d =$ 12.9 in., indicating the necessity of a 13- or a 14-in. pipe. *Ans.*

To Find Discharging Capacity.—For 15 miles between pumping stations with working pressures of 900 lb. per sq. in., what is the rate of flow for an 8-in. pipe, for an oil of specific gravity 0.92 and absolute viscosity 2.0?

Solution: The criterion $\frac{Qs}{du} = \frac{Q}{17.4}$. If, as a guess, Q is some 1000 bbl. per hr., the criterion value is about 58, indicating viscous flow. From $p = 19.1\frac{uQ}{d^4}$, or $Q = \frac{pd^4}{19.1u}$, $Q = 1220$ bbl. per

hr., p being 11.36 lb. per sq. in. per 1000 ft. The revised criterion value is 70, rather close to the critical velocity value.

If p were 20 lb. per sq. in. per 1000 ft., it would be probably a case of turbulent flow. With $\frac{(20 \times 2.31)}{0.92}$ or 50 ft. (of oil head) per 1000 ft., or $\frac{50}{2\sqrt{\sqrt{\frac{2}{0.92}}}} = 20.6$ ft. per 1000 ft. on Fig. 117 for 8-in. pipe, $Q = 2.3$ c.f.s. $= 2.3 \times 641 = 1470$ bbl. per hr. The criterion value is 80.5, so close to 90 that there would be no doubt as to whether viscous or turbulent flow would prevail. The viscous-flow formula shows 2140 bbl. per hr., or about 45 per cent greater than by the turbulent-flow formula. In a case of this sort, unless reliable tests on an oil of similar viscosity can be used as guides, it would be better to take the smaller value as the dependable capacity. This is particularly advisable since certain tests indicate that the critical velocity becomes less as the viscosity increases. (See *Lubrication*, July, 1920, issued by the Texas Company, New York.)

142. Flow of water in very small tubes follows a different law from that for ordinary-size pipes. The friction-head varies as the velocity, and is very sensitive to the temperature of the water. The experiments of Poisseuille (1846), Hagen (1854), Reynolds (1883), Coker and Clement (1903), and Saph and Schoder (1903) on glass, brass, and lead tubes with internal diameters ranging from 0.001 to 0.631 in. show an average value of about 500 for C in the formula $v = C\left(\frac{h_F}{L}\right)d^2\frac{(t+10)}{60}$, where h_F and L are friction-head and length, respectively, both expressed in same units, d the diameter in inches, t the temperature of the water in degrees Fahrenheit, and v the velocity in feet per second. The factor $\frac{(t+10)}{60}$ takes care of the variations in viscosity, making a more convenient formula for water than that of Poisseuille given in Par. 137. For the larger tubes this formula holds only at very low velocities, *viz.*, the velocity must be less than about $v = \frac{0.37}{d^{0.85}}$, v in feet per second, d in inches. This gives roughly the *critical velocity*, above which the flow changes from

"stream-line" to "eddy" motion and the friction-head varies as a higher power of v. The critical velocities from the above formula for various diameters are: 1 in., 0.37 ft. per sec.; ½ in., 0.66; ¼ in., 1.2; ⅛ in., 2.2; $\frac{1}{16}$ in., 4.0; $\frac{1}{32}$ in., 7.1. These are for usual temperatures (see Reynold's criterion (Par. 139)).

143. Flow of Light Crude Oils in Pipes.—The resistance to the flow of oils lighter than about 30°Bé. (sp. gr., 0.875) is not much different from that of water. The value 96 for C in the Chézy formula ($v = C\sqrt{Rs}$) is used in the oil fields in designing pipe lines for crude oil (42 to 43°Bé., sp. gr., 0.814 to 0.809). One per cent is added for each 3°Bé., *i.e.*, lighter oils flow more easily. As pumping is universal, diagrams based on the more practical form of the formula, *viz.*, $Q = 1.125d^{2.5}\sqrt{\frac{p}{L}}$ are used, in which Q is expressed in barrels (of 42 U. S. gal.) per hour and d in inches; p is the necessary pump pressure in pounds per square inch and L the length of line in miles. Friction increases with cold oils in winter and decreases in summer. Deposits of paraffin in the pipes conveying crude oil reduce the effective diameter and increase the friction. "Scrapers" are therefore driven through the pipes periodically to clean away the paraffin.

144. Flow of Gas or Air in Pipes.—For the ordinary flow of gas or air through pipes, whether highly compressed or not, where the total fall in pressure along the pipe line is only a few per cent of the total pressure, *experiments show the* formula $h_F = f\frac{L}{d}\frac{v^2}{2g}$ may be used. The values for the friction factor f may be found from the diagram (Fig. 107) for the class of pipe used. For ordinary practice the values for fairly smooth pipe are safe. It may be noted that this is the identical formula used for the turbulent flow of water, and derived from fundamental experimentally found laws applying to all fluids. Note, however, that the h_F is measured in feet height of the air or gas considered. (All other units are usually in the foot-second system as with water.)

If the fall in pressure is required in feet height of water, the value would be $h_F \times \frac{w_g}{w}$, where w_g = weight of a cubic foot of

gas and w equals the weight of a cubic foot of water. To convert to drop in pressure in pounds per square inch:

h_F in feet height of gas is equivalent to $\frac{h_F \times w_g}{144}$ lb. per sq. in.

When the total fall in pressure is relatively large or the temperature varies considerably, the difference in volumes according to Boyle's law must be taken into consideration. Although the *weight* of gas flowing per second is constant, the *volume per second* will vary along the pipe, and the velocity will increase as the pressure drops and the gas expands.

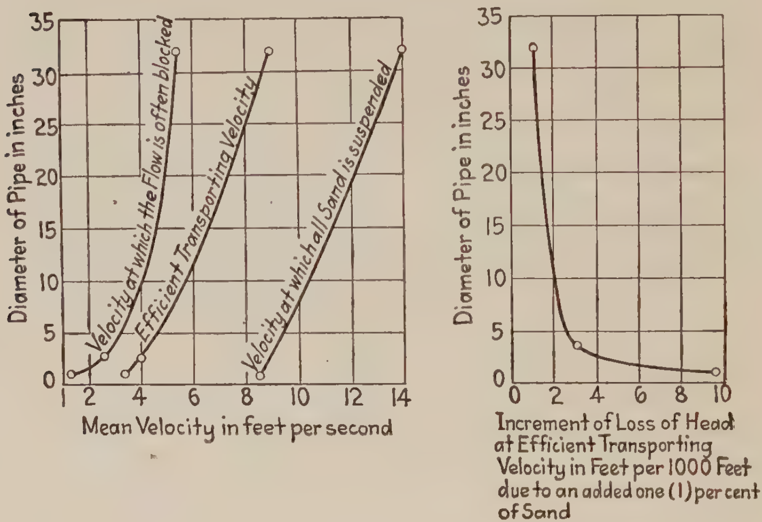


FIG. 133*A*.—Flow of water carryings and in pipes. (*From Trans. Am. Soc. C. E., Vol.* 57 (1906).)

145. Flow of Water Carrying Sand in Pipes.—In dredging and hydraulic excavation where the water flowing in a pipe carries sand (or other material), it is frequently necessary to make an estimate of the head lost due to friction of the combined material.

Experiments[1] show that three distinct kinds of flow exist:

1. When the velocities fall below a certain minimum the pipes frequently become blocked. This minimum velocity varies with the diameter of the pipe and with the kind of material being

[1] A compact discussion is given by Nora Stanton Blatch in *Trans. Am. Soc. C.E.*, Vol. 57, (Dec., 1906), p. 400–408.

transported. Figure 133*A* shows the average minimum velocities for coarse sand for various sizes of pipe.

2. For velocities above this minimum and up to an *efficient transporting velocity,* which varies from 3½ to 9 ft. per sec. (the lower value being for a 1-in. pipe and the upper for a 32-in. pipe), the increase in friction loss appears to be almost independent of the velocity but to vary with the per cent. of sand carried. At the efficient transporting velocity the loss of head due to friction is approximately given by the formula: Total loss in head in feet per 1000 ft. = loss of head due to water alone at the efficient transporting velocity *plus* a constant times the per cent. of sand carried. This constant is given in Fig. 133*A*. (Due regard must be given to the fact that this curve (Fig. 133*A*) is based on three sets of experiments only, and is therefore subject to further experimental verification.)

The efficient transporting velocity for one size of pipe is that velocity at which any given volume of sand per hour can be transported through a pipe with the least expenditure of power per unit volume of sand carried.

3. Above the efficient transporting velocity the friction loss increases over that for water alone at a rate depending directly on the per cent of sand carried. For rough estimates, the loss may be taken to vary with the square of the velocity starting with that which occurs at the efficient transporting velocity and calculated by the approximate rule in item 2 above.

CHAPTER XVII

DEVIATED FLOW—NOZZLE-TYPE WATER TURBINES

146. Pressure Due to Deviated Flow.—The case first considered is that of a free liquid stream turned aside from a straight path by a fixed (non-moving) solid, across the surface of which the liquid glides in a curved path with very little friction.

The change in direction of the stream is assumed to be brought about gradually rather than abruptly, consequently with very little or no back splash. Hence the liquid flows in the new direction with practically undiminished velocity.

These assumed ideal conditions are met, very nearly, by the flow in an easy bend or a smooth curve in a water pipe, hose line, or open channel; and by the action of a jet from a nozzle on a bucket or vane (stationary) of a Pelton-type of water wheel, the so-called "nozzle" or "impulse" wheels.

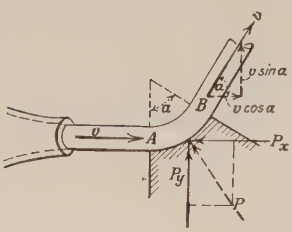


FIG. 134.

To turn aside a moving mass requires a force, because the *velocity in the original direction is changed.* In the case shown in Fig. 134, the force is the pressure of the fixed solid against the stream of liquid. For convenience, this force (resultant of pressures all along the solid) has been divided into two components P_x and P_y.

N. B.—The pressure of the water against the solid *is not* a push from the water approaching the curve, but is due solely to the mass of water at any instant passing over and being deviated by the solid.

If t represents the time required for the average particle of water to travel across the solid, and Q the volume rate of steady flow, then the mass of water actually being deviated at any instant is $\frac{Qw}{g}t$.

By Newton's law, Force = Mass × Acceleration; *i.e.*, the force in a given direction equals the product of the mass by the rate of change of velocity *in that direction*. In this case each particle of the flowing water in time t has its direction changed by angle a, and its velocity *in the original direction* is reduced from v to $v \cos a$ (see Fig. 134).

Hence the average acceleration (here retardation) $= \dfrac{v - v \cos a}{t}$

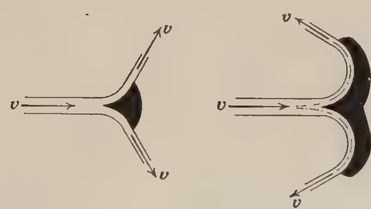
$P_x =$ Mass × Acceleration in the x direction $= \dfrac{Qwt}{g} \times \dfrac{v - v \cos a}{t}$.

or

$$P_x = (1 - \cos a)\frac{Qw}{g}v. \tag{77}$$

Similarly, at right angles to the original direction of the stream, the velocity is changed in time t from zero to $v \sin a$. Hence,

$$P_y = (\sin a)\frac{Qw}{g}v. \tag{78}$$



FIGS. 135 and 136.—Stationary vanes.

If A is the cross-sectional area of the stream before turning, then, since $Q = Av$, Eq. (77) may be written:

$$P_x = 2(1 - \cos a)A\frac{v^2}{2g}w, \tag{79}$$

which is the weight of a prism of liquid of cross-section A and of height $2(1 - \cos a)\frac{v^2}{2g}$. If $a = 90$ deg., $P_x = 2Aw\frac{v^2}{2g}$, or *twice* the pressure corresponding to the velocity-head considered acting hydrostatically on an area equal to the cross-section of the stream.

For $a = 180$ deg., P_x is a maximum, or

$$P_x = 2\frac{Qw}{g}v = 4Aw\frac{v^2}{2g}. \tag{80}$$

For solids shaped as in Figs. 135 and 136 with a "splitter," the y components neutralize one another due to the symmetrically divided stream.

Example: A 30-in. pipe on a trestle carries 60 c.f.s. water. It is required to compute the components of the pressure due to the deviation of the flow caused by a 45-deg. bend in the pipe.

Solution: The mean velocity $v = \frac{Q}{A} = \frac{60}{4.91} = 12.2$ ft. per sec. $P_x = \frac{60 \times 62.4}{32.2} \times 12.2(1 - 0.707) = 1420 \times 0.293$

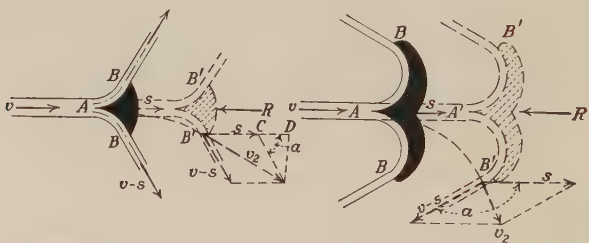
$= 416$ lb. *Ans.*

$$P_Y = \frac{60 \times 62.4}{32.2} \times 12.2 \times 0.707 = 1004 \text{ lb.} \quad \textit{Ans.}$$

$F \times \frac{L}{T}$

147. Pressure and Power Due to Jet Deviated by Moving Vane.—Since Power = Force × Velocity, it is obvious that a vane deflecting a stream can yield no power unless the vane itself moves away from the jet. It must move more slowly, however, than the water in the jet, otherwise none of the water could overtake the vane and be deflected by it.

In Figs. 137 and 138 vanes arranged to split the jet are shown moving with a velocity s away from the jets whose velocity $= v$.



Figs. 137 and 138.—Moving vanes.

At the point A the water is moving faster than the vane, and has a velocity *relatively to the moving vane* $(v - s)$, and at B the relative velocity is nearly the same, since friction is assumed to be very small.[1]

While the water is flowing from A to B, the vane moves ahead to position B', and the actual absolute path of the water is as shown by the curved dashed line AB' in the lower halves of Figs. 137 and 138. Note that this absolute path has an easier curvature than does the bucket.

In Fig. 137 it is seen that, although the water leaving the vane has a velocity *relatively to the vane* $= (v - s)$, the vane itself is moving forward with a velocity s, so that the *absolute velocity*

[1] The arrow marked R designates *a resistance*, or *load* (so many lbs.) which (if not too great) permits the vane, or bucket, to move but prevents it from accelerating beyond the speed s.

(or velocity relatively to the earth) of the water is shown by the diagonal v_2 of the parallelogram of exit velocities.

The velocity of the water has been reduced in the original direction of the jet from v to the projection of v_2, or to $B'D = s + \overline{CD} = s + (v - s) \cos a$; *i.e.*, the change (reduction) *in velocity* is $v - [s + (v - s) \cos a] = (v - s)(1 - \cos a)$.

If, as on page 284, it takes a time t for water to travel across the vane from A to B, the mass of water on the vane at any instant is $\frac{Q'wt}{g}$, where Q' is less than the Q from the nozzle, since one moving vane cannot intercept all of Q. But if a series of vanes, or buckets, be arranged on the circumference of a wheel (Fig. 139), all of the water coming from the nozzle will be intercepted, subject to proper design. Such water wheels are called *impulse wheels*. See also Fig. 143.

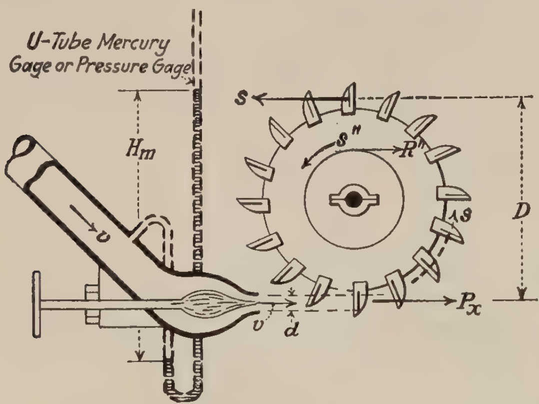


FIG. 139.

The average pressure P_x of the jet against the rim buckets will be

$$\frac{(v - s)(1 - \cos a)}{t} \frac{Qwt}{g} = (1 - \cos a) \frac{Qw}{g} (v - s)$$

and ideally the *power*, which $= P_x s = (1 - \cos a) \frac{Qw}{g} (v - s)s$. (81)

For a fixed head of water at the power plant and at the nozzle, the value of the jet velocity v also is practically fixed, but the speed s of the wheel depends on the resistance overcome, R'' of Fig. 139, due to some kind of load on the wheel such as a brake-band friction, belt tension, gear pressure,

armature drag, or other analogous resistance. If, as is usual, the wheel speed s is also fixed by the requirements of efficiency (as will be shown presently) and of constant electrical voltage from a driven generator, then the variation in the power supplied by the wheel must be provided by variations in the discharge Q acting on the wheel. To accomplish this, the wheel governor either deflects the nozzle slightly so that some of the jet does not strike the buckets, or moves the regulating needle (Figs. 139 and 141) so as to change the size of the jet, or does both.

Evidently, with a given jet-velocity v, the power as given by Eq. (81) is a maximum when $(v - s)s$, or $vs - s^2$ is a maximum. Placing the first differential of this with respect to s equal to zero, then $v - 2s = 0$, or $v = 2s$, or $s = \frac{v}{2}$, *i.e.*, for maximum power the speed of the buckets should be half of the velocity of the jet. Substituting $\frac{v}{2}$ for s in Eq. (81), maximum power $= \frac{1}{2}(1 - \cos a)Qw\frac{v^2}{2g}$. If a is made nearly 180 deg., or $\cos a$ nearly -1, power $= Qw\frac{v^2}{2g}$. If h is the total head acting on the nozzle, this would be Qwh, if there were no friction.

Actually, for the velocity of the jet from water-wheel nozzles, $v = 0.95$ to $0.99\sqrt{2gh}$. Other minor departures from ideal conditions include: (1) The "splitter" of the bucket causes some shock due to the abrupt change sidewise of a few degrees in the direction of the jet. (2) The tip of the bucket cuts into the jet before the bucket path is tangent to the jet axis. (3) The curved surface of the bucket offers some friction. As a net result of such departures from the ideal assumptions in the theory given above, the actual ratio of s to $\sqrt{2gh}$ (known in practice as the "speed-ratio," or ϕ) for "best" speed (that of greatest efficiency) of Pelton or nozzle type of wheels ranges from 0.43 to 0.47, averaging about 0.45.

Figure 142 is a typical curve showing how the horsepower and efficiency vary as the speed increases from zero to runaway (no-load) speed.

Note that the expression "speed ratio" does not mean the ratio of s to v, but means the ratio of s to $\sqrt{2gh}$, or the ratio of the mean bucket speed (for other types of wheels, the rim speed

of wheel) to the ideal spouting velocity of the water under the acting head on the nozzle.

Referring to Fig. 138, the kinetic energy of the water, as it leaves the buckets with the absolute velocity v_2, represents lost power, and $\frac{v_2^2}{2g}$ represents lost head as far as useful power is concerned. The smaller v_2 can be made, the greater is the efficiency of the wheel. But the angle a cannot be made 180 deg. because there must be room for the escaping water (at the relative velocity about $\frac{v}{2}$) without interference with succeeding buckets. Figure 140 shows horizontal sections through jet and bucket, and shows diagrammatically how the wheel speed

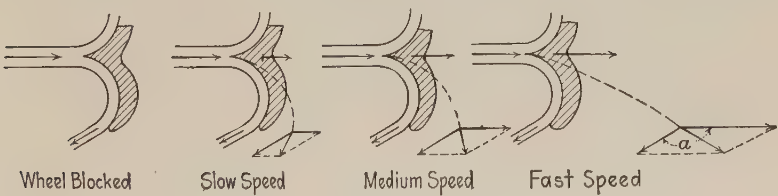


FIG. 140.

affects the value of v_2. If h represents the total head at base of nozzle referred to the level of jet, then the head available to the wheel for power[1] and losses in nozzle and bucket $= h - \frac{v_2^2}{2g}$.

In actual construction the splitter angle of the bucket is from 14 to 30 deg., each half of the jet being abruptly deviated half of this angle. The radial length of the bucket is from 2.3 to 2.9 $\times$ d, where d is the diameter of the jet. The whole width of the bucket is from 3.1 to $4d$, each half taking care of half of the jet. The exit angle of the bucket is made so that the angle a is from 4 to 7 deg. less than 180 deg., as far as mechanical layout is concerned (see Fig. 143).

Example: With 240-ft. total head, and allowing 15 ft. for loss by friction in a long supply pipe, what diameter is implied for an impulse wheel to run at 360 r.p.m. "best" speed?

Solution: Net head at wheel $= 240 - 15 = 225$ ft. Ideal spouting velocity $= \sqrt{2gh} = 120.3$ ft. per sec. Assuming speed ratio $= 0.47$, the peripheral bucket velocity $= 0.47 \times$

[1] The corresponding power is $Qw\left(h - \frac{v_2^2}{2g}\right)$. To get the *useful power*, available on the shaft, this must be decreased by losses due to mechanical friction and air resistance in addition to losses in nozzle and bucket, the total decrease being usually about 10 to 15 per cent.

$120.3 = 56.5$ ft. per sec. But rim velocity = circumference $\times$ r.p.s., or $56.5 = \pi D \times {}^{360}\!/_{60}$, whence $D = 3.0$ ft. *Ans.* This is not the extreme diameter of the runner but is the diameter on center line of jet.

The maximum permissible size of jet from the nozzle, and consequently the maximum power obtainable from such a wheel, are discussed on p. 292.

GENERAL CHARACTERISTICS OF WATER WHEELS

148. General.—To enable the engineer to select and specify intelligently concerning water wheels, useful relations between power, size, speed, etc. are obtainable from a consideration of the geometric and hydraulics properties. For this purpose a further examination will be made of the Pelton, or nozzle, type of water wheels (the so-called impulse wheels). Practically all of what immediately follows applies also to the turbine type of water wheel.

In planning a water power development, the main performance data of the water wheel are: (*a*) the *head* under which the wheel operates; (*b*) the useful *horsepower*[1] given by the wheel; and (*c*) the *speed* (revolutions per minute) at which the wheel is to operate (which speed is usually the speed of maximum efficiency).

149. Characteristic or Type Indexes. *Characteristic (or Specific) Speed.*—Assuming that the performance of a water wheel built after a certain model is known, it is established both by theory and experience that all wheels geometrically similar to this wheel are definitely related to it and to one another in power and speed, taking due account of the head under which they operate. It is the purpose of the following discussion to derive a distinctive index that shall clearly characterize all wheels made according to a certain model.

Suppose that a certain wheel operates under a head of h ft., giving (hp.) horsepower at speed of n, revolutions per minute (usually taken as the speed for maximum efficiency unless otherwise stated). Let D be the diameter of the wheel, d the diameter of the jet, in case of a nozzle wheel, and Q the discharge.

[1] Horsepower (on the shaft) $= e\dfrac{Qwh}{550}$ (ft.-lb.-sec. units), where e is the efficiency found by test.

If the head on the wheel in question be reduced to 1 ft., *the discharge* will change according to the law for the flow through orifices. Hence, if the head be divided by h (to get 1 ft.), the discharge is divided by $\sqrt{h}$, so that Q_1 (the discharge under 1 ft. head) $= \dfrac{Q}{\sqrt{h}}$.

The *power* varies as $Q \times h$. Hence, $(\text{hp.})_1$ (= the horsepower under 1-ft. head) $= \dfrac{(\text{hp.})}{\sqrt{h} \times h} = \dfrac{(\text{hp.})}{h^{3/2}}$.

The velocity of the water and the speed of the wheel both vary as $\sqrt{h}$, so that n_1(= the speed under 1-ft. head) $= \dfrac{n}{\sqrt{h}}$.

If, now, still under the 1-ft. head, the wheel be changed in size so that it gives 1 hp. instead of $(\text{hp.})_1$, obviously this is because the discharge has been reduced in the same proportion, because if the head remains constant the power varies directly as the discharge. But the discharge, in turn (under a constant head), varies as the cross-sectional area of the stream, which, for geometric similarity, varies as d^2 and as D^2, since d varies as D; or *vice versa*, D varies as $\sqrt{Q}$ or as $\sqrt{\text{hp.}}$

Consequently, the reduction of horsepower from $(\text{hp.})_1$ to 1, demands reduction of size from D to $\dfrac{D}{\sqrt{(\text{hp.})_1}}$ (head remaining constant at 1 ft.).

The rim speed (peripheral linear speed s), remains constant under constant head and speed ratio. Hence the *speed in revolutions per minute varies inversely* as the *circumference* or *inversely* as the *diameter*. Thus the reduction of horsepower from $(\text{hp.})_1$ to 1, which requires *reduction* of the area of the stream in the same proportion (*division* by the value $(\text{hp.})_1$), or *division* of the diameter by $\sqrt{(\text{hp.})_1}$, also requires *increase* of revolutions per minute from n_1 to $\sqrt{(\text{hp.})_1} \times n_1$.

Then the new speed $= \sqrt{\dfrac{(\text{hp.})}{h^{3/2}}} \times \dfrac{n}{\sqrt{h}} = \dfrac{\sqrt{\text{hp.}}}{h^{5/4}} n.$

This calculated index speed is called the "characteristic speed" N_s. In usual symbols, characteristic speed

$$N_s = \frac{\sqrt{\text{hp.}}}{h^{5/4}} \times (\text{r.p.m.}). \tag{82}$$

This is an index speed in revolutions per minute common to all

wheels built after the particular model, and involving the other main performance data, *viz.*, power and head.

In words, "*characteristic speed*" *is the speed in revolutions per minute at which a model of the type in question would run under a head of* 1 *ft. when of such a size as to give* 1 *hp.* It is based on the actual performance of a similar wheel and on simple geometric and hydraulic relations.

Example: What is the "characteristic speed" of a Pelton wheel which operates under a head of 1900 ft. at 375 r.p.m. and delivers 10,000 hp.? $N_s = 3.0$. *Ans.*

150. Power Limitations of Nozzle Water Wheels.—The significance of the general index expression derived in the previous paragraph will be made clearer by obtaining it in a less formal manner by dealing directly with the rather simple case of a nozzle water wheel.

It has been found by experience that the diameter d of the jet cannot be made larger than about $\frac{D}{9}$ (where D is the diameter of c. to center of buckets the wheel) without considerable reduction in efficiency.

As stated in Par. 147, the radial length of a well-designed bucket is a little less than $3d$, so that, with D (on center line of buckets) $= 9d$, nearly half of the wheel's extreme radius is occupied by the bucket, and there is a very considerable departure from straight-line motion of the bucket away from the jet. Accordingly, still larger buckets are ruled out.

Ordinarily, the commercial upper limit for the jet diameter is $\frac{D}{12}$ and the more usual value is $\frac{D}{15}$ to $\frac{D}{20}$ for jet with full gate opening (regulating needle drawn back to the limit) (see Fig. 141).

Starting with this practical limitation in discharging capacity, the corresponding upper limit of the index value, or "characteristic speed," for a nozzle type of wheel may be calculated.

For assumed 80 per cent efficiency of the water wheel at best speed (although 85 to 90 per cent are realizable in large plants carefully designed), the shaft hp. $= 0.8\frac{Qwh}{550} = \frac{Qh}{11}$ (foot-pound-second units). But $Q = Av = \left(\frac{\pi}{4}d^2\right)0.98\sqrt{2gh} = 6.17d^2\sqrt{h}$, for d and h in ft. If, for practical reasons, as stated above, d cannot

be made larger than $\frac{D}{9}$, this means that Q cannot be larger than $0.0762D^2\sqrt{h}$. Consequently, the hp. (which $= \frac{Qh}{11}$ at 80 per cent efficiency) cannot be greater than about $0.0069D^2h^{3/2}$, or, reversing the relation, the wheel diameter D cannot be less than about $\sqrt{\frac{\text{hp.}}{0.0069h^{3/2}}}$ or about $12\frac{\sqrt{\text{hp.}}}{h^{3/4}}$.

For a best-speed ratio of 0.47 (see Par. 147), the peripheral or "rim" speed of the wheel is $0.47\sqrt{2gh}$, and the r.p.m. $= \frac{0.47\sqrt{2gh} \times 60}{\pi D} = 72\frac{h^{1/2}}{D}$. But, if D must not be *less* than about

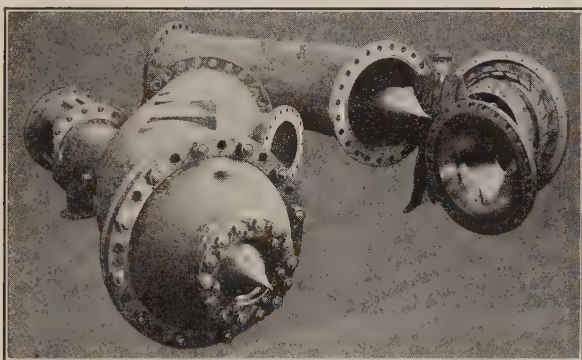


Fig. 141.—Large needle nozzles for Great Western Power Co.'s Caribou Plant, California. (*Courtesy Allis-Chalmers Mfg. Co.*)

$12\frac{\sqrt{\text{hp.}}}{h^{3/4}}$, the r.p.m. should not be *greater* than $72\frac{h^{1/2} \times h^{3/4}}{12\sqrt{\text{hp.}}} = 6\frac{h^{5/4}}{\sqrt{\text{hp.}}}$; *i.e.*, the *index value* 6 should not be exceeded by $\frac{\sqrt{\text{hp.}}}{h^{5/4}} \times$ r.p.m. In condensed form, if r.p.m. is not greater than $6\frac{h^{5/4}}{\sqrt{\text{hp}}}$, then $\frac{\sqrt{\text{hp.}}}{h^{5/4}} \times$ r.p.m. is not greater than 6 (all foot-pound-second units). In this expression, hp., h, and r.p.m. refer to the *actual* horsepower, head, and speed of the wheel.

Lower index values, *i.e.*, lower characteristic speeds than 6, namely, 3.5 to 4.5, correspond to the more usual maximum jet diameters of engineering practice, $d = \frac{D}{16.2}$ to $\frac{D}{12.6}$. Such index values are seen to be also the speeds in r.p.m. when hp. =

1 and $h = 1$ ft., a convenient imaginary condition, assuming that the 80 per cent. efficiency and 0.47 speed ratio hold for the 1-ft. head.

151. Specific Power.—If, in Par. 149, at 1-ft. head, the change in size of wheel be assumed to be such that the best speed in revolutions per minute changes from its value $n_1\left(= \frac{n}{\sqrt{h}}\right)$ to 1 r.p.m., this is accomplished by *multiplying* the wheel diameter

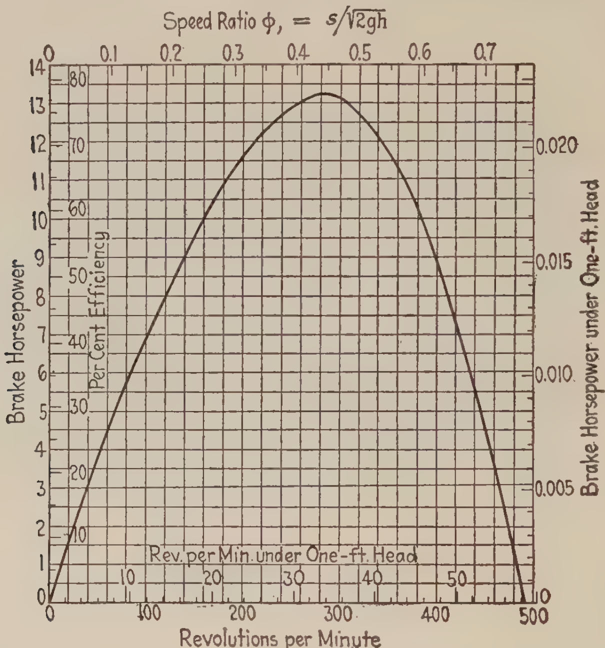


FIG. 142.

Test curve of a 24-inch Pelton nozzle water wheel at 0.9 gate, 70.6 ft. head, discharge = 2.124 c.f.s.

by the same ratio, because for similar wheels under the same head, r.p.m. varies as $\frac{1}{D}$. Such an increase in the linear dimensions of the wheel enlarges the discharge passageway of the nozzle and the cross-section of the jet by the *square* of the same ratio, and hence increases the discharge and the horsepower also by the square of the ratio. Then $(\text{hp.})_s = n_1^2 \times (\text{hp.})_1 =$

$$\left(\frac{n}{\sqrt{h}}\right)^2\left(\frac{\text{hp.}}{h^{3/2}}\right) = \frac{n^2}{h^{5/2}}(\text{hp.}) = \frac{(\text{r.p.m.})^2}{h^{5/2}}(\text{hp.}) \tag{83}$$

This calculated power is known as the *specific power* of the particular type of wheel built after the model in question.

Note that the *specific power* equals the square of the *characteristic speed*. (Query: Briefly, in your own words, why is this so?)

152. Range of Capacities of Present-day Nozzle Water Wheels.—For impulse wheels of the Pelton type, the characteristic speed may range from almost any desired lowest value up to 6 as a maximum, as explained in Par. 150. Usual practice in manufacture adopts 3.5 or 4.5 as the upper limit.

As an illustration of the very low specific speed possible, consider a Pelton wheel 10 ft. in diameter driven by a jet 1 in. in diameter. If proper provision is made to prevent clogging by screening out all fine débris from the water before it enters the nozzle, and if formation of frazil ice be prevented, there are no mechanical difficulties nor hydraulic disadvantages in building and operating such a wheel. In fact, for certain purposes the relatively slow speed in revolutions per minute may be desirable, and there may be no objection to the relatively high cost per horsepower when the total power is small.

Problem: Under a 400-ft. head, compute the revolutions per minute and horsepower to be expected from such a wheel as just discussed, assuming 85 per cent efficiency. *Ans.* 144 r.p.m., and 33.2 hp. What is the specific speed? *Ans.* $N_s = 0.463$ r.p.m.

If *two wheels* are mounted on a single shaft, the horsepower is doubled, and the "characteristic speed" of the double unit is increased. Two Pelton wheels, each with characteristic speed = 4 r.p.m., on a single shaft, would give a unit with value $4 \times \sqrt{2} = 5.6$ r.p.m. for characteristic speed. Two nozzles acting on a single wheel (a device sometimes employed) accomplishes the same object. Such compounding of wheels and nozzles adds to the mechanical difficulties of construction and governing, and results in some loss of efficiency.

Example: Can a Pelton wheel with single nozzle be obtained to utilize 10 c.f.s. under a head of 225 ft. and run at a speed of 600 r.p.m.? If not, what may be done?

Solution: Calculations may be made to ascertain either if the size of jet from the nozzle is too large $\left(\text{larger than } \frac{D}{9},\right)$ or if the characteristic speed is larger than 6 r.p.m.

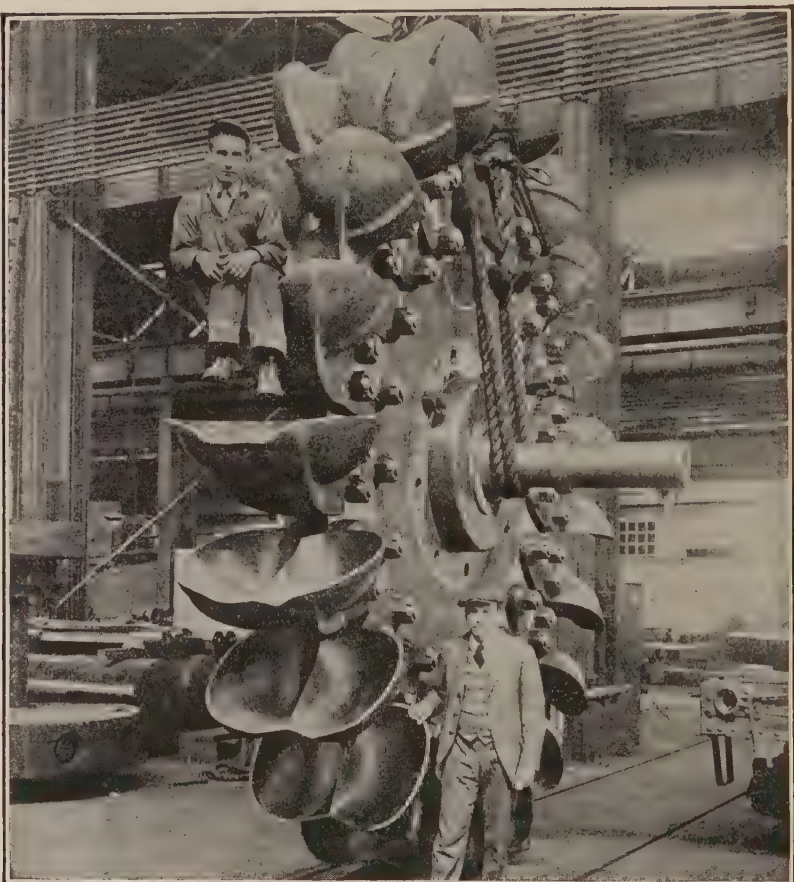


FIG. 143.—Pelton wheel—Caribou plant. Double-overhung units, 30,000 hp., 1008 ft. head, 171 r.p.m. (*Courtesy of Allis-Chalmers Mfg. Co.*)

The cross-sectional area A of jet $= \frac{Q}{v} = \frac{10}{0.98\sqrt{2g \times 225}} =$ $^{10}\!/_{118} = 0.0847$ sq. ft., or $d = 0.329$ ft. $= 3.95$ in. The rim speed of the wheel $=$ about $0.47 \times \sqrt{2g \times 225} = 56.6$ ft. per sec., and for 600 r.p.m., or 10 r.p.s. the wheel diameter would be $D =$

$\frac{56.6}{10\pi} = 1.80$ ft. This is $5.5 \times d$ (of jet) just calculated. Hence, for 600 r.p.m. the discharge demanded is too great.

Using the characteristic speed formula, $N_s = \frac{\sqrt{\text{hp.}}}{h^{5/4}} \times$ r.p.m., first calculate the implied hp. (at 80 per cent assumed efficiency) $= \frac{10 \times 225}{11} = 205$. Then $N_s = \frac{\sqrt{205}}{225^{5/4}} \times 600 = 9.85$ r.p.m. Note that $(225)^{5/4} = 225 \times \sqrt{\sqrt{225}}$, an easy slide-rule operation.

This value 9.85 is considerably above the maximum permissible 6 for nozzle wheels; hence, a single wheel with one nozzle is ruled out of consideration.

Assuming two wheels on one shaft, the power demand (for one wheel) is halved, and $N_s = 6.96$, still too high. If each of the two wheels on the same shaft had two nozzles, $N_s = 4.92$, a permissible value, although practically the plant would be too complicated.

If the assumed speed of 600 r.p.m. cannot be reduced, the power may be divided into two units, each having two wheels on a single shaft, in which case $N_s = \frac{\sqrt{205/4}}{225^{5/4}} \times 600 = 4.92$. This would be an expensive layout, but permissible if the 600 r.p.m. is necessary. If a speed of 300 r.p.m. is usable, however, a single wheel with $N_s = 4.92$ would be proper. Or, for a speed of 450 r.p.m., a double-runner unit, $N_s = 3.7$, could be used.

153. The Girard Impulse Wheel or Impulse Turbine.—At present (1927) practically all wheels used in the United States are either of the Pelton type of tangential nozzle wheel or are of the Francis type of turbine (inward or inward-axial flow). Before discussing the latter type of water wheel it may assist the student to consider an intermediate type.

In the Pelton tangential nozzle wheel the buckets are so shaped and so placed that there is very little loss of head and of power due to shock at entry (abrupt deviation of the jet at the splitter (Figs. 140 and 143) *no matter at what speed* the wheel is allowed to run. Entry conditions, therefore, do not enter (except to a minor degree) into the theory fixing the best speed. The matter was dismissed with the opening statements of Par. 146.

If a different arrangement of nozzle and wheel vanes, or buckets, is used, however (Fig. 144), the matter of avoiding shock at entrance to the rotor demands a wheel speed involving other factors than those considered in Par. 147. Figure 144 illustrates a *Girard wheel*, or impulse turbine, or turbine of free

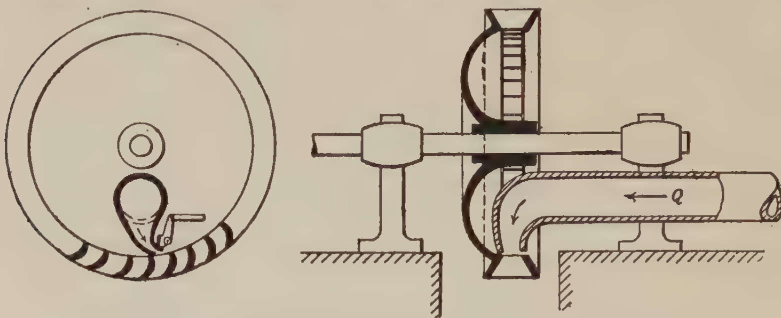


FIG. 144.—The Girard impulse wheel or turbine of free deviation.

deviation. It is still occasionally used in Europe for high heads, but is not used in the United States.

Figure 145 shows the velocity conditions. The water leaves the nozzle at point 1 with a velocity v_1 making an angle a with the tangent to the inner wheel circumference. The curved vane

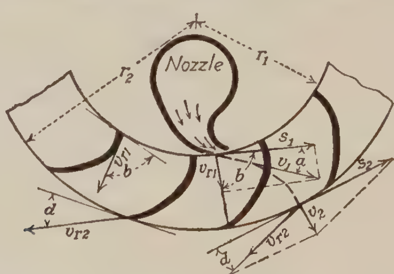


FIG. 145.—Diagram showing velocity relations of entry and exit of Girard wheel. The dashed line shows the path of the water from the nozzle to exit from the wheel.

or bucket makes a greater angle b. In order that the velocity of the water *relatively to the moving vane* may be in the direction of the vane, so as to avoid shock due to abrupt deviation, it is necessary to have the wheel and vane speed s_1 at entry, related to the jet velocity v_1 as shown in Fig. 145. The jet velocity,

in amount and direction, is the diagonal of a parallelogram whose sides are the tangential or rim wheel speed s_1 and the relative velocity along the vane v_{r1}, *i.e.*, the jet velocity is the resultant of the wheel velocity and the water velocity relatively to the wheel.

At the exit point 2 the wasted power is represented by the head in the remaining velocity v_2. To make v_2 small, the exit angle d of the vane must be kept small. It cannot be made zero since there must be some space between the vanes to discharge the water. An angle of 15 deg. is about as small as practically feasible. (Note again the easy curvature of the absolute path of the water through the runner, dashed line in Fig. 145.)

By trigonometry $v_2^2 = s_2^2 + v_{r2}^2 - 2s_2v_{r2} \cos d$. To make this a minimum $\left(\text{and hence, } \frac{v_2^2}{2g} \text{ also a minimum}\right)$, place $\frac{dv_2}{dv_{r2}} = 0$, whence $v_{r2} = s_2 \cos d$ (momentarily regarding s_2 as constant). But if the angle d is only about 15 deg., this gives $v_{r2} = 0.97s_2$, and the value of the short diagonal v_2 will not change appreciably by making $v_{r2} = s_2$.

The linear speed of the outer rim of the wheel is faster than that of the inner rim, and $s_2 = \frac{r_2}{r_1} \times s_1$. Also by centrifugal action there is more induced head at the outer rim than at the inner (see p. 322). In this case of free flow in the atmosphere the extra head due to the whirl accelerates the water as it flows from 1 to 2, so that $\frac{v_{r2}^2}{2g} = \frac{v_{r1}^2}{2g} + \left(\frac{s_2^2}{2g} - \frac{s_1^2}{2g}\right)$.

But if $v_{r2} = s_2$, or very nearly so, there remains simply $v_{r1} = s_1$. Hence the parallelogram of velocities at entry is a *rhombus*, v_1 being the diagonal (which bisects the angle b). The proper vane angle at entry $= b = 2 \times a$, and the proper inner rim speed $s_1 = \frac{v_1}{2 \cos a}$. If $v_1 = 0.97\sqrt{2gh}$ and $b = 40$ deg., (60 deg.), [80 deg.], the speed ratio for the inner rim is $\frac{s_1}{\sqrt{2gh}} =$ 0.52, (0.56), [0.63]. Evidently, as the angle b is made larger the speed ratio increases; *i.e.*, for a given head the wheel must run faster in order to avoid waste of power due to loss of head by shock of abrupt deviation of the path of the water at entry to runner.

This effect of entry angle of bucket on the speed of the wheel suggests the principal phase of the method used in turbines to secure "a high-speed wheel."

Problems

1. A stream of water carrying 2 c.f.s. at a velocity of 20 ft. per sec. is turned aside 45 deg. Compute the X and Y components (in pounds) of the total pressure of the stream against the stationary deflector due to the deviation. *Ans.* X, 22.7 lb.; Y, 54.7 lb.

2 The average flow in a stream is 10 sec.-ft. (c.f.s.). The height of fall is 40 ft. What is the theoretical horsepower available? 45.5

3. A 6-in. pipe is delivering water to a small turbine. In this pipe just before entering the turbine the pressure is 50 lb. per sq. in. If $Q = 2$ c.f.s., what is the theoretical horsepower? 26.2

4. A 12-in. old cast-iron water main 2 miles long leading from a reservoir to a town delivers 2,000,000 g.p.d. What is the horsepower lost in friction in the pipe? 26.

5. The velocity of the jet, diameter 1.2 in., of a Pelton wheel, diameter 6 ft., is 150 ft. per sec. The efficiency of the wheel when run at best speed is 75 per cent., referred to the sum of velocity-head and pressure-head (above atmosphere) at *base* of *nozzle*. The loss of head in the nozzle may be taken as one-twentieth of $\frac{(\text{jet velocity})^2}{2g}$. Compute the horsepower to be expected from the wheel when run at best speed. 36.8 Also, compute the proper diameter to be chosen for the pulley on the same shaft as the wheel such that a resistance of 800 lb. applied tangentially to the outer edge of this pulley shall absorb all the power of the wheel when running at best speed (see Fig. 139, $R'' = 800$). Finally, compute the value of the average working force representing the action of the jet on the wheel cups when the wheel is run at best speed. d=2.

6. A Pelton water wheel is guaranteed by the makers to develop 100 hp. (shaft hp.) at 350 r.p.m.—best speed—under 225-ft. head, giving an efficiency of 82 per cent. (usual meanings of these designations are to be understood). The base (*i.e.*, upstream end) of nozzle is 8 in. in diameter (circular). The speed ratio of wheel is 0.47. Show all calculations. Compute results accurately within one-half of 1 per cent, wherever possible. *Check results carefully.* (1) (*a*) What consumption of water by the wheel, in cubic feet per second, does this guarantee imply? (*b*) What is the implied diameter, in feet, of the wheel *i.e.*, diameter of circle of cup centers? (2) (*a*) In a precise test, a pressure gage at the base of the nozzle (at level of nozzle tip) should show what pressure (pounds per square inch) when the wheel is operating under the conditions of the guarantee? (*b*) What loss of head in 11,000 ft. of 24-in. supply pipe is involved, first, if pipe supplies

water for one such wheel; and, second, if pipe supplies water for two such wheels and in addition 3,000,000 gal. per 24 hr. for other purposes?

Ans. (1) (*a*) 4.77 c.f.s.
(*b*) 3.08 ft.
(2) (*a*) 96.2 lb. per sq. in.
(*b*) 3.96 ft., 30 ft.

7. A paddle wheel, undershot type, is driven by a river current flowing 4 miles per hr., and operates at 25 per cent efficiency. The paddles dip 2 ft. below the water surface and are 12 ft. long axially, so that the Q acting on the wheel $= Av = 24 \times 4 \times 5280/3600 = 24 \times 5.87 = 141$ c.f.s. The head acting on the wheel is the velocity-head of the current. The wheel is geared to a pump whose duty is to elevate water 60 ft. into a reservoir through 800 ft. of 2-in. "fairly smooth" pipe. The gearing is 75 per cent and the pump 50 per cent efficient. How many gallons per day of 24 hr. can be pumped? (1 c.f.s. = 646,300 U. S. g.p.d.) *Ans.* 50,500 g.p.d.

8. Water issues in steady flow from a nozzle, 6 in. in diameter at base and 1½ in. at tip, to operate a Pelton impulse wheel of 8-ft. diameter. The wheel is to be run at best speed and has then an efficiency of 80 per cent. This wheel is to operate a pump of 75 per cent mechanical efficiency, the function of the pump being to raise water a vertical height of 104 ft., at a rate of 6 c.f.s., through a fairly smooth pipe of 12 in. in diameter and 2000 ft. long, the water forming at the point of delivery a jet (in atmosphere) of 50 ft. per sec. velocity. Determine what unit pressure there should be at the base of the nozzle, that the wheel may be able to operate the pump. Then, with radius of 4 ft., find the speed (finally in revolutions per minute) of the wheel, and the resistance R' at edge of a pulley of radius 2 ft., to which the resistance of the pump is equivalent. The rate at which water is used by the wheel (volume per second) should also be determined.

9. Pelton wheel. Take loss of head in nozzle as one-twentieth of velocity-head in jet, but neglect velocity-head in long pipe (12 in. in diameter). Why? Base of nozzle is 12 in. in diameter, 0.72 cu. ft. of water is to be used per second, and the head h is 300 ft. Compute the necessary diameter of jet and determine such a radius r for the circle of cup centers that the wheel shall have a speed of 300 r.p.m. when giving maximum power. If on test, at best speed, the efficiency is found to be 85 per cent, compute the horsepower of the wheel at that speed, assuming the efficiency to be based on the conditions at the base of the nozzle. Also, find pounds of resistance that could be steadily overcome at best speed at periphery of pulley, 2 ft. radius, on same shaft.

10. By means of a single hydroelectric unit directly connected to a generator, 3000 hp. are to be produced under 900-ft. head at a speed not to exceed 450 r.p.m. Using a Pelton wheel with speed ratio = 0.47, velocity coefficient of nozzle = 0.97, efficiency = 80 per cent., what diameter will be required? Check your result by observing whether the ratio of the jet and wheel diameters are within the usual limits. *Ans.* 4.79 ft.

11. If the buckets of a turbine wheel (blocked, not running) be assumed to deflect the water 180 deg. with 10 c.f.s. flowing through the wheel at 100 ft. per sec. velocity, how many pounds pressure are exerted? *Ans.* 3870 lb.

12. Explain why "specific power" equals "specific or characteristic, speed" squared.

13. If a Pelton wheel is to operate at 300 r.p.m. best speed under a head of 225 ft.: (*a*) What is the proper diameter in feet? (*b*) For these conditions, what is the largest permissible size of jet? *Ans.* (*a*) 3.6 ft.
(*b*) 0.4 ft.

14. In a Pelton wheel, the jet of water flows with 200 ft. per sec. velocity and carries 4 c.f.s. What pressure in pounds is exerted if the stream is deviated 135 deg. (= 180 deg. − 45 deg.) by a stationary bucket?
Ans. 2650 lb.

15. Nozzle wheel. If v = jet velocity, s = bucket speed, and a = total deflection angle of the buckets (between 140 and 180 deg.), (1) make a properly labeled sketch showing the various velocities concerned in the theory. State, as in the progressive steps of the theory, equations as follows: (2) For the pressure exerted on one moving bucket. (3) For the power of one bucket. (4) For the power of the wheel. (5) Derive the expression for maximum power from the wheel. (6) Explain what is meant by "speed ratio." (7) Avoiding entirely compounded formulas, if the actual performance of a wheel is to be 128 hp. under 64-ft. head at 100 r.p.m., what are the horsepower and the speed of this same wheel under 1-ft. head? Show calculations. (8) Avoiding entirely compounded formulas, calculate the revolutions per minute of a similar wheel under 1-ft. head yielding 1 hp. (9) Avoiding entirely compounded formulas, calculate the horsepower of a similar wheel under 1-ft. head running at 1 r.p.m. (10) Explain briefly what it is that limits the power obtainable from a certain diameter of Pelton wheel and of turbine under a fixed head. *Ans.* (7) 12.5 r.p.m.; 0.25 hp.
(8) 6.25 r.p.m.
(9) 39.1 hp.

CHAPTER XVIII

WATER TURBINES

154. Recapitulation of Velocity Conditions in "Impulse" Wheels.—The previous discussion of the principles of action of the Pelton and Girard wheels has shown that the pressure, causing the wheel to rotate and to overcome continuously the resistance of the load on the shaft, is due to *deviation* of the stream of water from a direction tangential, or nearly so, to a direction radial or nearly so. This deviation is accomplished for both types of wheels by curved vanes, or buckets (themselves rotating), with their discharge ends directed as nearly 180 deg. opposite the direction of rotation as is practicable. Inevitably associated with such a deviation is the gradual reduction of the absolute velocity of the water as it flows towards the exits of the passageways between the curved vanes. This reduction is seen to be caused by the combination of the motion of the vanes forward and the motion of the water along the vanes backward (relatively to the vane).

In both the Pelton and the Girard wheels the relative velocity of the water leaving the bucket is approximately equal to the linear speed of the bucket but in nearly an opposite direction, thus reducing the resultant absolute velocity as far as possible for a given small value of the angle d (Fig. 145). Hence, since the *exit* conditions for the two types of nozzle, or impulse, wheels are substantially the same, the reason for the higher speed of the Girard type of wheel must lie in the *entry* conditions, in particular, the fabricated angles a and b, necessitating increased wheel speed to avoid shock due to abrupt deviation. Corresponding structural features at entry to a *turbine* runner are also chiefly responsible for the still higher speed ratios of certain types of turbines, as shown later.

Impulse and Reaction, Not Impact.—The student should note carefully that a case of *impact* in which fast-moving water strikes

slow-moving water and suffers shock as it is abruptly retarded is not being considered. On the contrary, velocities, angles, and speeds are so adjusted as to produce a *gradual* retardation. To deviate the water requires a pressure from the vane on the water. The equal and opposite *reaction* pressure of the water against the vane is what drives the wheel. In this sense all modern water wheels are reaction wheels, including so-called impulse wheels.

155. Development of Water Turbines.—If the Girard wheel be so arranged as to receive water around the entire inner circumference from a whole circle of adjacent nozzles, there would result an *outward flow turbine* with "free deviation," *i.e.*, flow through runner under atmospheric pressure. Such a turbine would give greatly increased power approximately in proportion to the amount of water discharged. The speed for greatest power would remain practically the same as with a single nozzle.

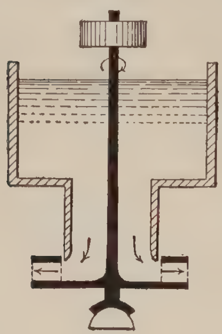


FIG. 146. FIG. 147.

FIG. 146.—"Cadiat's" turbine, the rotor with its curved vanes shown below. "Combe's" turbine was similarly arranged but with the supply pipe delivering the water upward from below the wheel. In neither of these early wheels were there any guide vanes to direct the water against the rotor.

FIG. 147.—Sketch showing how Fourneyron, a French engineer, 1826, improved Cadiat's turbine by adding guide vanes inside the rotor, a type formerly much used in Europe and the United States.

Actual turbine development followed a somewhat similar path.[1] A primitive type of turbine was Barker's mill arranged somewhat like a revolving lawn sprinkler. Another early arrangement is shown in Fig. 146. Fourneyron, in 1826, added the inner guide vanes (Fig. 147), thus producing the modern turbine as far as general action is concerned. Up to about 1900, Fourneyron

[1] "The American Mixed-flow Turbine and Its Setting," *Trans. Am. Soc. Civil Eng.*, vol. 85, pp. 1237–1356, (1922), is an excellent account.

outward-flow turbines were widely used in Europe and America. Howd, in 1838, patented the inward-flow turbine (reversing the direction of the flow of the Fourneyron turbine) and Francis in 1849 further improved this type (Fig. 148).

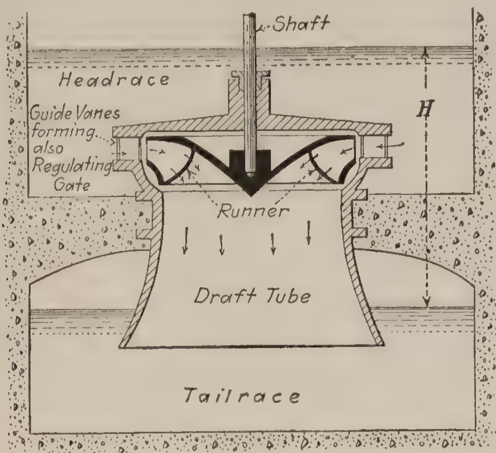


FIG. 148.—Diagrammatic vertical section of inward flow turbine.

FIG. 148A.

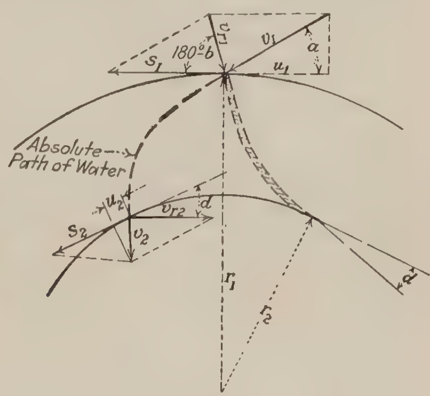


FIG. 148B.

FIG. 148A.—Diagrammatic horizontal section of guide vanes and runner of inward flow turbine.

FIG. 148B.—Velocity diagrams at inlet and exit of turbine (Fig. 148A) to large scale.

Later improvements of the inward-flow turbine (now the type mostly used) have been along the lines of increased discharging capacity and increased revolutions per minute for a given size, at the same time securing by refinements in design greater effi-

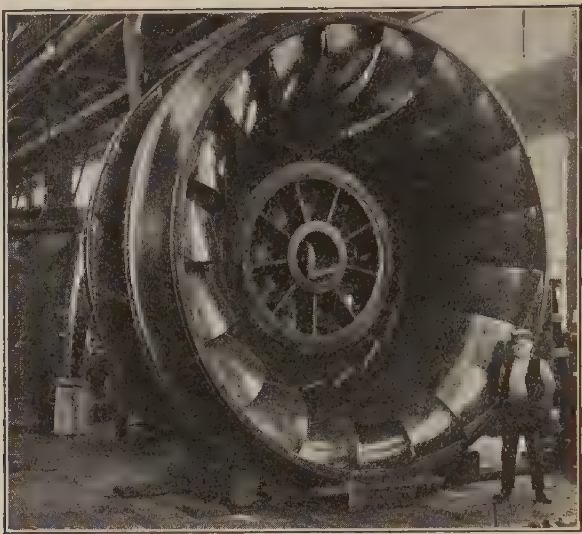


FIG. 149.—Runner of 70,000 hp. Francis turbine. See Fig. 150.

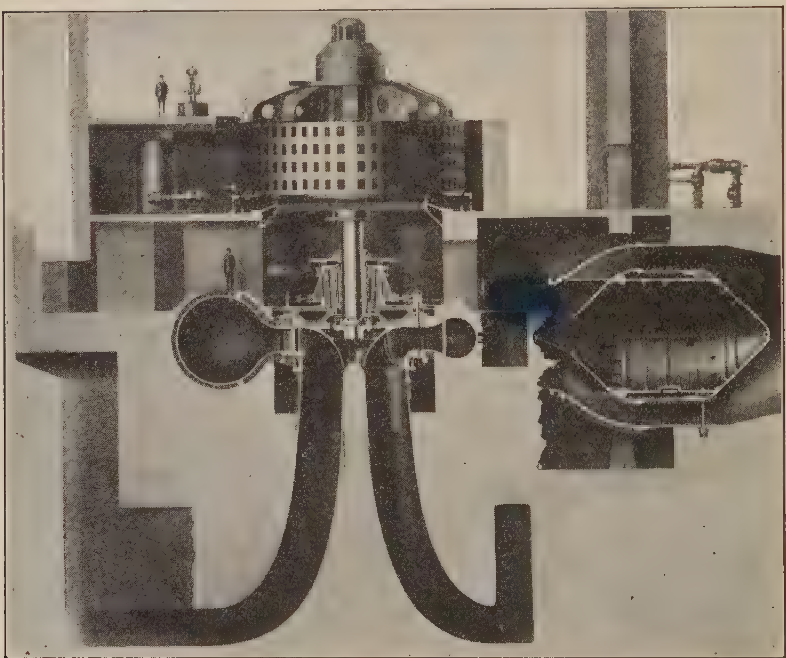


FIG. 150.—70,000 hp. Francis turbine, Niagara Falls Power Co. Head, 213.5 ft.; speed, 107 r.p.m.; efficiency 93.8 per cent. The runner is shown in Fig. 149. (*Courtesy Wm. Cramp & Sons, Philadelphia.*)

ciencies for the runner itself and for the whole installation from head race to tail water. The maximum size of water turbines has increased enormously, recent units installed at Niagara Falls yielding 70,000 hp. with 93 per cent efficiency (Figs. 149 and 150).

156. Draft Tube.—Modern turbines almost never discharge freely into the air, and seldom freely under water. The wheel is usually placed some distance (commonly 5 to 20 ft.) above the

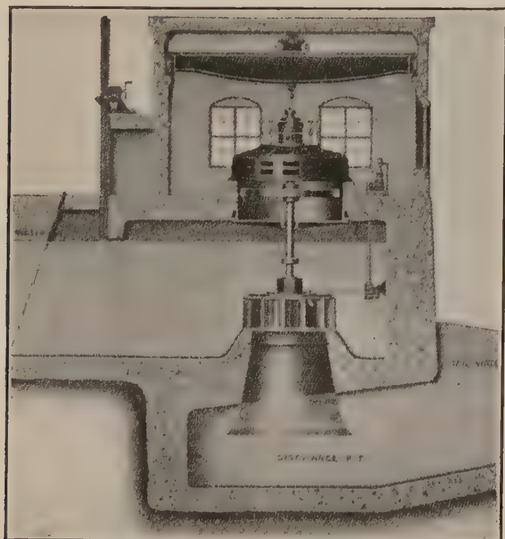


FIG. 151.—Typical low head installation. (*Courtesy James Leffel & Co., Springfield, O.*)

tail-water surface. The discharge takes place into an air-tight *draft tube* with its lower end below the tail-water surface (Figs. 148, 150, and 151). Thus, there is a *suction* pressure-head at the exits of the runner equal to the height above tail water, *less* friction loss, and *plus* any reduction in velocity-head due to the flaring shape of the draft tube (see Bernoulli's theorem, p. 142, and examples).

For high-speed turbines (with high discharge for a given diameter) the exit velocity v_2 is relatively high, and careful attention to the design of the draft tube to convert this energy into useful power becomes of increasing importance.

157. Turbine Theory. *Assumed Conditions.*—In the basic theory of the flow of water through a turbine and the associated power and speed, it is assumed that the resistance, or load, applied on the shaft of the wheel is neither too large nor too small, but is of such an amount that the required volume of water to keep the wheel rotating at best speed is the discharge for which the wheel was primarily designed. This normal discharge for "full load" is assumed to be such that, when the wheel is running at best speed, there is no sudden change in the direction of the absolute path of the water as it leaves the converging passageways between the guide vanes and as it enters the moving buckets of the runner.

The passageways between the buckets (also usually converging from entry to exit) are assumed to be designed, as far as possible, so the water flows without eddies. This implies, since the passageways *flow full*, that all the water moves *forward*, whether rubbing on the "front" concave curve of one bucket or on the "back" convex curve of the adjacent bucket. It also implies that the total areas of cross-sections of passageways between guide vanes and between buckets at entry to runner be related to the velocities v_1 and v_{r1}, so that the product is constant as demanded by the equation of continuity.

158. Miscellaneous Comments.—It requires very careful designing, closely guided by test results of previously built successful turbines, to achieve a close approximation to such idealized flow through the runner passageways *for one particular discharge*. This discharge, as stated above, should correspond to normal operation at full load, but since it is usually desired to have the wheel capable of responding to a momentary "overload" of from 10 to 20 per cent., the runner must be able to take more water than for full load, if the guide vanes are opened further to full gate. Many wheels, moreover, operate much of the time with less than full load, say at half-gate or less. Naturally, there is more eddying and the wheel has a lower efficiency under the off-normal conditions, although at full gate there will be more power than at normal gate; *e.g.*, if 20 per cent. more water is forced through the runner the power is increased, say 15 to 17 per cent. If the wheel carries less than about one-quarter

full load there is much eddying and the efficiency of the wheel is very low.

To summarize, it is assumed that for some particular gate opening, usually one giving from 75 to 90 per cent of the full-open discharge, all passageways are designed to secure a practical minimum of friction, and, when running at best speed, to have a minimum loss due to sudden changes of direction or of cross-section of the stream flowing progressively through the converging passageways between the guide vanes, between the revolving curved buckets of the runner, and through the draft tube.

159. Tangential Velocity Components.—If all the elemental pressures of the flowing water against the curved buckets are imagined to be subdivided into components, radial and tangential, it is obvious that only the tangential pressures can cause the wheel to rotate. Hence only the tangential components of the changes of velocity (accelerations) accompanying the pressures need be considered. Thus, (Fig. 148*B*), at entry, $u_1 = v_1 \cos a$.

160. Theoretical Power.—An expression for the theoretical power given up to the wheel will now be derived in accord with the assumptions made.

In general, Force = Mass × acceleration
= Mass × rate of change of velocity.

Also, Power = Force × linear speed (of body acted on by the force).

During each second, a mass of water $\frac{Qw}{g}$ flows into the runner entry. If the tangential component of the velocity of the water, u_1, were continuously reduced to zero at the entry circumference, without shock or friction, then during each second a *total tangential force* $\frac{Qw}{g}v_1 \cos a$ would be exerted (distributed around the runner).

Let $u_1 = v_1 \cos a$ (Fig. 148*B*).

Then the tangential force of the water on the wheel under these idealized conditions would be $\frac{Qw}{g}u_1$.

The corresponding *power* is $\frac{Qw}{g}u_1 s_1$ *under entry velocity and speed conditions.*

From the runner exit, likewise, a mass of water $\frac{Qw}{g}$ flows out during each second with a velocity v_2, the tangential component of which is u_2. If this were continuously reduced to zero at the exit circumference, during each second a *force* $\frac{Qw}{g}u_2$ would be exerted, and the corresponding *power* is $\frac{Qw}{g}u_2s_2$ *under exit velocity and speed conditions.*

But the last-mentioned power *escapes* from the runner. Hence, the power given up to the runner between entry and exit is the difference between the entering power and the escaping power with reference to the wheel; *i.e.*,

$$\text{Power} = \frac{Qw}{g}(u_1s_1 - u_2s_2). \tag{84}$$

Even in the best-designed and operated wheels, a small part of this power is lost in friction against the buckets and in eddies caused by the curvilinear flow and small departures from idealized flow. The rest of the power of Eq. (84), except what is lost by mechanical friction of bearings, etc., is available on the shaft.

As to the *manner* in which this power is given up to the wheel, it need be observed only that a continuous reaction pressure is exerted by the water on the curved runner vanes, the design assumed to give easy entrance, easy absolute path through the wheel, and favorable exit conditions. Also, the design must provide sufficient size of passageways in the runner to discharge the desired amount of water with the relative velocity prevailing in the runner.

Equation (84) is in no way invalidated by the fact that the curved passageways of the runner are usually gradually reduced in cross-sectional area between entry and exit, acting as a set of nozzles to increase the *relative velocity* v_{r1} to the higher value v_{r2}, in harmony with the reduction of pressure-head between entry and exit of runner.

Equation (84) does not involve the *pressure* of the water at either entry or exit from the runner. The prime reason for this is that the runner is not at all designed nor adapted to utilize pressures directly (like the piston of a pump). The pressures appear indirectly in the velocities.

161. Best Speed of Turbines.—Having found an expression for the power of the turbine runner, an expression for the best speed—speed for maximum power for the particular runner assuming most advantageous gate opening (opening between guide vanes)—may now be found.

Mathematically, the condition for maximum power would be given by placing the first differential of power with respect to wheel speed equal to zero. But, practically, this calculus method is not feasible even for the simpler case of the Girard wheel, although for the still simpler case of the Pelton type of wheel it is an available scheme (see p. 288).

The student should review the arguments used for the Girard wheel on pp. 298–299 concerning: (1) avoidance of shock at entry, and (2) securing minimum wasted energy at exit of runner.

Assuming an inward-flow turbine, if it is specified that the water leaving the runner (Fig. 148*B*) shall have a *radial* direction (to avoid whirling in the draft tube beyond the runner),[1] then, from the parallelogram of velocities at exit,

$$v_2 = s_2 \tan d; \frac{v_1}{s_1} = \frac{\sin (180 \text{ deg.} - b)}{\sin (b - a)} = \frac{\sin b}{\sin (b - a)};$$

and $s_2 = \frac{r_2}{r_1} s_1$, since both s_1 and s_2 refer to speeds of the runner at different distances from the center.

Equating the two expressions for power, (see p. 289),

$$Qw\left(h - \frac{v_2^2}{2g}\right) = \frac{Qw}{g}(u_1 s_1 - u_2 s_2), \qquad (85)$$

where u_1 and u_2 are the tangential components of the absolute velocities of the water at entry and exit.

But $u_1 = v_1 \cos a$; and $u_2 = 0$ (since angle of v_2 with tangent $= 90$ deg.). Hence, the second term on the right side of the above equation disappears.

Canceling Qw, and substituting for u_1,

$$h - \frac{v_2^2}{2g} = \frac{v_1(\cos a)s_1}{g}, \text{ or } h - \frac{s_2^2}{2g}\tan^2 d = \frac{s_1^2 \cos a}{g} \frac{\sin b}{\sin (b - a)},$$

or

$$h - \frac{s_1^2}{2g}\left(\frac{r_2}{r_1}\right)^2 \tan^2 d = \frac{s_1^2}{g} \frac{\sin b \cos a}{\sin (b - a)}.$$

[1] The escaping head $\frac{v^2}{2g}$ is a minimum, and the available power is a maximum, when v_2, the short diagonal of the parallelogram of velocities at exit, is smallest. This occurs, practically, when $v_{r2} = s_2$. (Note that when s_2 increases, wheel running faster, v_{r2} decreases, and *vice versa*, somewhat as s and $(v - s)$ are interrelated for the Pelton wheel. See p. 286.) But v_2 does not change appreciably from this minimum value if its direction is made *radial*, to avoid whirling, *i.e.* $u_2 = 0$. For this condition v_{r2} is a little larger than s_2.

It is convenient to substitute for the speed of the outer rim, *i.e.*, for the s_1 on the left side of the above equation, its equivalent $\phi\sqrt{2gh}$, where ϕ is the *speed ratio* of the wheel (see p. 288). Then

$$h\left[1 - \phi^2\left(\frac{r_2}{r_1}\right)^2 \tan^2 d\right] = \frac{s_1^2}{g}\frac{\sin b \cos a}{\sin (b - a)}. \qquad (86)$$

The term in brackets, coming from the left side of Eq. (85), represents the portion of h available to the runner to produce power, and therefore may be called e_h, the hydraulic efficiency,"[1]

or $$e_h = \left[1 - \phi^2\left(\frac{r_2}{r_1}\right)^2 \tan^2 d\right].$$

Sub e_h
mult. x g ain

Hence, $$s_1 = \sqrt{e_h gh} \times \sqrt{\frac{\sin (b - a)}{\sin b \cos a}}.$$

To make this equation of practical utility, the value of ϕ (the speed ratio) must be assumed, subject to revision, or, preferably,

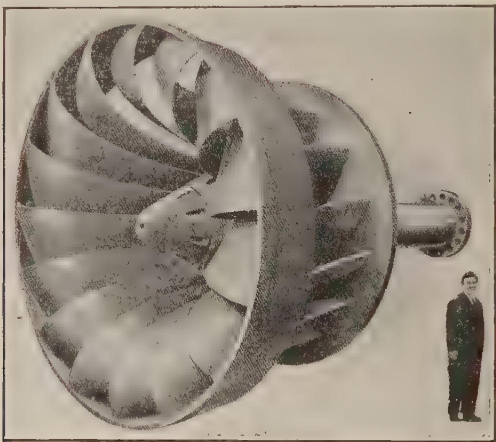


FIG. 152.—One of five cast iron turbine runners, Wateree, Charlotte, N. C., each 20,000 hp. under 75 ft. head at 100 r.p.m. (*Courtesy Allis-Chalmers Mfg. Co.*)

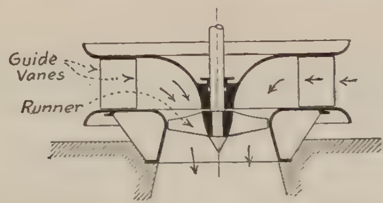
a tentative value of the whole expression in brackets must be assumed, based on actual performance of other wheels.

Note that this expression for best speed for the entry rim of a particular type of runner is a constant $\times \sqrt{2gh}$, as found also

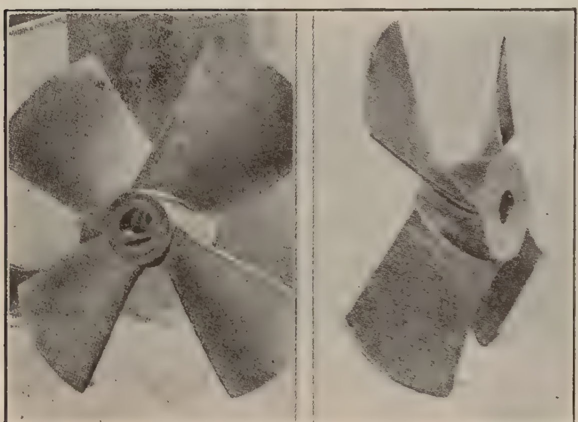
[1] Prof. S. J. Zowski uses this expression in *Eng. News*, Jan. 6, 1910, p. 21, in "A Rational Method of Determining the Principal Dimensions of Water Turbine Runners."

for the Pelton and Girard types, although different ranges of the constants are involved. Practice firmly supports theory in regard to this constant value of the speed ratio for the same turbine under different heads, and for turbines of different sizes built after the same model, all at the same relative gate opening.

162. Actual Speed Ratios of Turbines.—Depending mostly on the angles used, which, in turn, are dependent on the dis-



(a)



(b)

FIG. 153.—(a) Diagrammatic section through propeller type runner and guides. (b) Two views of propeller type runner for high-speed, low-head turbines. (*Courtesy Allis-Chalmers Mfg. Co.*)

charging capacities desired for a given diameter, the actual speed ratios of turbines at best speed range from about 0.60 to 0.90, and to over 2.00, as the types range from the pure inward (radial) flow turbine (Fig. 149) through the bell-shaped (mostly axial paralleling discharge) high-speed turbine (Fig. 152) to the propeller-type or suction turbine giving extreme high speed (Figs. 153 and 154).

163. Discharge Capacities of Turbines. *Low Limit.*—Theoretically, a turbine of a given diameter can be made with extremely small passageways. Practically, the small spaces would become clogged by débris in the water. Independently of this the friction would increase too much, and the cost per horsepower would run up unduly. The practical extreme low limit (corresponding to characteristic speed 10 r.p.m.) corresponds to a discharge about three times as great as for a Pelton wheel of the

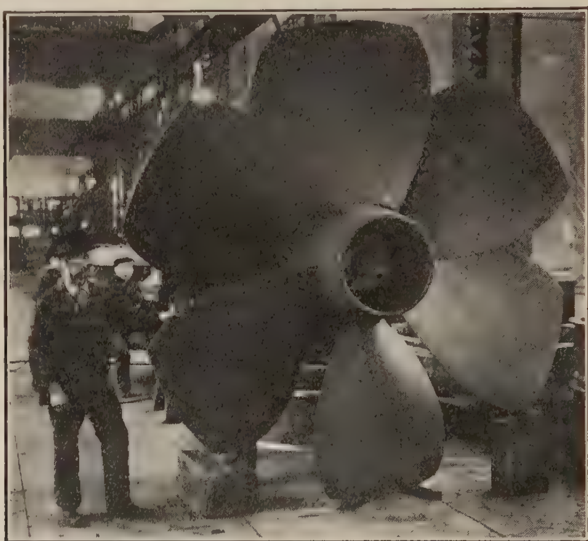


FIG. 154.—High-speed propeller type runner for low heads. See Fig. 155.

same diameter but with *maximum* size of jet, $d = \frac{D}{9}$. The turbine discharge, however, is divided among numerous passageways (at least 12 to 20 even for small diameters), so that the individual spaces are relatively very small for turbines near the low limit.

Upper Limit.—Extreme bell-shaped turbines with specific speed about 100 to 120 r.p.m. can be made to discharge, at high power efficiency, about 90 per cent as much water as for an ideal simple circular orifice of the same diameter as the nominal (entry) diameter of the runner. Propeller or suction runners of

the same diameter as the *extreme diameter* of the high-speed type just discussed are readily made of equal discharging capacity.

Runner Passageways.—If due account is taken of the centrifugally induced heads (see p. 322), in modifying the difference of pressure-heads between entry and exit of the runner, the calculations for necessary cross-sectional areas of the runner passageways to discharge the desired amount of water are made as for a simple nozzle, using, however, the *relative* velocities.

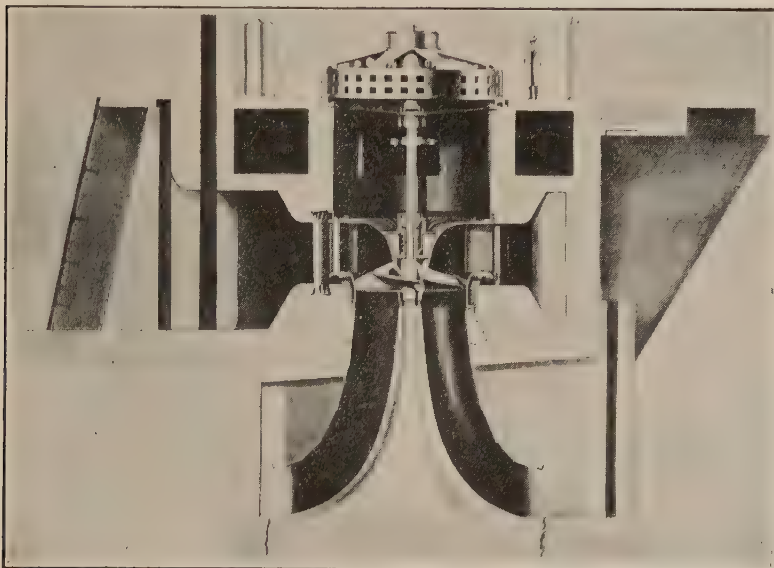


Fig. 155.—Typical setting of low head, high speed unit, using propeller type runner. See Fig. 154. (*Courtesy Wm. Cramp & Sons, Philadelphia.*)

Thus the wheel designer has at his command a means of making the pressure at runner entry any desired value, above or below atmospheric, to suit the rate of flow and the exit suction-head which is fixed by the chosen draft-tube arrangement.

164. Characteristic Speeds. *For Turbines.*—The characteristic speed ranges from about 10 or 12 to 100 or 120, although the recently developed propeller runners have values ranging from 100 to 250 r.p.m. The low values, 10 to 20, are for almost pure inward-flow or Francis types of turbines (see Fig. 148). These values are the resultant indexes combining (1) the dis-

charge coefficient of the particular type of turbine considered as an orifice; (2), the speed ratio; (3) the efficiency.

As shown in Par. 150 for the Pelton wheel, definite magnitudes for these three factors fix the characteristic-speed index value, which is constant for any one model.

165. Off-normal Operating Conditions. (*a*) *Part-load Operation.*—It is seldom practically feasible to operate a turbine always at, or near, normal full load. With electrical generation the load usually varies considerably from hour to hour and even from minute to minute, ranging each day perhaps from hours at a small part-load to short periods of overload.

(*b*) *Off-speed Operation.*—Nor is it possible always to operate at best speed. Several conditions may force a turbine to run at other than its best speed for the head acting. In the common case of a low-head water power plant, the flood-flow water ordinarily rises more below the dam than above it, thus reducing the available head. Also during periods of extreme low water the head may be reduced because the turbines use more water than is flowing in the river, and thus the pond level is lowered below the crest of the dam. In either case the revolutions per minute of the wheels and electric generators must be kept constant. Hence the wheels must run faster than the best speed proper for the reduced head. The governor opens the guide-vane gates beyond the position for normal head in order to give the increased discharge demanded by the lowered head and the slightly reduced efficiency at the too-high speed ratio. The opposite situation sometimes occurs with a high-head plant and a long pipe line, causing considerable loss of head because rather too small in diameter for the full water consumption of the complete water power plant with all wheels in operation, and yet of ample diameter with very little loss of head for the amount of water used by the few wheels at first installed. If these wheels are designed for the head conditions of the complete plant, and are operated at the corresponding speed but under the temporarily higher head, they will run at lower than best speed unless some scheme of throttling is adopted in the pipe line.

166. Characteristic Curves.—Therefore, since more or less importance attaches to the performance of a water wheel at

part load, or overload, and at speeds below and above best speed, it is desirable that the engineer when selecting the wheel have data on horsepower, and efficiency from tests of a similar wheel at various speeds with various gate openings. To make

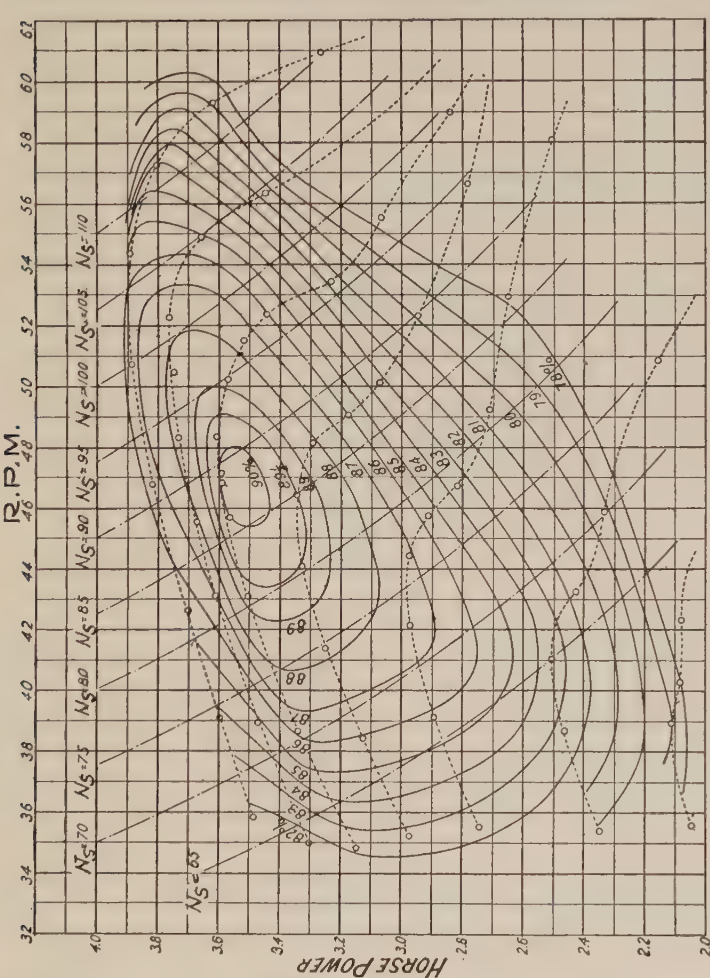


FIG. 156.—Characteristic performance curves of a turbine. Data found in test are reduced to correspond to 1 ft. head on turbine. (*Courtesy S. Morgan Smith Co., York Pa.*) See Prob. **12**, p. 321, for additional description.

such data compact and readily comparable with similar data on other wheels, it is customary to plot the values and draw "characteristic curves" showing equal-efficiency "contours" with the ordinate horsepowers and abscissa speeds reduced to a 1-ft. head condition[1] (see Fig. 156). Another scheme is to plot

[1] See p. 291.

with Q_1 as abscissae, speed ratio ϕ as ordinates, and to draw both equal-efficiency and equal $(\text{hp.})_1$ curves.

It is beyond the scope of this book to enter into further discussion of the practical features of general water power engineering or of specialized turbine performance and design.

Problems

1. A certain turbine, under operating conditions, discharges 0.80 as much water as an ideal orifice of the same diameter as the nominal diameter of the turbine, *i.e.* $Q = 0.80\frac{\pi}{4}D^2\sqrt{2gh}$. The speed ratio is 0.87, and the efficiency is 85 per cent. (*a*) Without using the derived formula for N_s, but solely by use of simple fundamental relations, compute the characteristic speed for this model of wheel. (*b*) What are the diameter and the best speed (r.p.m.) of a 1000-hp. wheel of this type under a 60-ft. head? (*c*) If a speed of 600 r.p.m. were desired (60-ft. head) and the 1000 hp. to be divided between two units, what type of wheel should be selected? (Find characteristic speed by formula directly.) (*d*) If the speed ratio of the wheel in (*c*) is 0.80 and the efficiency 85 per cent, what is the ratio of wheel's discharge to that of an ideal orifice of the same diameter as wheel?

Ans. (*a*) 93 r.p.m.
(*b*) 2.1 ft., 492 r.p.m.
(*c*) 80.3 r.p.m. (characteristic speed).
(*d*) 0.705.

2. An inward-flow turbine running at best speed, 19-in. diameter, on test gives 33.6 b.hp. at 610 r.p.m. under 98.0-ft. head at turbine and discharges 4.42 c.f.s. (*a*) Find the speed ratio. (*b*) Find the characteristic speed. (*c*) Find percentage efficiency from data of test. Now, with the same turbine under the regular operating head of 225 ft., find the (*d*) b.hp.; (*e*) best r.p.m.; (*f*) discharge. On the basis of 80 per cent efficiency attainable with a better wheel (developing same b.hp. under the same operating head of 225 ft.), find (*g*) the wasted hp., and (*h*) the wasted water (cubic feet per second) involved in the continued use of the low-efficiency wheel, which wastage might be saved by the use of the better wheel. (*i*) Capitalize this wastage on basis of \$75 value per hp. per year and 6 per cent interest.

Ans. (*a*) 0.637.
(*b*) 11.46 r.p.m.
(*c*) 68.4 per cent.
(*d*) 117.0 hp.
(*e*) 925 r.p.m.
(*f*) 6.70 c.f.s.
(*g*) 20 hp.
(*h*) 0.98 c.f.s.

3. If the turbine of problem 2 is replaced by a Pelton wheel giving 100 b.hp. at 80 per cent efficiency under 225-ft. head at 300 r.p.m. best speed,

the value of the speed ratio ϕ being 0.45, (*a*) find the diameter of the wheel. In emergency cases it would be necessary to supply the wheel with water from a lower reservoir giving a head at the wheel of 150 ft., but the speed would have to remain as 300 r.p.m. With this condition find (*b*) the value of the speed ratio ϕ. (*c*) For 100 b.hp. find the amount of water used when thus operated at 150-ft. head (making use of the curve (Fig. P.79), showing the relation between the efficiency and the speed ratio to determine the efficiency of this case).

Ans. (*a*) 3.44 ft.
(*b*) 0.55.
(*c*) 8.52 c.f.s.

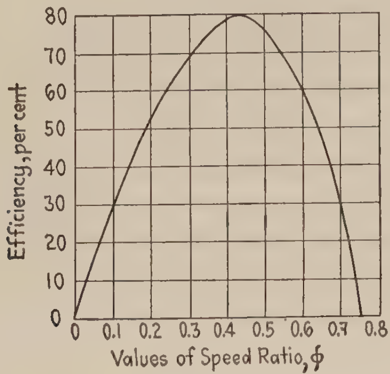


FIG. P.79.

4. During a test a 31-in. Wellman-Seaver-Morgan (inward-flow) turbine gave the following performance:

Head	17.4 ft.
Part-gate opening	0.873
R.p.m	204.2
Q	53.85 c.f.s.
B.hp	92.77
D	31 in.

(*a*) From this compute the efficiency and speed ratio. (*b*) Assuming the same wheel to operate at the same speed ratio and the same gate opening under a head of 50 ft., what will be the new r.p.m., Q, b.hp.? Check these values by recomputing from them the efficiency and speed ratio.

Ans. (*a*) 87.2 per cent., 0.823.
(*b*) 346 r.p.m., 91.2 c.f.s., 452 hp.

5. Barge Canal Lock water power, Seneca Falls, N. Y. On a river with 2370 c.f.s., it is possible to construct a power plant utilizing 48-ft. head. The desired r.p.m. = 180, and 90 per cent efficiency may be assumed. From the average equipment of manufacturers it is decided that a charac-

teristic speed of 76.7 r.p.m. would be best for the plant and the following data are found in catalogue or handbook:

HEAD 1 FT.; CHARACTERISTIC OR SPECIFIC SPEED 76.7

Diameter inches	Horsepower	Revolutions per minute
54	7.8	27.4
57	8.7	26
60	9.6	24.7
63	10.6	23.5
66	11.6	22.5

(*a*) How many water wheels will be necessary in the power plant? For this do not use tables. (*b*) What diameter of wheel will be selected? Why?

Ans. (*a*) Four.
(*b*) 57 in.

6. Explain what particular features establish the upper limit of approximately 6 r.p.m. for characteristic speed of nozzle wheels and 10 to 12 r.p.m. as the lower limit for characteristic speed of turbines.

7. A turbine tested at 16-ft. head shows 100 hp. at 200 r.p.m. best speed. With same wheel at same gate opening at 64-ft. head, compute: (*a*) the new best speed and (*b*) the new hp. at 64-ft. head. (*c*) Does the speed ratio change?

Ans. (*a*) 400 r.p.m.
(*b*) 800 hp.
(*c*) No change.

8. Preferably do not use formula for characteristic speed. Given turbine with characteristic speed n_s = 100 r.p.m., efficiency = 90 per cent., speed ratio = 0.85, what per cent is the Q used by turbine of the Q through ideal orifice of the same diameter D as the nominal turbine diameter?

Ans. 91.8 per cent.

9. Given dependable Q, in river, = 10,000 c.f.s. at head = 25 ft. If 85 per cent water-wheel efficiency can be counted on, (*a*) is it feasible to develop this power by two wheels with r.p.m. not less than 50, the wheels having specific speed not greater than 100 r.p.m.? (*b*) What is diameter of wheel if speed ratio is 0.85?

Ans. (*a*) Yes.
(*b*) 13.0 ft.

10. (*a*) If a turbine (with propeller-type runner) shows 80 per cent efficiency and runs at 150 r.p.m. (best speed ratio = 1.07), under 1-ft. head, giving 1 hp., as calculated from test by proper laws, what percentage of water is used as compared to an ideal circular orifice of the same diameter? (*b*) Using a single turbine of type described in (*a*), 4-ft. diameter, (1) what hp., and (2) what r.p.m. are available when operating under 16-ft. head?

Ans. (*a*) 146 per cent.
(*b*) (1) 857 hp.
(2) 164 r.p.m.

11. If a certain type of modern water turbine, when imagined reduced in size so as to give 1 hp. under 1 ft. head, gives maximum efficiency at best speed 50 r.p.m. (= the characteristic speed), why is the "specific power" (or hp. under 1-ft. head for similar wheel of size to run at best speed of 1 r.p.m.) = 2500? Use simple fundamental relations, not a complex formula.

12. Each dotted line on Fig. 156 represents test data for a particular opening of the turbine gate, and is, in fact, like the summit of curve in Fig. 142, p. 294, using the bottom-inner and right-outer scales. See p. 291 for reduction of r.p.m. and hp. to 1-ft. head condition.

It is required to calculate the abscissae values for the curve ($N_s = 80$) corresponding to the ordinate values 4.0, 3.5, 3.0, 2.5, and 2.0, respectively, as typical of the calculations necessary for drawing the several N_s curves.

CHAPTER XIX

CENTRIFUGAL PUMPS

167. Centrifugally Induced Head.—If an inverted T-shaped assembly of pipes (Fig. 157) is provided with indicator glass columns, and, having been filled with water, is rotated on a central axis, the centrifugally induced heads are as shown by the instantaneous photograph (Fig. 158). The curve of induced heads is a true parabola, the theory of which will be given below. In a liquid rotating in an open vessel, the free upper surface of the liquid assumes the shape of the corresponding paraboloid of revolution.

Theory.—Within a body of rotating liquid imagine a horizontal slender prism of the liquid as a free body (Fig. 159). Let its cross-sectional area be dA and its length r. The outer end rotates with a linear (tangential) speed s and the center of gravity at the midpoint of the prism has a linear speed $\frac{s}{2}$. Let h be the difference between the heads at the axis and the outer end, caused by the centrifugal effect.

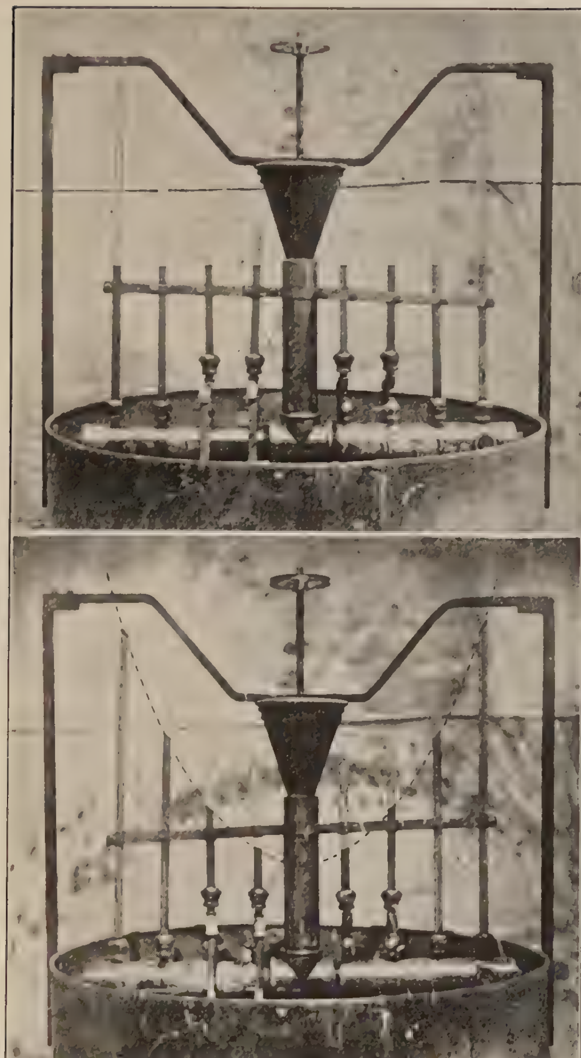
The general mechanics expression for centrifugal force is $\frac{M s_{cg}^2}{r_{cg}}$, in which M is the mass of the rotating body, s_{cg} the tangential speed of its center of gravity, and r_{cg} the distance from the axis of rotation to its center of gravity. Applying this to the imagined horizontal prism of liquid,

$$dA \times h \times w = \frac{\left(\frac{r\,dA\,w}{g}\right)\left(\frac{s}{2}\right)^2}{\frac{r}{2}},$$

whence

$$h = \frac{s^2}{2g}. \tag{87}$$

In words, the extra induced head due to whirl at any point in a body of rotating liquid equals the velocity-head corresponding to the peripheral or tangential speed of the point.



Figs. 157 (above) and 158 (below).—**Centrifugally induced head.** Fig. 158 is an instantaneous photograph showing equilibrium *during rotation*, in contrast to conditions *at rest*, Fig. 157.

168. Action of Centrifugal Pumps.—The above statements give the fundamental principle of the centrifugal pump. If water is fed into the central pipe of the apparatus shown in Fig.

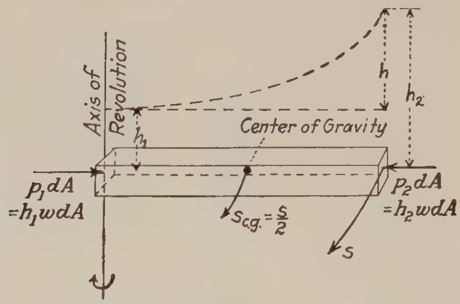


Fig. 159.

158, and if the rotation is fast enough, water is thrown from the outer glass columns at a higher level. This would be a crude representation of the action of a centrifugal pump.

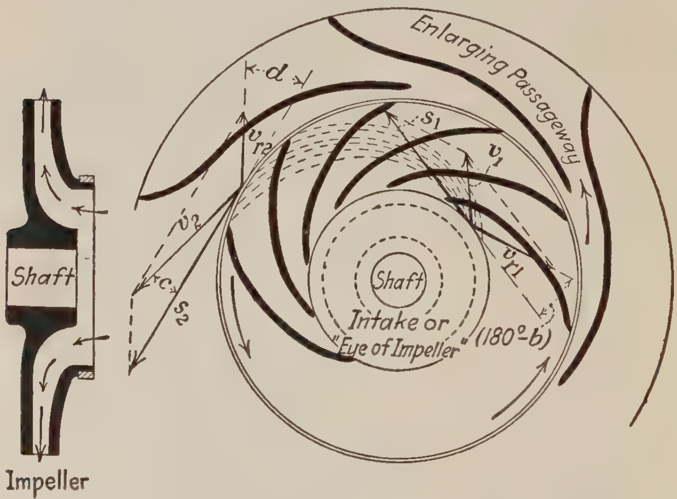


Fig. 160.—Diagrammatic section showing impeller vanes and enlarging passageways of a high head turbine pump.

The universal method adopted in actual centrifugal pumps is to cause the whirling of the liquid by an impeller (a specially shaped paddle wheel), rotating within a casing arranged to receive water at the center in order to feed it into the central space

or "eye" of the impeller at the low "suction" pressure and to discharge the "whirled" water at the high delivery pressure from the passageway surrounding the circumference of the impeller.

169. Obstacles to Achievement of High Efficiencies.—The velocity diagrams at entry and at exit of the impeller are shown in Figs. 160 and 161. For the simple "volute" centrifugal pump (Fig. 162), there are no guides outside the impeller to give the axially entering water its proper direction with velocity v_1. At

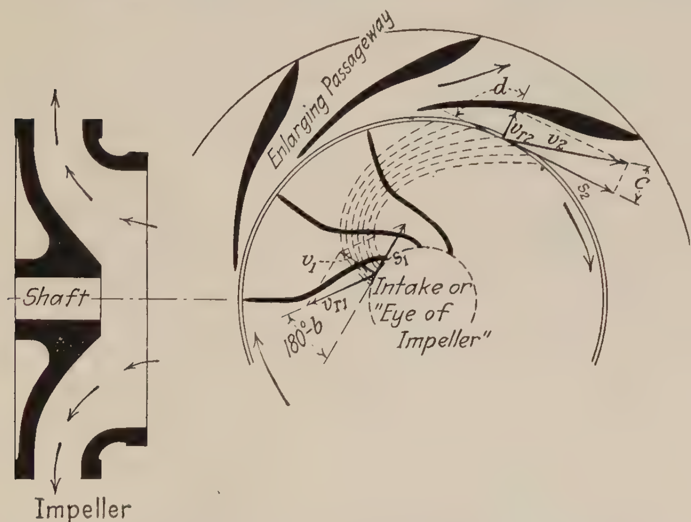


FIG. 161.—Diagrammatic section showing impeller vanes and enlarging passageways of a low head, high capacity turbine pump.

exit there are no guides to receive the water in the direction of v_2, except as the spiral passageway outside the impeller furnishes a crude guidance.[1] Consequently, with this type of pump, there inevitably occurs a considerable loss of head, which may be expressed as $C\frac{v_2^2}{2g}$, where C is a combined, or lump, coefficient deduced from experimental measurements.

As noted previously for turbines, the smaller the angle d, for a constant relative velocity v_{r2}, the smaller is the absolute velocity

[1] By the interposition between the impeller and the spiral of a hollow ring passageway (of same width as the impeller passageways at exit (Fig. 163), conditions are much improved.

v_2 at exit, and the less is the wasted power. Ideally, the value of v_2 should be the same as the velocity in the discharge pipe, but the service demanded of the pump renders this impossible. The angle d cannot be reduced much below 20 deg. without abnormally cutting down the size of the impeller passageways and hence reducing the discharge. If compensation were assumed by increasing the relative velocity v_{r2}, this would imply an increase in v_2 for approximately fixed value of s_2, the rim speed demanded by the head pumped against. A high value for the relative

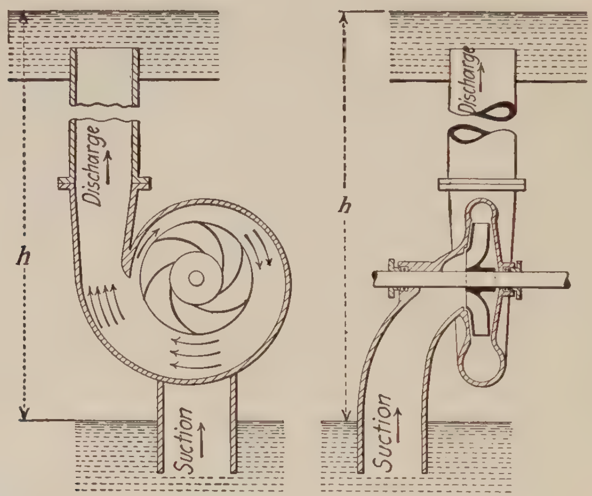


FIG. 162.—Diagrammatic sections of a volute centrifugal pump.

velocity v_{r2} along and between the impeller vanes is in itself undesirable because it greatly increases the friction loss.

The situation outside the impeller, even at its best, with enclosed impeller and gradually enlarging passageways (Figs. 160 and 161), resembles the divergence beyond the throat of a Venturi meter for which the loss of head is perhaps one-quarter to one-half of the throat velocity-head in a comparable case. For the pump impeller, with v_2 ranging from about 0.6 to 0.8 of rim speed s_2, and the exit velocity-head $\frac{v_2^2}{2g}$ from about one-third to two-thirds of $\frac{s_2^2}{2g}$, or, roughly, the same fraction of the head

h pumped against, it is seen that a loss of head at exit, ranging from about one-tenth to one-third of the whole head, is to be expected. It is a common saying in hydraulics that water can be speeded up with little loss of energy but cannot be made to convert velocity-head back into pressure-head nearly so efficiently.

With the centrifugal pump the case is different than with the turbine, although one is sometimes called the reverse of the other. The student will note, in comparing the exit velocity diagrams of pump and turbine, that for the latter it is the fortunate possibility of having both a small value of the angle d and approximate equality between v_{r2} and s_2 that enables v_2 to be reduced to a small value. The pump has comparatively unfavorable conditions at both entrance and exit.

Nevertheless, by careful analysis of test results on numerous pumps showing improved performance, and guided by available reasonable special theory and the general principles of hydraulics, engineers have brought centrifugal pumps to a remarkable state of development for a wide range of uses. In the larger sizes, efficiencies of 70 to 80 per cent are common.

170. Shut-off Head, *or Head of Impending Delivery.*—For the device shown in Fig. 158, the induced head at the extremity above the head at the center is truly $\frac{s_2^2}{2g}$. In the actual centrifugal pump, (casing filled with water, impeller revolving), but with valve closed in discharge pipe, the induced head may be greater or less than $\frac{s_2^2}{2g}$, depending on which particular group of departures from ideal conditions dominates. Usually it is from 1.00 to $1.15 \times \frac{s_2^2}{2g}$ although it is in some cases lower, down to $0.85\frac{s_2^2}{2g}$.

In any installation of a centrifugal pump it is, of course, of prime importance that the shut-off head of the pump (at the available speed of the driving motor or engine) be greater than the maximum static head to be pumped against, no matter what the characteristics of the pump when discharging water. Otherwise, pumping could not begin.

171. Head Pumped against.—As in Par. 160, the ideal power of the impeller is $\frac{Qw}{g}(u_2s_2 - u_1s_1)$, of which term the part

$\frac{1}{g}(u_2 s_2 - u_1 s_1)$ represents the ideal head. (As for turbines, u_1 and u_2 are the *tangential components* of the absolute velocities of the water v_1 and v_2 at entry and exit of the impeller.) If v_1 is assumed to be radial, so that its tangential component $u_1 = 0$, the ideal head is $\frac{1}{g} u_2 s_2$. For any given pump at full normal discharge, u_2 may be considered as a certain ratio[1] of s_2, and the actual head supplied by the pump is less than this ideal because of losses, so that the actual head $= K\frac{s_2^2}{2g}$ where K is a function of the combined or lump friction coefficient C and the angle d.[2]

Viewed from the outside, the head supplied by the pump must be sufficient to overcome the static or elevation head and all losses of head at the rate of discharge prevailing, also any terminal velocity-head not included in the static head. The value of K, for pumping at normal discharge rate, varies considerably for different types of pumps. For the ordinary run of centrifugal pumps, K ranges from about 0.75 to 0.90, but it may be as low as 0.60 or as high as 1.25. Small values of the angle d and too few vanes in the impeller tend to cause low values of K; *i.e.*, such

[1] Professor I. P. Church in "Hydraulic Motors" deduces, for maximum efficiency, $\frac{u_2}{s_2} = \sin d$; and for the maximum efficiency itself $e_{max.} = 1 - \left(\frac{C}{c\text{osec } d + 1}\right)$.

[2] If, in the ideal head expression $\frac{1}{g} u_2 s_2$, $v_2 \cos c$ is placed for u_2 and $v_2 \cos c$ solved for in the trignometric relation $v_{r2}^2 = s_2^2 + v_2^2 - 2s_2v_2 \cos c$, then the ideal head supplied by the pump $= \frac{s_2^2}{2g} - \frac{v_{r2}^2}{2g} + \frac{v_2^2}{2g}$. Actually, some additional fraction of the relative velocity-head $\frac{v_{r2}^2}{2g}$ will be lost in the impeller and only part of the absolute issuing velocity-head $\frac{v_2^2}{2g}$ will be converted into pressure-head, as stated in Par. 169. Therefore, the question whether the head supplied by the pump is greater or less than the "whirling head" $\frac{s_2^2}{2g}$ depends on the relative magnitude of the subtracted multiple of $\frac{v_{r2}^2}{2g}$ and the added fraction of $\frac{s_2^2}{2g}$. (Even with test data available, it is difficult properly to separate these losses of head or rationally to make them include all losses.)

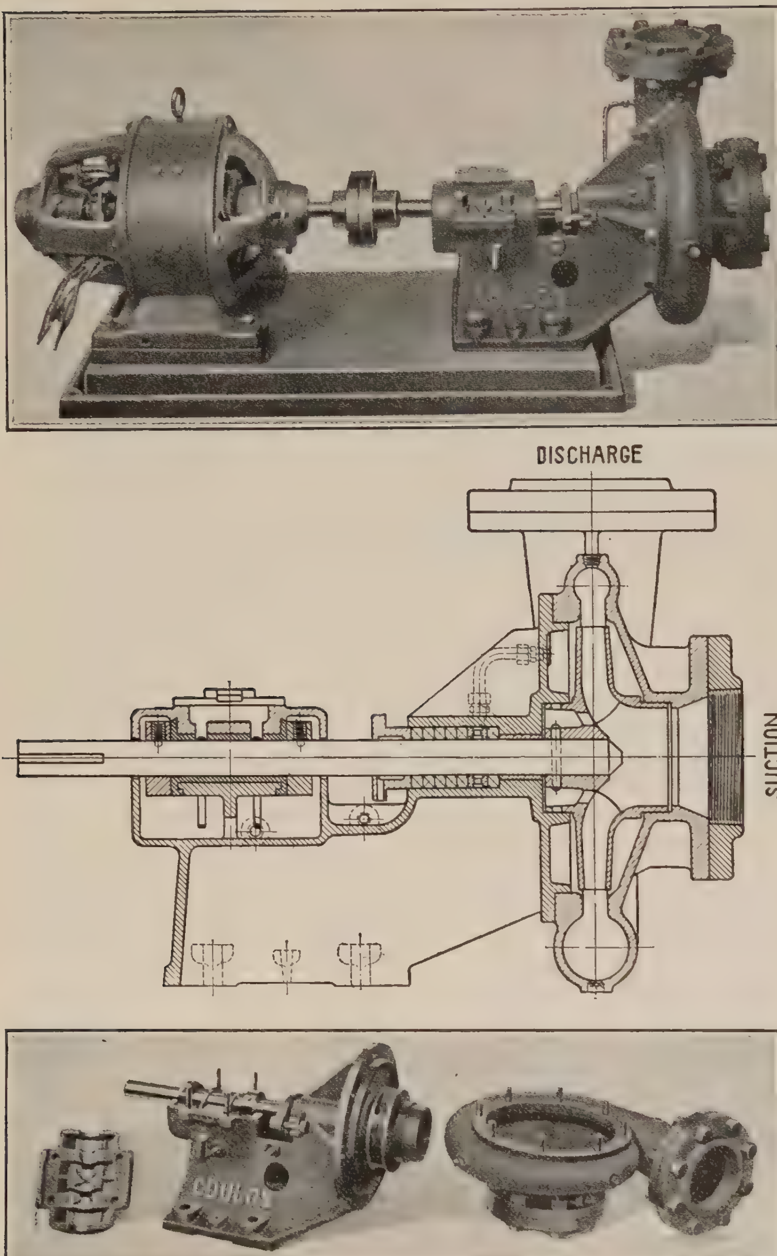


Fig. 163.—Centrifugal pump, single stage, single suction, enclosed impeller. Above: Unit direct connected to electric motor. Intermediate: Axial section through pump. Below: Pump with casing and bearing cap removed. *(Courtesy Goulds Pumps, Inc., Seneca Falls, N. Y.)*

conditions require high-rim speeds for a given head or high *speed ratios*, $\frac{s_2}{\sqrt{2gh}}$. The speed ratios corresponding to the values of $K = 0.60$, 0.75, 0.90, and 1.25 mentioned above are, respectively, 1.29, 1.15, 1.05, and 0.88.

Test results on a similar pump form the only reliable guide for close estimates of performance.

172. Effect of Variation in Speed.—A given centrifugal pump may be used for different heads by varying the speed, since generated head tends to vary approximately as the square of the speed. The discharge, however, also varies as the speed changes, the law of variation depending on the ratio of the friction-head to the static head in the pipe line. If practically all the head is used up in friction in the pipe line, there being relatively small actual lift, then the commonly stated rule that the discharge varies directly as the speed is approximately applicable. In this case the power required varies nearly as the cube of the speed; but if the head is practically all static, with little frictional loss between pump and end of discharge line, the discharge increases and decreases much faster than does the speed. This is because there is little added head and the additional power is represented mostly by the increase in discharge. Figure 164 shows characteristic curves for the same pump at three speeds.

173. Constant Speed. *Relations between Discharge, Head and Efficiency.*—Power is very commonly supplied to a centrifugal pump by an induction electric motor on the same shaft, the only variations in speed being due to small changes in voltage, and slip of motor. Typical test results are shown in Figs. 164 and 165. The curve showing the variation in the head pumped against (*i.e.*, produced by the pump) as the discharge increases from zero to maximum (at the particular speed) is commonly called the *pump characteristic*, but the other characteristic curves of required shaft horsepower and of efficiency are usually plotted also because they supply desired auxiliary information concerning the pump performance.

It is usually desired to pump with as little power as possible. Hence, the usual or average operating range of the discharge rate and head should be at, or near, the summit of the efficiency

curve. If either the head pumped against or the discharge rate is subject to variation, as frequently occurs in practice, the effect on the other of these two factors and also the effect on the efficiency may be learned from the characteristic curves (see Fig. 166). Most pump manufacturers maintain testing laboratories and can supply performance curves of their pumps. In many cases the makers can supply special impellers with the proper terminal vane angles to give desired characteristics. It is possible to have *rising, flat,* or *falling characteristics, i.e.,* as the discharge increases from zero and approaches normal capacity

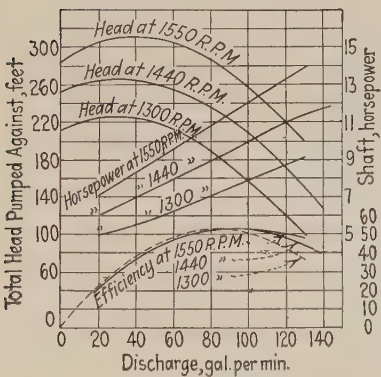


FIG. 164.—Characteristic curves test of a 2 inch 4 stage turbine centrifugal pump. Impellers 9⅝ in. diam. (circum. 2.52 ft.). Test by Authors.

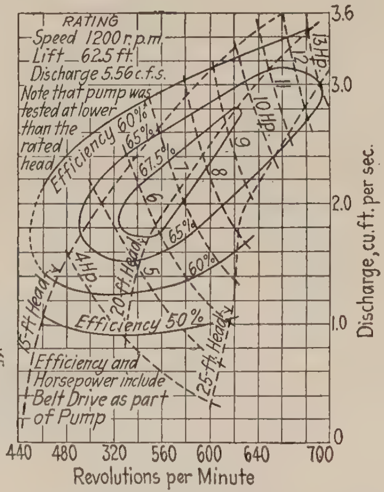


FIG. 165.—Performance curves from test of a 10 inch DeLaval centrifugal pump single stage. Test by E. W. Schoder.

(speed remaining constant) pumps are available for which the head increases, remains constant, or decreases.

Centrifugal pumps are used for all sorts of liquids and mixtures, such as heavy oils, pulps, sludges, dredged material ranging from fine silt and sand to large gravel. It is entirely beyond the scope of this book to enter into the many engineering considerations involved, but it is obvious that the power required to handle very viscous liquids is greater and the efficiency is less than for freely flowing liquids like water and light oils.[1]

[1] Goulds Pumps, Inc., Seneca Falls, N. Y., in *Bull.* 126 by R. L. Daugherty, give detailed test results on very heavy oils.

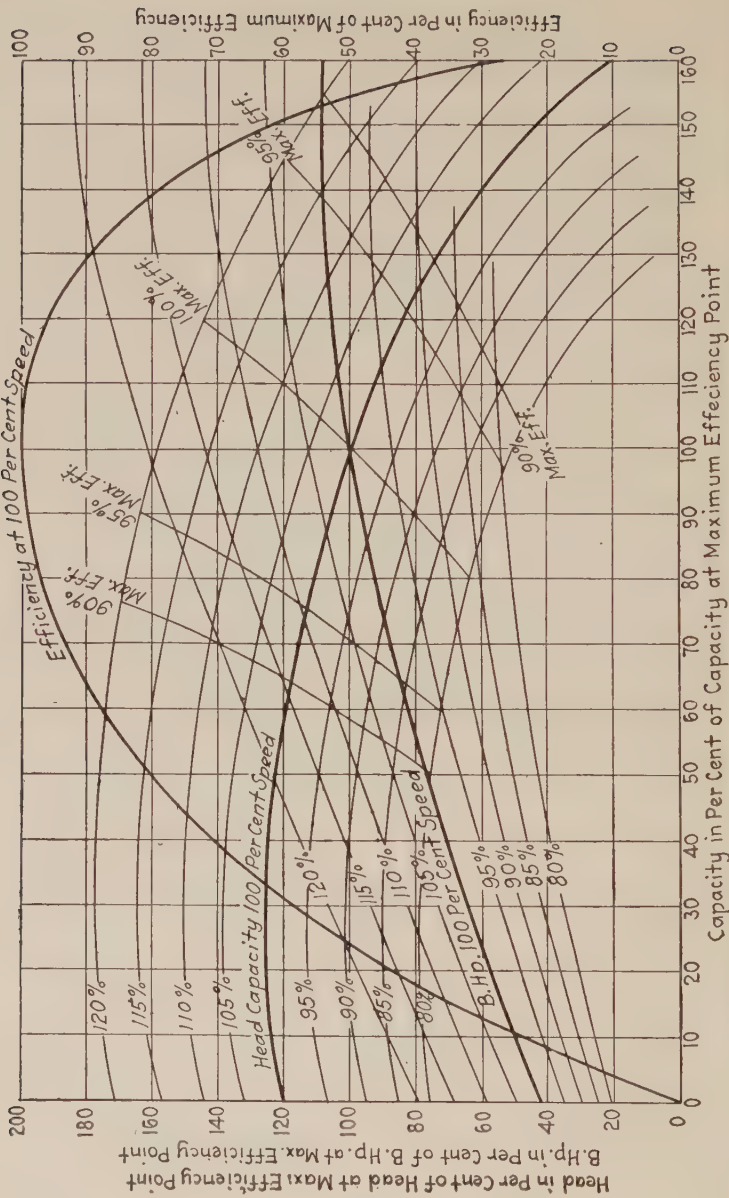


FIG. 166.—(See explanation, p. 333.) General performance characteristic curve, centrifugal pumps. (*Courtesy Worthington Pump and Mach. Corp., New York.*)

174. General Characteristics of Centrifugal Pumps.—The test results shown in Fig. 164 indicate similarity in performance at different speeds. This is to be expected from the analogies between centrifugal pumps and turbines. When such curves are analyzed from tests on many pumps, all of a general type but of various capacities, the similarity is so pronounced that, for comprehensiveness, it is found convenient to express values as percentages. Figure 166 is a typical plotting of this sort.

On Fig. 166, 100 per cent means the capacity of the pump under consideration at point of best efficiency. The heavy line "Head Capacity 100 Per Cent Speed," shows how the head decreases as the capacity increases when running at constant speed. For example, suppose the pump has a normal capacity of 1000 g.p.m. at 100-ft. head. If the head drops to 60 per cent of normal = 60 ft., the capacity will increase to 140 per cent of normal = 1400 g.p.m. Likewise, if the head increases to 120 per cent = 120 ft., the capacity will be only 60 per cent = 600 g.p.m. All these points are located by noting where the heavy "Head Capacity" curve intersects the horizontal and vertical lines, showing the percentages of head and capacity for varying conditions. The light lines parallel to the heavy line show the changes due to changes in speed when increased or decreased by 5 per cent steps. For example, the above pump with the speed decreased 20 per cent (80 per cent line) will, at 1000 g.p.m. (100 per cent capacity), deliver against a head of only 50 per cent = 50 ft.

The heavy line "b.hp. at 100 Per Cent Speed" shows how the horsepower increases as the capacity increases, and the light parallel lines show how the horsepower changes as the speed varies by 5 per cent steps.

The heavy line "Efficiency at 100 Per Cent Speed" shows how the efficiency drops as the capacity increases or decreases from the normal (100 per cent) point. The sloping lines marked "100 Per Cent Maximum Efficiency," etc., show how the head and capacity must both be varied (simultaneously) if the efficiency is not to drop. For example, with 10 per cent speed increase, if the head becomes 120 per cent and the capacity 110 per cent, the intersection is at the 100 per cent line; in other words, the efficiency is still at a maximum.[1]

In the last illustration, the horsepower required, of course, varies as $Q \times h$. If the efficiency remains at a maximum (*i.e.*, 100 per cent of 70, 75, or 80 per cent, or of whatever the actual maximum efficiency of the pump is shown to be by tests), the required horsepower is $1.10 \times 1.20 = 131$ per cent., of normal.

[1] Taken, by permission, from the Instruction Book of the Worthington Pump and Machine Corporation, New York.

APPENDIX A

BRIEF NOTES ON HYDRAULIC MEASUREMENTS

The Pitot Tube (Figs. 78 and 78*A*.)—Figures 167 and 168 illustrate representative Pitot tubes of a compact type adapted for insertion through a small hole in the wall of a pipe. Such tubes have coefficients less than 1 because of the suction effect at the pressure openings, due to the shape of the tube causing the flowing water to be swerved away from the openings instead of flowing perpendicularly past them as for the wall opening in Fig. 78*A*. For tubes as in Fig. 167, $V = 0.84\sqrt{2gh}$, *i.e.*, the indicated water column difference h is about $\left(\frac{1}{0.84}\right)^2$, or 42 per cent greater than the h_v of Fig. 78*A*. Very slight changes in the shape of a compact-type side-pressure-opening tube as in Fig. 168, or in the location of the pressure openings, cause a change in the coefficient of the tube. For Fig. 168, $v = 0.84$ to $0.88\sqrt{2gh}$, depending on location of side openings and shape of tube; for precise work such tubes should be rated. The use of a single opening tube in connection with a "wall piezometer" (*i.e.*, either a single hole as Fig. 78*A*, a pair of diametrically opposite holes, or a ring piezometer) that transmits only the pressure-head of the water affords the combination of the simplest and most easily constructed instrument with the greatest certainty of indications without necessity of special calibration in case the tube is constructed a little differently from some model. No coefficient is necessary, *i.e.*, $v = \sqrt{2gh}$ for a tube that has its impact opening

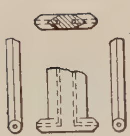


Fig. 167.


Fig. 168.

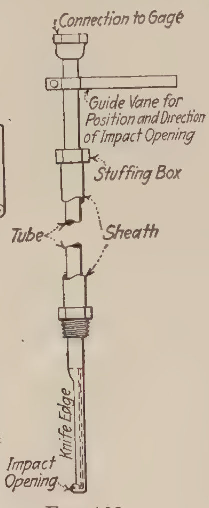


Fig. 169.

projecting a little upstream from the body of the tube (see Fig. 169). The pressure holes in the pipe should be about 1 in. upstream from the impact point of the tube, so that the flow past them will be undisturbed by the presence of the tube in the pipe. If the inside of the pipe is tuberculated, a single tap in the pipe wall may give erroneous pressure indications.

For abnormal flow in curved passages, or wherever a wall opening is subject to centrifugal action due to the flow not proceeding parallel to the wall, a two-opening tube is most convenient, as also where many tests in different pipes are to be made.

A sheath, or elongated stuffing box (Fig. 169), into which the point of the tube can be drawn back out of the pipe, is very convenient and is practically essential for high pressures where the flow in the pipe cannot be shut off. A corporation cock (½, ¾, 1 in. or larger, to suit the tube) may be permanently tapped into the pipe. The sheath should have a union coupling adapted for connection to the corporation cock. This gives a shutoff that allows a ready insertion of the tube at any future time. A ¼-in. tap and a gage cock are suitable for the pressure holes when a single-opening tube is used.

Use of the Pitot Tube in Pipes.—*The distribution of velocities* in the cross-section of a pipe with flowing water is affected by upstream disturbances and by the roughness of the pipe's interior surface. If some 50-diameter length of straight pipe without tees, valves, etc. extends upstream from the place of measurement, normal distribution prevails. For this the velocity at the wall of the pipe is about one-half the center velocity, and the ratio of the mean velocity to the center velocity (*the pipe coefficient*) averages about 0.83, the *velocity curve* being approximately a semiellipse with the points of mean velocity at about one-fourth the radius from the wall toward the center (see Fig. 170).

Discharge Measurements. *The Traverse.*—Various local effects cause departures from the above-stated normal conditions. If the mean velocity deduced from $0.83 \times v_{\text{cen.}}$ does not agree with that found at the normal mean velocity points within some 3 per cent, or if the greatest possible precision is required, a complete **traverse** should be made; *i.e.*, the velocity should be found

at points distributed along the whole diameter. If this is done for various mean velocities and for two diameters perpendicular to each other, the ratio of mean to center velocities for the particular pipe and cross-section may be determined precisely, and thereafter observations with the Pitot-tube set on the center only will suffice for accurate discharge measurements.

The 10-point method of making a **traverse** consists in observing with the Pitot-tube impact opening set on each of the intersections of a diameter with the circumferences of the odd numbered of 10 concentric circles dividing the cross-section into 10 equal areas. The inner wall of the pipe is the tenth circle (see Fig. 170). Since each point of observation controls one-tenth of

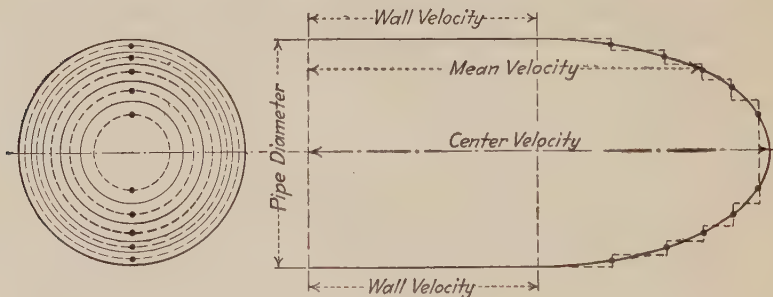


Fig. 170.

the cross-sectional area, it is necessary only to add the calculated velocities and divide by 10 to obtain the mean velocity. The so-calculated mean velocity is theoretically only 0.30 per cent too high and may be considered practically exact for most measurements. If 20 points are used, the error in summation is theoretically only 0.10 per cent too high (see *Proc. Am. Soc. Mech. Eng.*, p. 517, May, 1908).

The float gage consists of a hollow float of non-corrosive metal or other substance, either spherical or cylindrical with convex or conical top and bottom. A light, rigid stem extends vertically upward, having an index mark that is guided along the edge of a graduated scale or is attached to an automatic recording mechanism. For **great range of levels,** a graduated tape attached to the float and passing over a pulley or system of pulleys, and with a suitable counterweight, makes a sensitive indicator. The tape

need be marked only every foot, if it passes the edge of a fixed auxiliary scale with subdivisions.

For precise work the float should be confined against lateral movement by a vertical pipe with internal diameter $\frac{1}{4}$ or $\frac{1}{2}$ in. larger than the float. *For the greatest accuracy,* where errors of 0.001 to 0.004 ft. are inadmissible, the float should be made at least 1 in. smaller than the pipe and should be loosely guided (at the stem or by rounded radial fins on the float itself) to keep it away from the pipe so as *to avoid capillary lifting.* To lessen oscillations due to surges and waves, the lower end of the float pipe is capped and one or more small openings are made near the bottom to allow equalization of levels inside and outside. The area of the openings may be some 1 to 3 per cent of the float-pipe section. For standing liquids in open tanks or reservoirs the float pipe may be placed either inside or outside the container, as is most convenient. For flowing water in open channels it is placed outside the channel and communication is made by a small pipe, which must end perpendicularly to the inside wall of the channel and without projecting into the channel; *i.e.,* its end should be flush with the inside wall, so as to avoid suction effect (see Fig. 171).

The Hook Gage, Point Gage, Plumb-bob Gage (Fig. 171).—These all are devices for locating the height of a liquid surface by observing its contact with the "point" attached to the movable graduated scale of the gage. The point gage has a sharpened point directed downward in place of the hook of the hook gage. For these gages the scale usually is engraved on a square brass rod. A steel or bronze tape is used for the *plumb-bob gage.* The "point" should not be a "needle point," but a conical point with a vertex angle of 90 to 120 deg., so as to produce a maximum of optical effect for slight variations from perfect contact of the point with the liquid surface. The use of a *stilling box,* such as the float pipe above described, is desirable, especially for the hook gage (see Fig. 171).

In use, the *hook-gage* point is moved up from beneath until it just pierces the liquid surface. The *point gage* and *plumb-bob gage* are manipulated by lowering from above until a "bubble" shows contact with the liquid surface. If used for flowing water

or on a large surface of a tank or reservoir, where there are waves or surges, and where a stilling box is not available, the gage would be set so that the average time durations during which the point is alternately submerged and exposed shall be as nearly equal as the observer can estimate. Even with a stilling box there may be surges of small amplitude, and several sets of observations may be necessary to assure a determination within 0.001 ft.

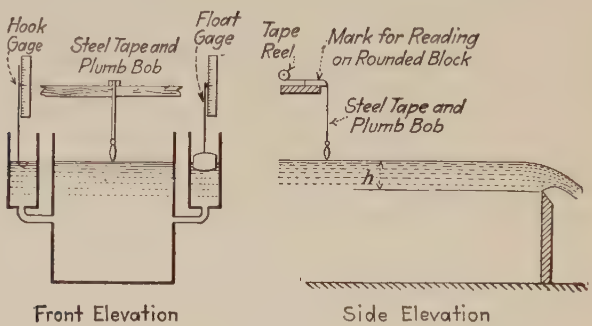


Fig. 171.

The **float gage** is the best of the gages mentioned **for sensitiveness and accuracy.** It automatically shows the small variations of water surface level, and the observer needs merely to watch the index on the float stem. Any desired degree of damping out of minor or temporary fluctuations of water level may be accomplished by changing the size of the opening or by regulating a shutoff cock in the connecting pipe.

Whenever there is a perceptible vibration of the water surface, maximum and minimum readings should be taken, corresponding to the crests and troughs of the waves. The average will give the mean level in most cases.

APPENDIX B

TIME REQUIRED TO FILL AND EMPTY TANKS, CANAL LOCKS, RESERVOIRS, ETC., THROUGH ORIFICES OR PIPES UNDER VARIABLE HEAD

The problem first considered is that of a basin, such as a canal lock, which is filled through submerged "ports" or culverts communicating with an extensive body of water whose surface remains at a constant level while the level inside the lock chamber changes from maximum value to zero as the filling or the emptying progresses (see Figs. 172 and 174).

Of similar nature is a tank emptying through an orifice or pipe with free discharge into the atmosphere or submerged discharge

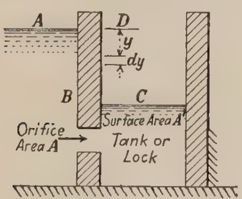


FIG. 172.

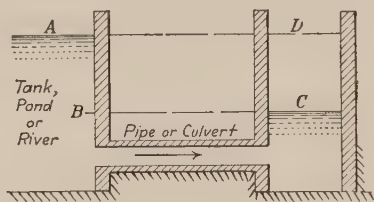


FIG. 173.

into a large pond. The discharge thus occurs under the action of a *variable head.*

The Theory.—In general, *time required* to fill or empty equals volume divided by rate, or $t = \frac{\text{Vol.}}{Q}\cdot$ But since the head and hence the rate of discharge are variable, this must be written in the differential form for a thin horizontal slice of the volume to be filled or emptied.

Let the horizontal cross-section of the prismatic volume be A' and the vertical thickness of the slice (Fig. 172) be the differential dy. Then $d(\text{Vol.}) = A'dy$, and $dt = \frac{(-A'dy)}{Q}$. (The *minus sign* is used because as t increases y decreases.) But

$Q = Av = AC\sqrt{2gy}$, where A is the cross-sectional area of the pipe or orifice, C is the coefficient of discharge, and y is the variable difference of levels. Hence

$$dt = \frac{A'}{AC\sqrt{2g}}(-y^{-\frac{1}{2}}dy), \text{ whence } t = \int_h^o \frac{A'}{AC\sqrt{2g}}(-y^{-\frac{1}{2}}dy).$$

Note the limits. Hence, $t = \dfrac{A'}{AC\sqrt{2g}} 2h^{\frac{1}{2}}$, or $t = \dfrac{2(A'h)}{AC\sqrt{2gh}} =$

$$\frac{2 \times \text{Volume}}{\text{Initial } Q}, \qquad (B\text{-}1)$$

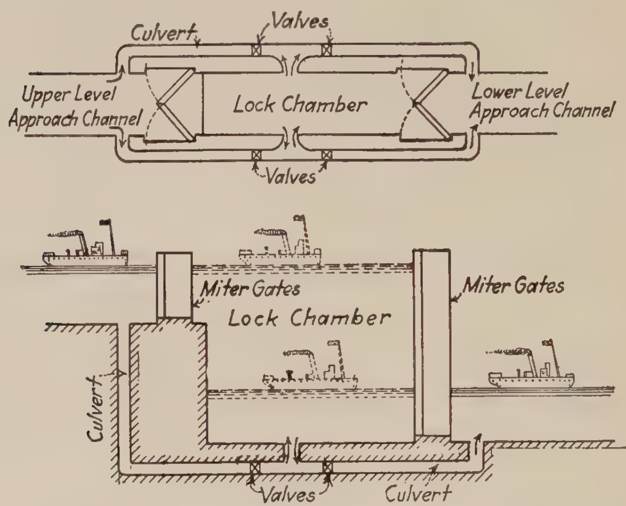


FIG. 174.—Diagrammatic plan and longitudinal vertical section of a canal lock. The arrangement of the culverts is not at all a fixed matter, but the principles involved are the same, no matter what the particular arrangement or special design.

a value easily remembered; but note that this is *only for a prismatic tank or reservoir.*

As for the value of C for a culvert (Fig. 173), note that (from example 3, p. 201), where $h = \left(0.5 + f\frac{L}{d} + \ . \ . \ . \ + 1\right)\frac{v^2}{2g}$,

$$v = \sqrt{\frac{1}{0.5 + f\frac{L}{D} + \ . \ . \ . \ + 1}}\sqrt{2gh},$$ the first radical being the value of C.

Comments.—In the above derivation it is assumed that C does not vary. Actually there is a small variation. Figure 80

shows a slight increase in C as the head decreases, and Fig. 107 shows that the Chézy coefficient and hence the orifice C decrease somewhat as the rate of flow decreases, especially at very low velocities.

However, there is the offsetting practical point that, for all ordinary requirements, the two water surfaces need not come to *exactly* a common level, but need merely approach very close to such a common level. For example, the miter gates of canal locks can be operated (closed or opened as the case may be) when the difference of levels is still as much as 1 or 2 in.; *i.e.*, a little before absolute equality of levels is established. Also, in emptying a tank or reservoir, say for cleaning or repair purposes, it is not a question of draining the tank to the last drop, because preliminary work can start while there is still a depth of a few inches of liquid remaining. Hence the time as calculated by the above formula may be taken as practically exact (subject, of course, to uncertainties in value of C and errors in A, A' and h).

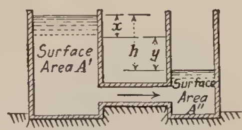


FIG. 175.

It is important to prevent whirling or vortex motion in such a case as emptying a tank through an orifice in the bottom, otherwise the rate of discharge is much diminished. An everyday example is the case of a lavatory wash bowl. If the hand is placed so as to make a vertical radial fin to prevent whirl, the rate of discharge is seen to be increased. Side openings and nonsymmetrical location of orifices are arrangements tending to decrease whirling.

Time Required for Two Communicating Prismatic Tanks to Come to a Common Level (Fig. 175).—As the level in one tank descends, the level in the other tank rises, so that $A'x = A''y$; but $h = x + y$, and $y = \frac{A'}{A''}x$.

Hence,
$$h = x + \frac{A'}{A''}x = x\left(1 + \frac{A'}{A''}\right) = \frac{A' + A''}{A''}x,$$

or
$$x = \left(\frac{A''}{A' + A''}\right)h, \text{ and } dx = \left(\frac{A''}{A' + A''}\right)dh.$$

Now, as before, $dt = \dfrac{d(\text{Vol.})}{Q} = \dfrac{-A'dx}{AC\sqrt{2gh}}.$

The alternative value $-A''dy$ may be written for d (Vol.) in the numerator. Substituting for dx its value as stated above, and integrating with limits as in the previous case,

$$t = \text{either } \frac{A''}{A' + A''}\frac{2A'h}{AC\sqrt{2gh}}, \text{ or } \frac{A'}{A' + A''}\frac{2A''h}{AC\sqrt{2gh}}, \qquad (B\text{-}2)$$

the form of the expression depending on whether the upper tank is regarded as emptying or the lower tank as filling, the time being the same in either case.

Each of these two expressions for the time required is seen to be a constant times (a volume divided by a rate of discharge). If $A' = A''$, *i.e.*, if the tanks are of equal cross-section, $t = \frac{A'h}{AC\sqrt{2gh}} = \frac{\text{Vol. of one tank}}{\text{Initial } Q}$, or twice as fast as if one tank were replaced by an extensive pool. This is what would be expected, since the volume to be emptied or filled is one-half as great, and the range of heads is the same.

Non-prismatic Volumes.—For regular geometric shapes, or any volume where the surface area can be expressed as a function of h, an equation for dt may be written in terms of h and dh. If integration is possible, a formula may be derived.

Thus to empty a *V-trough* through an orifice in the bottom, or through a horizontal pipe leading from the bottom, requires $t = \frac{4}{3}\frac{\text{Vol.}}{\text{Initial } Q}$, the $\frac{4}{3}$ being comparable with the 2 for a prismatic tank. For an inverted cone, or a pyramid-shaped vessel (upside down), the factor is $6/5$. For a sphere it is $8/5$; for paraboloid of revolution, resembling an egg-cup, it is $4/3$, as for the V-trough (although the volume and hence the time itself are different).

Problem for the Student: Check the statement for the time required for a V-trough by working out the theory leading up to the final formula.

The Law of Variation between *time* and *head* (total difference of levels) *for a prismatic tank is* seen to be that *time varies* as *the square root of the head* (see Eqs. (B-1) and (B-2). Hence, the *last* quarter of a canal lock will empty or fill in half of the total time, so that the *first* three-quarters will fill or empty in the other half of the total time. The *last* one-ninth of a lock requires one-third of the time, etc.

Note that, in effect, the times for two tanks of same horizontal section are under comparison, the total head in each case being the range from initial head to zero.

For non-prismatic containers the volume does not vary directly as the head. Thus, for a V-trough the volume varies as the square of the remaining head, and the law is: time varies directly as $h^{3/2}$.

Irregular reservoirs may be divided up, for calculation purposes, into a number of horizontal slices whose areas are known from topographical surveys. The time for each layer may be computed by assuming that the discharge is that due to the average head for the particular layer, and then these fractional or partial times may be summed up.

Reservoirs Emptying over Weirs.—If the reservoir is discharging over a weir, the rate of discharge $Q = CLh^{3/2}$, where C for practical purposes may be considered constant. Applying the general principles as above, the time required to empty a pool from an upper level h_1 ft. above the crest to a lower level h_2 ft. above the crest is found to be $t = 2\frac{A'}{CL}\left[\frac{1}{\sqrt{h_2}} - \frac{1}{\sqrt{h_1}}\right]$, where A' is the area of the pond.

In most cases correction must be made for rate of inflow. In certain cases the area of the pool is obscured by backwater effects.

Problems

1. The terminal lock of the Barge Canal at Troy, N. Y., is 45 ft. wide and 520.5 ft. long. The upper, or dam crest, level is 14.33 ft. above mean sea level, and the lower level will be taken at its low value, 1 ft. above mean sea level. In filling and in emptying the lock chambers water flows through twin culverts, one on each side of the lock. The "basket-handle" culvert cross-sectional area is 57.75 sq. ft. for each culvert. (*a*) The lock operator states that he can empty the full-length lock in just 5½ min. Compute the discharge coefficient of the culverts. (*b*) Given that time varies as $\frac{\text{Volume}}{\sqrt{h}}$, and that volume varies as h (for different parts of the same prism), how long does it take to empty the last one-third of the lock? (*c*) How long for the first 5 ft.?

Ans. (*a*) 0.56.
(*b*) 3 min. 10.5 sec.
(*c*) 69 sec.

2. To empty a wedge-shaped tank through an orifice in the bottom requires 4/3 times the time required under initial rate of discharge. If the

water initially is 6 ft. deep, how long (what fraction of the whole time) will it take to empty the upper 3 ft.? *Ans.* 0.646 t.

3. (a) Compute the proper diameter (in inches) for a short pipe with length L = four times its diameter, screwed into the side of a tank at the bottom with the inner end of the pipe just flush with the inside face of the tank, in order that the 60 ft. of water in the tank may be emptied in 1 hr. Tank is rectangular 4 ft. by 6 ft. (b) What would be the proper diameter of pipe in inches in (a) if the tank contained only 10 ft. of water and the time of emptying was 10 min.?

4. What fractions of the total time of emptying a canal lock will it take to empty (a) the *first* three-fourths of the lock? (b) The *last* one-half? (c) The *last* one-ninth?

5. A tank 36 ft. high is emptied through a hole in the bottom in 8.4 min. How long did it take for the water level to drop the upper 11 ft.? The tank is circular in cross-section and is full at the start.

6. It is desired to fill a canal lock, 150 ft. long, 30 ft. wide, with a lift of 9 ft., in 5 min. through twin circular culverts or pipes. Each culvert can be constructed so that its discharge coefficient will be 0.55. What should be the diameter of each culvert? *Ans.* 3.6 ft.

7. What is the proper diameter in inches for a smooth, well-rounded orifice (circular) in the bottom of a rectangular tank containing water 3 ft. deep and holding 24 cu. ft. so as to empty the tank in 30 sec.? *Ans.* 4.64 in.

8. If it takes exactly 1.50 min. to empty the *first* quarter of a canal lock, how many minutes will it take to empty the *last* half of the lock? If desired, a lock 100 ft. long, 30 ft. wide, and 12 ft. deep may be assumed as a concrete case. *Ans.* 7.95 min.

9. What *fraction of the whole time* of emptying a canal lock will it take to empty the (a) *last* quarter? (b) *First* three-quarters? (c) *First* eight-ninths? (d) In filling the lock, what fraction of the whole time will it take to fill 99 per cent of the lock? *Ans.* (a) $\frac{1}{2}$.

(b) $\frac{1}{2}$.

(c) $\frac{2}{3}$.

(d) $\frac{9}{10}$.

APPENDIX C

BACKWATER

General Notes.—As noted at the end of Chap. IX, the construction in a river of a weir, dam, bridge piers, or any form of obstruction will raise the water level upstream from the obstruction. The water thus held above the original or natural level is known as *backwater,* the extra elevation is called *height of backwater,* and the distance upstream to the place where backwater disappears is called *extent of backwater.*

The design (and hence the cost) of the structure causing the backwater is often dependent on the permissible elevation of the water surface at critical points upstream. The engineer must investigate the probability of flooding valuable property and of causing associated interferences with the "natural" conditions.

Measured Data Assumed Available.—It is assumed that the original surface elevations of the water at various points along the stream are known for the particular *known* rate of discharge for which the backwater due to the dam is to be computed. These data enable the original surface falls (Fig. 176), h_A, h_B, h_C, etc. to be computed. Also it is assumed that proper surveys of the bed of the river and of the adjacent valley have been made so that the depth, width, area, wetted perimeter, and hydraulic radius can be found at the various cross-sections for all stages of the river that will be involved after the backwater is caused. Especially it is assumed that sets of hydraulic elements (v, Q, R, and s) at several stages of river surface have been measured on the stream in question.

The Theory of Backwater (Fig. 176).—A short distance upstream from the dam the height of backwater (above original water surface for the rate of discharge in question) is $h_0 + h_W$. In words, it equals the height of the crest above the original surface plus the weir-head (by proper weir formula) required to

discharge the assumed rate of flow over the dam. (If the crest does not project above the original surface, then h_0 is negative, and the dam will be "submerged.")

The surface fall in reach A (the first reach upstream from the dam), which originally was h_A, is now h_A', and it is this new value that is to be computed. In general, by Bernoulli's theorem with a friction term added,

$$h_A' = h_F' - \left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right), \qquad (C\text{-}1)$$

where h_F' is the new loss of head or new surface fall *due to friction only*. Let s_A' be the *new slope due to friction only* as calculated for the mean hydraulic radius and the mean velocity of reach A, using the proper roughness index n. Then $h_F' = L_A s_A'$.

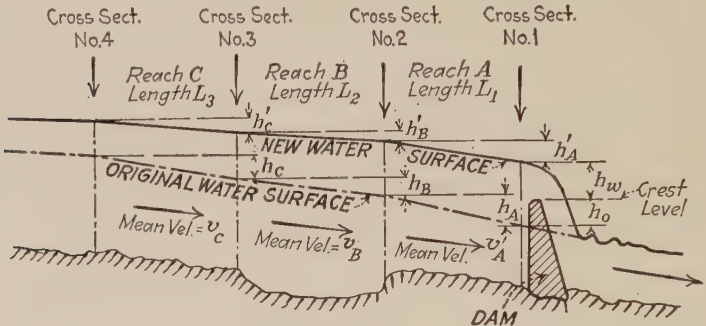


FIG. 176.

Approximately, if the difference of velocity-heads $\left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right)$ be neglected, there results simply

$$h_A' = h_F'. \qquad (C\text{-}2)$$

This approximation makes estimated backwater somewhat too high, but the total error will seldom exceed a few inches. Moreover, it is on the safe side.

To Find Height of Backwater at Any Given Point Upstream from the Dam.—Usually the points would be chosen where gages have been maintained, or where the river slope changes.

Example: Assume that the crest of a dam is 6 ft. above the original water suface with 7400 c.f.s. flowing on an average 6 ft. deep in a river 300 ft. wide with a surface slope = 0.00065. Re-

quired to compute the height of backwater at a point 2000 ft. upstream from the dam, it being previously calculated that a head 3.8 ft. above the crest of the dam is necessary for the given discharge, thus making the total depth of water 15.8 ft. just upstream from the dam, and the height of backwater 9.8 ft.

Solution: Before it is possible to compute the new surface slope, an approximate value of the mean hydraulic radius in the 2000-ft. stretch must be available. But this in itself involves the new slope. A first approximation is made as follows: Using the law of variation derived from the Manning formula, $v = \frac{1.486}{n} R^{2/3} s^{1/2} = KR^{2/3} s^{1/2}$; then, $s = \frac{v^2}{K^2 R^{4/3}}$. But $v = \frac{Q}{A} = \frac{Q(\text{per foot width})}{\text{depth}} =$, approximately, $\frac{Q(\text{per foot})}{R}$. Hence $s = \frac{Q(\text{per foot})^2}{K^2 R^{10/3}}$, *i.e.*, the slope s (for a constant Q) varies inversely as $R^{3.33}$.

The new R_1' (near the dam) $= \frac{Area}{w.p.} = \frac{300 \times 15.8}{300 + (2 \times 15.8)} = \frac{4740}{331.6} = 14.3$ ft. For the first rough approximation assume that $R_1' =$ the mean R_A' for stretch A. The original $R_A = \frac{300 \times 6}{300 + (2 \times 6.0)} = \frac{1800}{312} = 5.77$ ft., and the original velocity $v_A = \frac{Q}{A} = \frac{7400}{1800} = 4.11$ ft. per sec. The new velocity $v_A' = \frac{7400}{4740} = 1.56$ ft. per sec. Then, approximately, $\frac{s_A}{s_A'} = \left(\frac{R_A'}{R_A}\right)^{3.33} = \left(\frac{14.3}{5.77}\right)^{3.33} = 2.47^{3.33} = 20.6$. Hence, roughly, the new slope $= \frac{0.00065}{20.6} = 0.000032$. In 2000 ft. this approximate new slope converges towards the original slope $2000 \times (0.00065 - 0.000032) = 2000 \times 0.00062 = 1.24$ ft., so that the approximate upstream depth $= 14.3 - 1.2 = 13.1$ ft., and the corresponding $R_2' = \frac{300 \times 13.1}{300 + (2 \times 13.1)} = \frac{3930}{326} = 12.04$ ft.; also, $v_2' = \frac{7400}{3930} = 1.88$ ft. per second.

As for the proper value of Kutter's n, the data on the original stream show $v_A = 4.11$ ft. per sec. The diagram (Fig. 129)

shows $n = 0.030$ for this velocity, with $R = 5.77$ ft. and $1000 \times s = 0.65$. Using this value for n, the diagram gives (with the mean $R_A' = 13.2$ ft. and the mean $v_A' = 1.72$ ft. per sec.) the value for the corrected slope in reach A, *viz.*, $s_A' = \frac{0.026}{1000} = 0.000026$. Although this slope differs 19 per cent from the approximated value 0.000032, the effect on the backwater calculation is small. Thus, the corrected convergence of the new and original slopes $= 2000 \times (0.00065 - 0.000026) = 2000 \times 0.000624 = 1.25$ ft.

As for the neglected velocity-heads, the difference $= \frac{1.72^2 - 1.56^2}{64.4} = 0.008$ ft., thus justifying the omission. The total accumulated error for the entire extent of backwater would be $\frac{4.11^2 - 1.56^2}{64.4} = 0.22$ ft.

At the 2000-ft. station the desired backwater height (above original river surface) $= 9.8 - 1.25 = 8.55$ ft. *Ans.*

Calculations for stretch B may now be made in the same manner as just done for stretch A. Thus, stretch after stretch, the new surface profile is delineated.

To Find Distance Upstream from Dam for a Given Height of Backwater.—Proceed upstream from the dam by successive stretches (according to judgment) such that the upstream depth of the stretch (above the idealized average bottom) is some 5 to 20 per cent less than the downstream (known) depth. Then, with known values of average R and v, the surface slope is found by diagram for the stretch in question, and the convergence of this slope and the bottom slope is found by subtraction. Dividing the chosen difference of depths by the rate of convergence, the desired length is obtained.

Comments: Since the new river conditions above the dam involve depths of water greater than any previously existing for the same rate of discharge, it is impossible to know in advance exactly the proper value of n to use. Hence arises the necessity (if a fair approximation is demanded in the calculations) of making a basic series of measurements of elevations, slopes, and discharges covering several widely separated stages of the river.

From such data the rate of change (if any) of n with R (and perhaps with s also) may be found, as well as the law of variation of s with R. Large errors may result if the computations are based entirely on an open-channel formula with assumed roughness index factor.

The question as to the maximum flood to be provided for demands very careful consideration. Recently, many valuable data and graphical analyses have been published on frequency and intensity of floods.[1]

Idealized Solution.—If the section be assumed rectangular and quite wide compared to the depth, so that practically $R = d$, and if the bottom be assumed of uniform slope, then by calculus, applying Eq. (C-1) in differential form to one of the infinite series of short reaches, of length dL, upstream from the dam, the following backwater equation results from the integration:

$$L = \frac{1}{s}\left[(d_1 - d_2) + \left(d_0 - 2\frac{v_0^2}{2g}\right)(\phi_2 - \phi_1)\right], \qquad (C\text{-}3)$$

in which the symbols have meanings as indicated in the small sketch of Fig. 177, and where ϕ is a transcendental function of the depth ratio $\frac{d_0}{d}$, ϕ_1 being the value for the downstream section where the ratio is $\frac{d_0}{d_1}$, and ϕ_2 the value for the upstream section where the ratio is $\frac{d_0}{d_2}$. The numerical values of the "backwater function" ϕ for the range of $\frac{d_0}{d}$ ratios likely to be used in practice are given by the curves plotted in Fig. 177.

The backwater equation may also be written in another form, $\left(\text{noting that } v_0 = C\sqrt{Rs}, \text{ hence } 2\frac{v_0^2}{2g} = \frac{C^2Rs}{g} = \frac{C^2ds}{g}\right)$, thus:

$$L = \frac{d_1 - d_2}{s} + d_0\left(\frac{1}{s} - \frac{C^2}{g}\right)(\phi_2 - \phi_1). \qquad (C\text{-}4)$$

Instead of using the Chézy formula, if the Manning formula is used, *viz.*, $v = \frac{1.486}{n}R^{2/3}s^{1/2}$, or (for a chosen n) $v = KR^{2/3}s^{1/2}$ then instead of C^2 the value $\sqrt[3]{d_0}K^2$ is placed in Eq. (C-4).

[1] "Flood Flows," *Trans. Am. Soc. Civil Eng.*, vol. 77, p. 564; also vols. 84 and 85.

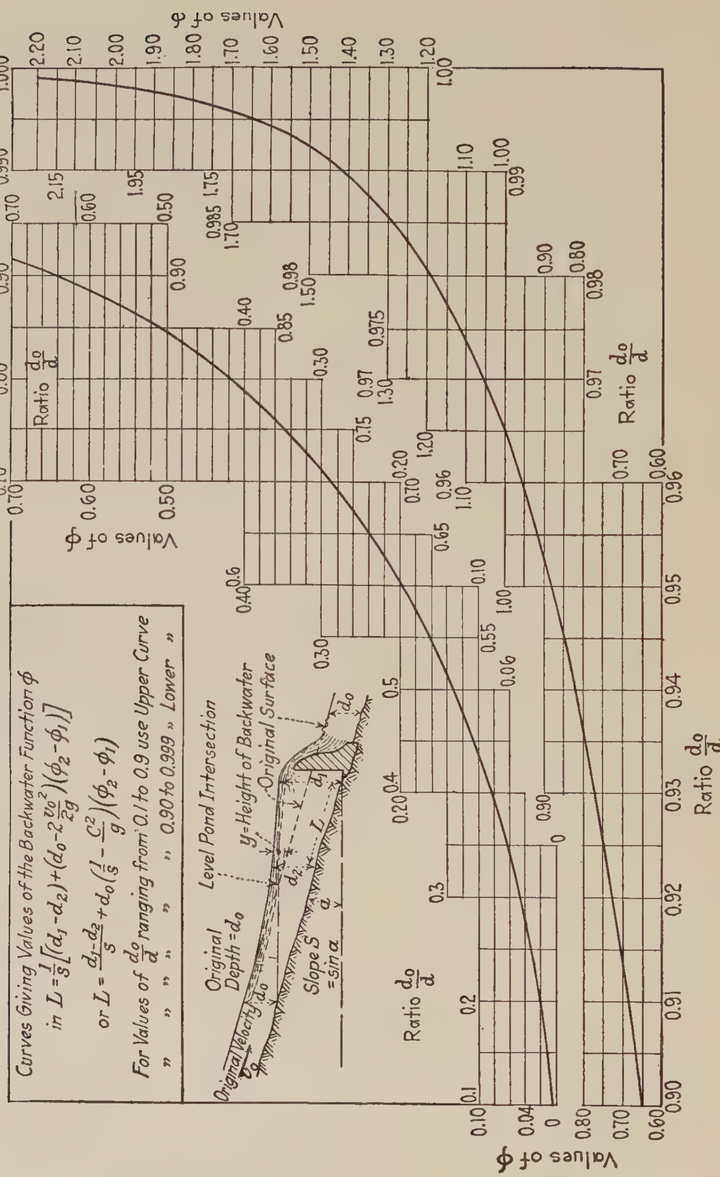

Curves Giving Values of the Backwater Function ϕ
in $L = \frac{1}{s}\left[(d_1 - d_2) + \left(d_o - 2\frac{v_o^2}{2g}\right)(\phi_2 - \phi_1)\right]$
or $L = \frac{d_1 - d_2}{s} + d_o\left(\frac{1}{s} - \frac{C^2}{g}\right)(\phi_2 - \phi_1)$
For Values of $\frac{d_o}{d}$ ranging from 0.1 to 0.9 use Upper Curve
" " " " 0.90 to 0.999 " Lower "
Original Velocity v_o
Original Depth $= d_o$
Level Pond Intersection
y = Height of Backwater
Original Surface
Slope S = sin α
Ratio $\frac{d_o}{d}$
Values of ϕ

FIG. 177.

Example: Assuming the 1.25-ft. decrease in backwater height found in the previous example as the chosen difference of depths $d_1 - d_2$, required to find the corresponding upstream length L.

Solution: The values of the functions ϕ_2 (referring to the ratio $\left.\frac{d_0}{d_2} = \frac{6}{15.8 - 1.25} = \frac{6}{14.55} = 0.412\right)$ and $\phi_1\left(\text{referring to } \frac{d_0}{d_1} = \frac{6}{15.8}\right.$ $= 0.380)$ are found from Fig. 177 to be 0.0874 and 0.0738. Eq. (C-3) gives $L = \frac{1}{0.00065}\left[1.25 + \left(6 - 2\frac{4.11^2}{64.4}\right)(0.0874 - 0.0738)\right]$

$$= 1536[1.25 + (5.46 \times 0.0136)] = 1536 \times 1.324$$
$$= 2035 \text{ ft.} \quad \textit{Ans.}$$

This is a close check on the 2000 ft. of the previous example.

In cases where the original slope of the river changes within the backwater region, the use of the idealized calculus solution is barred out, as also where the width is not fairly uniform.

Backwater Problem: Assuming a rectangular canal with bottom on uniform grade to be deep enough so that a weir 12 ft. high can be and is built in it, such that a head = 5.2 ft. is caused over the weir, how far upstream (L = ? ft.) will it be where the depth is just 1.5 ft. less than at the weir ? Use average of end cross-sections and solve by diagrams. Take $n = 0.017$. Same Q as in problem 29, Chap. XV. *Ans.* 12,600 ft.

APPENDIX D

WATER HAMMER

Water Hammer.—This is the series of shocks, sounding like hammer blows, produced by suddenly checking the flow of water in a pipe. If a valve, turbine gate, or faucet is suddenly closed, the kinetic energy of the arrested column of water is expended, if no relief devices are provided, in compressing the water and in stretching the pipe walls. Starting at the suddenly closed valve, a wave of increased pressure is transmitted back through the pipe with constant velocity and intensity. The shock pressure is not concentrated at the valve, but if a bursting pressure is produced it may show its effects near the valve simply because it acts there first. The velocity of the pressure wave for ordinary cast-iron pipe, 2 to 6 in. in diameter, is about 4200 ft. per sec.; for a 24-in. pipe it is about 3300 ft. per sec.; it depends on the elasticity of the metal and upon the ratio of its thickness to the diameter of the pipe. If the pipe were perfectly rigid the velocity would be that of sound through water, about 4700 ft. per sec.

The increase of pressure is proportional to the destroyed velocity of flow and to the speed of propagation of the pressure wave. This increase is about 60 lb. per sq. in. for each foot per second of extinguished velocity for 2- to 6-in. pipes, and about 45 lb. per sq. in. for each foot per second for 24-in. cast-iron pipe. These increases of pressure will be attained only in case the valve is closed in less time than one round trip of the pressure wave.

When the pressure wave has traveled upstream to the end of the pipe where there is a reservoir or a larger main (the whole pipe then being under increased pressure with checked flow throughout), the elasticity of the compressed water and that of the distended pipe reverse the flow at that end of the pipe, and a wave of normal pressure (that of the reservoir or main) travels downstream, the flow being progressively reversed as the compressed water expands. When this wave of normal pressure reaches

the valve, the kinetic energy of the column of water with reversed flow tends to create a vacuum at the valve. There the reversed flow is checked and the checking proceeds progressively upstream accompanied by a wave of subnormal pressure. When this wave reaches the upstream end (the whole pipe then being under subnormal pressure), the greater normal pressure in the reservoir or large main starts flow *into* the pipe, and a wave of normal pressure and forward flow travels downstream. When this wave reaches the valve there is forward flow throughout the pipe, the conditions being the same as when the valve was suddenly closed, and a wave of increased pressure and of checked flow again starts upstream. A complete cycle of pressure waves and reversals of flow occupies the time required for two round trips. The amplitude of the pressure vibrations becomes less with succeeding cycles because of friction, but the time interval remains constant.

If a high-pressure wave, in its travel through the pipe, enters a branch pipe with a closed, or "dead" end, there will be almost a doubling in the increase of pressure when the wave strikes the closed end. In some pipe systems *dangerous water-hammer pressures* are built up, for if the back wave from a branch pipe with dead end has access to another branch the high pressure may receive further augmentation.

As the intensity of the excess pressure in the "hammer" wave depends on the amount of "extinguished" velocity, the same excess pressure is produced by suddenly reducing the velocity from 7 to 4 ft. per sec. as by entirely stopping a velocity of 3 ft. per sec. If the flow is not checked rapidly, so that the wave from the first movement of the gate has time to travel upstream to the end and back again several times while the checking is in progress, the excess pressure is very much reduced. Hence, the wisdom of using slow-closing valves on long pipe lines.

The excess pressure and the speed of the pressure waves are given by the formulas: $p = v\sqrt{\frac{Ew}{g}}$; $s = \sqrt{\frac{Eg}{w}}$, and also

$$p = v\sqrt{\left(\frac{w}{g}\right)\frac{(EE't)}{(tE' + dE)}};\quad s = \sqrt{\left(\frac{g}{w}\right)\frac{(EE't)}{(tE' + dE)}}$$

(see proof, pp. 356–357).

In these formulas p is the excess pressure intensity and s the speed of transmission of the pressure wave through the water in the pipe. The first two simpler formulas consider the pipe as perfectly inelastic. The last two take into account the elasticity of the metal of the pipe. v is the extinguished velocity in feet per second, w the weight of 1 cu. ft. of water, $g = 32.2$, E the bulk modulus of elasticity of water = about 300,000 lb. per sq. in., E' the linear modulus of the pipe metal = about 30,000,000 lb. per sq. in. for steel, t the thickness of the pipe metal, and d the internal diameter of the pipe. The same system of units should be used throughout. If the foot-pound-second system is used, the above values for E and E' must be multiplied by 144.

Relief Devices.—Adequately proportioned *air chambers* on pumping mains and surge tanks on water power supply pipes serve to *absorb* almost entirely the *shock of water hammer*. Means must be provided for replacing the compressed air in the air

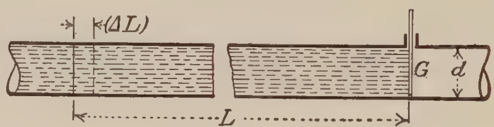


FIG. 178.

chambers, as this is soon absorbed by the water. Relief or safety valves with adjustable springs are not so good for water-hammer shocks, and are more likely to be out of order.

Proof of formulas for intensity of excess pressure due to sudden closure of a valve in a long pipe.[1]

1. Considering the Pipe to be Rigid (see Fig. 178).—The "bulk modulus" of elasticity E of water is the ratio of p to $\frac{\Delta V}{V}$ where ΔV represents the reduction in volume of the total volume V due to any fluid pressure of intensity p.

Hence,
$$E = p\frac{V}{\Delta V} \qquad (D\text{-}1)$$

Let p be the *excess pressure* due to stopping suddenly the flowing water by closing the gate G, after which closure the mass of water in a length L has been brought to rest and compressed

[1] Adapted from "Hydraulic Motors," by Prof. I. P. Church.

into a smaller length $L - \Delta L$ (*i.e., a wave of compression* travels the length L in time T).

In general, *Force* = *Mass* × *Acceleration*. The velocity of the whole mass is reduced from v to o in the time T.

Therefore, $$pA = \frac{LAw}{g} \times \frac{v}{T},$$

or $$p = \frac{Lw}{g}\frac{v}{T}. \qquad (D\text{-}2)$$

From Eq. (D-1),

$$E = p\frac{V}{\Delta V} = p\frac{AL}{A(\Delta L)} = p\frac{L}{(\Delta L)} \text{ or } L = E\frac{(\Delta L)}{p}. \qquad (D\text{-}3)$$

Substituting Eq. (D-3) in (D-2), $p = \dfrac{E(\Delta L)}{p}\dfrac{w}{g}\dfrac{v}{T}$. $\qquad (D\text{-}4)$

But $\dfrac{\Delta L}{T} = v$, because during the time when the wave was traveling up the pipe the water at the upper end was moving down the pipe with a velocity v a distance ΔL.

Hence, from Eq. (D-4),

$$p^2 = \frac{Ew}{g}v^2 \text{ or } p = v\sqrt{\frac{Ew}{g}}. \qquad (D\text{-}5)$$

For p in pounds per square inch and v in feet per second, Eq. (D-5) gives $p = 63v$; *i.e.*, 63 lb. per sq. in. for each foot per second of extinguished velocity.

Combining Eqs. (D-3) and (D-2) $\dfrac{Lw}{g}\dfrac{v}{T} = \dfrac{E(\Delta L)}{L}$.

Multiplying both sides by $\dfrac{1}{T}$ and equating $\dfrac{L}{T} = s$ (= the velocity of the compression wave) and $\dfrac{\Delta L}{T} = v$,

then, $$s^2 = \frac{gE}{w} \text{ or } s = \sqrt{\frac{gE}{w}}. \qquad (D\text{-}6)$$

Example 1: What is the velocity of the compression wave through water, assuming the pipe rigid and $E = 294{,}000$ lb. per sq. in. for near freezing temperature?

$s = 4700$ ft. per sec. *Ans.* (This is the velocity of sound in water.)

Example 2: What is the maximum excess pressure due to an instantaneous closure of a valve on a pipe line, if the water were flowing at the rate of 5 ft. per sec. (pipe rigid)? $p = 316.$ lb. per sq. in. *Ans.*

2. Considering the Elasticity of the Pipe (Assumed Free to Expand) (see Fig. 179).

As in Eq. (D-1), so here, $E = \dfrac{pAL}{A(\Delta L) - 2\pi r(\Delta r)(L - \Delta L)}$. ($D$-7)

Equation (D-2) applies here also, $p = \dfrac{Lw}{g}\dfrac{v}{T}$. ($D$-8)

Since $\dfrac{L}{T} = s$, the velocity of the wave of compression, this reduces to

$$p = \frac{vsw}{g}. \qquad (D\text{-}8a)$$

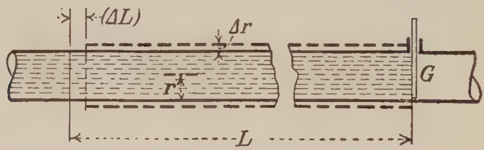


FIG. 179.

If t is the thickness of the pipe wall, the hoop tension

$$p' = \frac{pr}{t}. \qquad (D\text{-}9)$$

If the circumference stretches by length Δl and if E' is Young's modulus for the metal,

$$\frac{E'}{p'} = \frac{2\pi r}{\Delta l}, \text{ or } p' = \frac{E'(\Delta l)}{2\pi r}. \qquad (D\text{-}10)$$

Combining Eqs. (D-10) and (D-9), $E' = \dfrac{pr2\pi r}{(\Delta l)t}$;

by proportion $\dfrac{(\Delta l)}{2\pi r} = \dfrac{\Delta r}{r}$, or $(\Delta l) = 2\pi(\Delta r)$;

hence, $E' = \dfrac{pr^2}{t(\Delta r)}$, or $\Delta r = \dfrac{pr^2}{tE'}$. ($D$-11)

Substituting this in Eq. (D-7) and placing $A = \pi r^2$,

$E = \dfrac{p\pi r^2 L}{\pi r^2(\Delta L) - 2\pi r(L - \Delta L)\dfrac{pr^2}{tE'}}$. Next, canceling and substituting from Eq. (D-8) the value of $L = \dfrac{pgT}{wv}$, and neglecting ΔL when subtracted from L,

$$E = \frac{p^2 g}{\dfrac{(\Delta L)wv}{T} - \dfrac{2rp^2 g}{tE'}}; \text{ but as before } \frac{L}{T} = v.$$

Therefore, $$v = \frac{p^2 g}{vw}\left(\frac{1}{E} + \frac{2r}{tE'}\right). \qquad (D\text{-}12)$$

But $$p = \frac{vsw}{g}, \text{ and } 2r = d.$$

Hence, $$s = \sqrt{\frac{g}{w}\left(\frac{EtE'}{tE' + dE}\right)}, \tag{D-13}$$

and $$p = v\sqrt{\frac{w}{g}\,\frac{EtE'}{tE' + dE}}. \tag{D-14}$$

But $\sqrt{\frac{gE}{w}}$ is the velocity of the compression wave considering the pipe as rigid $= 4700$ ft. per sec. (also equals the velocity of sound wave in water).

Therefore, from (D-13)

$$s \text{ (in ft. per sec.)} = 4700\sqrt{\frac{E'}{E' + 294{,}000\frac{d}{t}}}. \tag{D-15}$$

E' is to be expressed in pounds per square inch, and d and t must both be in the same units. In general, having found the value of s for any particular pipe from Eqs. (D-13) or (D-15), it may be substituted directly into Eq. (D-8a) which is fundamental.

Example 3: What maximum excess pressure would be caused by the sudden closing of a valve on a 24-inch cast-iron pipe line if the water was flowing with a velocity of 5 ft. per sec.? Assume $E' = 14{,}000{,}000$, and that the walls of the pipe are 1 in. thick.

$$s = 4700\sqrt{\frac{14{,}000{,}000}{14{,}000{,}000 + 294{,}000 \times \frac{24}{1}}}$$

$$\therefore s = 4700\sqrt{\frac{14{,}000}{14{,}000 + 7060}}$$
$$= 3830 \text{ ft. per sec.}$$

From Eq. (D-8a), the excess pressure $p = \frac{vsw}{g}$ in pounds per square foot.

Hence, $p = \frac{5 \times 3830 \times 62.4}{32.2 \times 144} = 258$ lb. per sq. in. *Ans.*

It will be noted that the excess pressure due to sudden closure of a valve is independent of the length of the pipe. However, the length of the pipe must be taken into account in determining the time in which the valve must be closed to be equivalent to an instantaneous closure. If the valve is closed before the compression wave travels up the pipe and back again the excess

pressure is the same as for an instantaneous closure; *i.e.*, if the valve is closed in a time equal to or less than $\frac{2L}{s}$.

When the Time of Closure is Greater than $T = \frac{2L}{s}$.

Joukovsky[1] deduced that the maximum excess pressure is reduced in the proportion that the greatest permissible time for maximum excess pressure $\left(\frac{2L}{s}\right)$ is less than the actual time.

Example 4: If the pipe line of Prob. 3 is (*a*) 1000 ft., (*b*) 10,000 ft. long, within what time must the valve be closed to cause the maximum excess water pressure?

Solution: (*a*) T for maximum excess pressure $= \frac{2L}{s} = \frac{2 \times 1000}{3830} = 0.52$ sec. *Ans.*

(*b*) 5.2 sec. *Ans.*

Example 5: If the valve in problem 4*b* were closed in 15 sec., what excess pressure would be produced?

Solution: If valve is closed in 5.2 sec. p (maximum excess) = 258 lb. per sq. in. (see Prob. 3). Therefore,

$$\text{excess pressure} = \frac{5.2}{15} \times 258 = 89 \text{ lb. per sq. in.} \quad \textit{Ans.}$$

[1] *Proc. Am. Water Works Assoc.*, 1904, p. 396.

APPENDIX E

TABLE OF EXPERIMENTS ON PIPE FRICTION

VALUES OF m AND n ($h_F = mv^n$) FROM EXPERIMENTS h_F. HEAD LOST IN FEET PER 1000 FT. OF PIPE. v = VELOCITY IN FEET PER SECOND
(These data are plotted on Fig. 116)

No.	Diameter, inches	Values of m per 1000 ft.	Values of n	Material	Experimenter	Reference	Notes
1	0.370	22.4	1.78	Rubber hose	Saph and Schoder	*Trans. A.S.C.E.*, vol. 51, p. 302, Dec., 1903	
2	0.519	18.2	1.82				
3	0.254	42.0	1.81				
4	1.042	8.20	1.93	Galvanized iron pipe	Saph and Schoder	*Ibid.*, p. 296	
5	0.850	9.05	1.80				
6	0.626	13.20	1.86				
7	0.486	17.70	1.91				Ave. of 40 and 70°F.
8	0.350	35.2	1.96				
9	2.09	2.67	1.75	Seamless drawn brass pipes. Temp. water 55°F.	Saph and Schoder	*Ibid.*, pp. 294–295	B-C section
10	1.498	3.86	1.77				
11	1.237	5.07	1.75				
12	1.054	6.00	1.76				
13	0.832	8.50	1.74				
14	0.630	11.65	1.75				
15	0.498	16.35	1.74				Ave. of A-B and C-D
16	0.376	22.2	1.75				
17	0.322	27.2	1.75				
18	0.282	32.2	1.74				A-B section
19	0.261	36.1	1.74				
20	0.222	44.5	1.73				
21	0.180	58.0	1.75				
22	0.151	67.1	1.77				
23	0.107	101.3	1.78				
24	8.006	0.59	1.91	Cast iron asphalted	Schoder and Gehring	*Eng. Rec.*, vol. 58, p. 241, Aug. 29, 1908	Hub and spigot joints
25	6.075	0.60	1.94	Wrought iron			Screw flange joints
26	5 943	0.80	1.85	Spiral riveted asphalted			Flange joints
27	5.871	0.97	1.92	Spiral riveted galvanized			Flange joints
28	5.066	0.99	1.91	Wrought iron			Flange joints
29	5.013	0.92	1.79	Seamless brass			Flange joints
30	4.078	1.18	1.95	Wrought iron			Screw coupling
31	4.084	1.34	1.85	Spiral riveted asphalted			Flange joints
32	3.120	1.62	1.91	Wrought iron			Screw coupling
33	2.067	3.15	1.98	Wrought iron			Screw coupling
33*a*	2.067	3.72	1.97	Galvanized iron	Schoder	Not published	
34	4.0	1.99	1.72	Wood stave pipe	E. A. Moritz	F. C. Scobey on "Flow of Water through Wood Stave Pipe," *Bull.* 376, U. S. Dept. Agr. Table 3, pp. 38–39.	Only those experiments given a high rating by Scobey used.
35	5.0	1.27	1.90		E. A. Moritz		
36	6.0	0.87	1.81		E. A. Moritz		
37	6.0	0.91	1.88		E. A. Moritz		
38	8.0	0.745	1.85		E. A. Moritz		
39	8.0	0.528	1.72		E. A. Moritz		
40	8.0	0.800	1.77		F. C. Scobey		
41	12.0	0.556	1.74		E. A. Moritz		
42	14.0	0.341	1.73		E. A. Moritz		
43	14.0	0.296	1.87		E. A. Moritz		

VALUES OF m AND n ($h_F = mv^n$) FROM EXPERIMENTS. h_F = HEAD LOST IN FEET PER 1000 FT. OF PIPE. v = VELOCITY IN FEET PER SECOND.—(*Continued*)

No.	Diameter, inches	Values of m per 1000 ft.	Values of n	Material	Experimenter	Reference	Notes
44	18.0	0.215	1.70		E. A Moritz		
45	22.0	0.161	2.18		E. A. Moritz		Only those experiments given a high rating by Scobey used.
46	31.0	0.158	1.70		J. S. Moore	F. C. Scobey on "Flow of Water through Wood Stave Pipe," *Bull.* 376, U. S. Dept. Agr., Table 3, pp. 38–39	
47	31.0	0.230	1.53		J. S. Moore		
48	44.5	0.129	1.70		T. A. Noble		
49	54.19	0.081	1.72	Wood stave pipe.	T. A. Noble		
50	55.75	0.052	1.70		E. A. Moritz		1 year old
51	55.75	0.055	1.75		E. A. Moritz		1½ years old
52	72.5	0.067	1.62		Marx, Wing and Hoskins		1 year old
53	72.5	0.047	1.97				3 years old
54	144.0	0.020	1.89		F. C. Scobey		
55	162	0.010	2.14		F. C. Scobey		
56	12	0.741	1.83		F. C. Scobey #5		
57	16	0.346	1.98		F. C. Scobey #10		
58	19.7	0.372	1.86		F. C. Scobey #16		
59	19.7	0.248	1.98		F. C. Scobey #17		
60	20.0	0.212	1.91		F. C. Scobey #18		
61	20.0	0.219	1.90		Fanning #19	*Bull.* 852, U. S. Dept. Agr., Table 4, p 25	
62	24.0	0.298	2.08	Concrete pipe	F. C. Scobey #21		
63	29.9	0.227	2.05		F. C. Scobey #22		
64	30.0	0.139	1.86		F. C. Scobey #23		
65	36.0	0.050	2.17?		F. C. Scobey #29*b*		
66	42.0	0.071	1.89		F. C. Scobey #30		
67	63.5	0.079	2.40?		F. C. Scobey #35		
68	86.6	0.033	1.91		Budeau #36*a*		
69	174.0	0.0111	2.11		Moore #39*a*		Rondout tunnel
70	174	0.0120	2.03		Moore #40*a*		Wallkill tunnel
71	216	0.0082	1.93		Johnston #41*a*		Ontario tunnel
72	0.502	17.4	1.74		H. Smith, Jr.	Hydraulics, pp. 300–301	
73	0.746	9.8	1.76	Glass	H. Smith, Jr.		
74	0.917	7.4	1.78		H. Smith, Jr.		
75	1.955	3.3	1.81		Darcy	*Ibid.*, p. 226	
76	2.08	2.81	1.75	Smooth brass	G. S. Williams	*Trans. A.S.C.E.*, vol. 47, 1902	
77	1.067	5.98	1.69		Du Buat	H. Smith, Jr., "Hydraulics," p. 217, 219	
78	1.42	4.38	1.71	Tin	Bossut		
79	2.14	2.75	1.71		Bossut		
80	0.242	35.0	1.56		Reynolds	*Trans. Royal Soc.*, 1883	
81	0.242	29.5	1.64		Reynolds		
82	0.498	15.5	1.71		Reynolds		
83	0.55	12.6	1.76	Lead	Darcy	H. Smith, Jr., "Hydraulics," p. 225	
84	1.064	6.8	1.78		Darcy		
85	1.614	4.25	1.78		Darcy		
86	2.50	2.4	1.77		Leslie		
87	0.985	8.0	1.71		Iben	*Ibid.*, p. 229	
88	5.30	0.93	1.53	Lead and earthenware	Couplet	*Ibid.*, p. 214	
89	1.058	0.53	1.84	Lead	Bossut	*Ibid.*, p. 218	

VALUES OF m AND n ($h_F = mv^n$) FROM EXPERIMENTS. h_F = HEAD LOST IN FEET PER 1000 FT. OF PIPE. v = VELOCITY IN FEET PER SECOND.—(*Continued*)

No.	Diameter, inches	Values of m per 1000 ft.	Values of n	Material	Experimenter	Reference	Notes
90	1.055	7.4	1.76	Sheet iron (coated)	Darcy	*Ibid.*, p. 226	
91	3.25	1.54	1.81		Darcy		
92	7.71	0.59	1.78		Darcy		
93	11.22	0.39	1.81		Darcy		
94	0.480	27.9	1.83	Wrought iron (new)	Darcy	*Ibid.*, p. 225	
95	1.046	7.1	1.92		Darcy		
96	1.556	4.2	1.85		Darcy		
97	1.01	6.2	2.05		Ehmann		
98	0.627	12.6	1.74		H. Smith, Jr.	*Ibid.*, pp. 299–300	
99	1.046	6.6	1.76		H. Smith, Jr.		
99a	1.052	6.6	1.76		H. Smith, Jr.		Plotted as one point
99b	1.050	6.85	1.82		H. Smith, Jr.		
100	3.22	1.56	1.98	Cast iron (new)	Darcy	*Ibid.*, pp. 227–228	
101	5.40	0.79	1.97		Darcy		
102	7.40	0.62	1.96		Darcy		
103	19.68	0.22	1.86		Darcy		
104	48	0.059	1.89		Stearns	*Ibid.*, p. 237	
105	12	0.388	1.78		Williams, Hubbell and Fenkell		Ave. of two pipes
106	16	0.212	1.86				
107	16.47	0.270	1.77		Lampe	*Ibid.*, p. 234	
108	30	0.126	1.73		Coffin		
109	4.01	1.59	1.94		Iben	*Ibid.*, p. 229	Ave. of four pipes
110	5.99	1.2	1.93		Iben		
111	12.01	0.44	1.85		Iben		Ave. of four pipes
112	20.00	0.223	1.85		Iben		
113	3.97	1.77	1.73		Ehmann		
114	7.95	0.68	1.89		Ehmann		
115	9.90	0.45	1.79		Ehmann		
116	9.95	0.59	2.01		Ehmann		
117	1.97	4.66	1.87		Ehmann		Ave. of two pipes
118	1.43	4.15	1.84	Cast iron cleaned	Darcy	*Ibid.*, p. 227	
119	3.15	1.87	1.98		Darcy		
120	9.64	0.545	1.98		Darcy		
121	48.0	0.0565	1.93		Fitzgerald		
122	1.40	9.8	1.99	Cast iron tuberculated	Darcy	*Ibid.*, pp. 227–228	
123	3.13	3.45	1.94		Darcy		
124	9.56	0.90	1.99		Darcy		
125	11.69	0.38	2.00		Darcy		
126	48	0.0875	2.03		Fitzgerald		
127	48	0.084	2.00	Cast iron old	Fitzgerald		
128	9.55	0.70	1.93		Ehmann		
129	5.3	1.62	2.25		Meunier		28–30 years
130	7.86	1.00	1.68		Meunier		
131	23.6	0.276	1.78		Meunier		
132	35.4	0.124	1.95		Meunier		1 year
134	1.26	9.05	1.92	Wood bored	H. Smith, Jr.	*Ibid.*, p. 301	
135	14.18	0.295	1.74	Wood stave	Adams	Bull. 376, U. S. Dept. Agr.	
136	3.00	2.46	1.88	Riveted sheet iron	Giltner and Ketchum		

Values of m and n ($h_F = mv^n$) from Experiments. h_F = Head Lost in Feet per 1000 Ft. of Pipe. v = Velocity in Feet per Second.—(*Continued*)

No.	Diameter, inches	Values of m per 1000 ft.	Values of n	Material	Experimenter	Reference	Notes
137	10.92	0.51	1.81	Riveted sheet iron	H. Smith, Jr.	H. Smith, Jr. "Hydraulics," p. 308	
138	12.66	0.365	1.90		H. Smith, Jr.		
139	14.75	0.270	1.94		H. Smith, Jr.		
140	72	0.060	1.92	Riveted steel	Marx, Wing and Hoskins		
141	72	0.076	1.85		Marx, Wing and Hoskins		
142	38	0.099	2.00		Kuichling	"115 experiments on riveted pipes"	
143	47.4	0.090	2.00		Herschel		
144	36	0.176	1.67		Herschel		
145	42	0.116	1.87		Herschel		
146	42	0.110	1.93		Herschel		
147	47.5	0.120	1.82		Herschel		
148	103.0	0.0355	2.09		Herschel		
149	144	0.0162	2.15	Brick	Benzenberg		
150	31.5	0.100	1.88	Cement lined	Bazin	*Trans. A.S.C.E.*, vol. 47, p. 252	Ave. of two pipes
151	2.48	1.94	1.86	Rubber-lined hose	J. R. Freeman	*Trans. A.S.C.E.*, vol. 21, pp. 303–482	Hose D
152	2.67	1.70	1.92	Rubber-lined hose	J. R. Freeman		Hose E
153	2.48	2.08	1.84	Rubber-lined hose	J. R. Freeman		Hose O
154	2.60	4.11	1.88	Unlined linen hose	J. R. Freeman		Hose L
155	2.30	5.40	1.82	Unlined linen hose	J. R. Freeman		Hose N
156	4.08	1.48	2.03	Tile, hard-burned clay	Yarnell and Woodward	Bull. 854, U. S. Dept. Agr., p. 37	
157	5.03	1.05	1.97				
158	6.22	0.819	1.83	Tile, soft-burned clay			
159	8.22	0.617	1.99	Tile, hard-burned clay			
160	10.03	0.384	1.99	Tile, vitrified			
161	11.82	0.318	1.99	Tile, vitrified			
162	3.95	1.865	1.85	Tile, concrete			
163	4.96	1.077	1.92				
164	5.97	0.856	2.01				
165	7.90	0.567	2.04				
166	9.92	0.500	1.96				
167	11.90	0.345	2.50				
168	60	0.085	2.09	Cast iron (8 years old)	G. H. Fenkell	*Trans. A.S.C.E.*, vol. 51, p. 330	8215 ft. long v ranges from 0.18 to 1.1 ft. per sec.
169	0.364	77.0	1.702	Wrought iron	Rowland		
170	0.364	52.0	1.788				
171	0.623	35.6	1.554				
172	0.623	14.1	1.82				
173	1.048	7.4	1.84				
174	1.048	11.1	1.68				
175	1.048	11.8	1.69				
176	0.277	39.5	1.93	No. 1 Black wrought iron, new.	E. W. Schoder	Not published	Velocity ranged from 0.40 to 35 ft. per sec.
177	0.277	85.0	1.97	No. 2 Black wrought iron, new.			
178	0.277	56.0	1.96	No. 3 Black wrought iron, new.			
179	0.277	70.0	1.95	Combined Nos. 1, 2, and 3			

INDEX

U

V

W

Z

Hydraulic Formula.

(1) $p = wh$

(2) $P = wA\,h_{cg}$ ..... $\square$ .. $P = \dfrac{wh^2}{2}$

Chezy's $V = C\sqrt{RS}$

Manning's $V = \dfrac{1.486}{n} R^{2/3} S^{1/2}$

Bazin's $V = \left(\dfrac{158}{1 + \dfrac{m}{\sqrt{R}}}\right)\sqrt{RS}$